You have learned how to perform operations on whole numbers.

In Chapter 1, you will:
- Write algebraic expressions.
- Use the order of operations.
- Solve equations.
- Represent relations and functions.
- Use conditional statements and counterexamples.

**KY Program of Studies**

**HS-AT-S-VEO1** Students will write expressions, equations, inequalities and relations in equivalent forms.

**HS-NPO-S-PNO4** Students will justify the solution steps in simplifying expressions or solving an equation.

**Why?**

**SCUBA DIVING** A scuba diving store rents air tanks and wet suits. An algebraic expression can be written to represent the total cost to rent air tanks and wet suits. This expression can be evaluated to determine the total cost for a group of people to rent the air tanks and wet suits.
Get Ready for Chapter 1

Diagnose Readiness  You have two options for checking Prerequisite Skills.

Text Option  Take the Quick Check below. Refer to the Quick Review for help.

Quick Check

Write each fraction in simplest form. If the fraction is already in simplest form, write simplest form. (Lesson 0-4)

1. \(\frac{24}{36}\)  2. \(\frac{34}{85}\)  3. \(\frac{36}{12}\)
4. \(\frac{27}{45}\)  5. \(\frac{11}{18}\)  6. \(\frac{5}{65}\)
7. \(\frac{19}{1}\)  8. \(\frac{16}{44}\)  9. \(\frac{64}{88}\)

10. ICE CREAM  Fifty-four out of 180 customers said that cookie dough ice cream was their favorite flavor. What fraction of customers was this? (Lesson 0-5)

Quick Review

EXAMPLE 1

Write \(\frac{24}{40}\) in simplest form.

Find the greatest common factor (GCF) of 24 and 40.

factors of 24: 1, 2, 3, 4, 6, 8, 12, 24
factors of 40: 1, 2, 4, 5, 8, 10, 20, 40

The GCF of 24 and 40 is 8.

\[
\frac{24}{8} \div \frac{40}{8} = \frac{3}{5}
\]

Divide the numerator and denominator by their GCF, 8.

EXAMPLE 2

Find the perimeter of the figure.

\[
P = 2\ell + 2w
\]

\[
= 2(12.8) + 2(5.3)
\]

\[= 25.6 + 10.6 \text{ or } 36.2 \quad \text{Simplify.}
\]

The perimeter is 36.2 feet.

EXAMPLE 3

Find each product or quotient. (Lesson 0-5)

14. \(6 \cdot \frac{2}{3}\)  15. \(4.2 \cdot 8.1\)  16. \(\frac{3}{8} \div \frac{1}{4}\)
17. \(5.13 \div 2.7\)  18. \(3\frac{1}{5} \cdot \frac{3}{4}\)  19. \(2.8 \cdot 0.2\)
20. CONSTRUCTION  A board measuring 7.2 feet must be cut into three equal pieces. Find the length of each piece.

Online Option  Take a self-check Chapter Readiness Quiz at glencoe.com.
Get Started on Chapter 1

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 1. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

Expressions, Equations, and Functions

Make this Foldable to help you organize your Chapter 1 notes about expressions, equations, and functions. Begin with five sheets of grid paper.

1 Fold each sheet of grid paper in half along the width. Then cut along the crease.

2 Staple the ten half-sheets together to form a booklet.

3 Cut nine lines from the bottom of the top sheet, eight lines from the second sheet, and so on.

4 Label each of the tabs with a lesson number. The ninth tab is for the properties and the last tab is for the vocabulary.

New Vocabulary

<table>
<thead>
<tr>
<th>English</th>
<th>Español</th>
</tr>
</thead>
<tbody>
<tr>
<td>algebraic expression</td>
<td>expression algebraica</td>
</tr>
<tr>
<td>variables</td>
<td>variable</td>
</tr>
<tr>
<td>term</td>
<td>término</td>
</tr>
<tr>
<td>power</td>
<td>potencia</td>
</tr>
<tr>
<td>coefficient</td>
<td>coeficiente</td>
</tr>
<tr>
<td>equation</td>
<td>ecuación</td>
</tr>
<tr>
<td>solution</td>
<td>solución</td>
</tr>
<tr>
<td>identity</td>
<td>identidad</td>
</tr>
<tr>
<td>relation</td>
<td>relación</td>
</tr>
<tr>
<td>domain</td>
<td>dominio</td>
</tr>
<tr>
<td>range</td>
<td>rango</td>
</tr>
<tr>
<td>independent variable</td>
<td>variable independiente</td>
</tr>
<tr>
<td>dependent variable</td>
<td>variable dependiente</td>
</tr>
<tr>
<td>function</td>
<td>función</td>
</tr>
<tr>
<td>nonlinear function</td>
<td>función no lineal</td>
</tr>
<tr>
<td>deductive reasoning</td>
<td>razonamiento deductivo</td>
</tr>
<tr>
<td>counterexample</td>
<td>contraejemplo</td>
</tr>
</tbody>
</table>

Review Vocabulary

additive inverse • p. P11 • inverso de la adición a number and its opposite
multiplicative inverse • p. P18 • inverso multiplicativo two numbers with a product of 1
perimeter • p. P23 • perímetro the distance around a geometric figure

KY Math Online glencoe.com

- Study the chapter online
- Explore Math in Motion
- Get extra help from your own Personal Tutor
- Use Extra Examples for additional help
- Take a Self-Check Quiz
- Review Vocabulary in fun ways
Variables and Expressions

Why?

Cassie and her friends are at a baseball game. The stadium is running a Dime-A-Dog promotion where hot dogs are $0.10 each. Suppose \( d \) represents the number of hot dogs Cassie and her friends eat. Then \( 0.10d \) represents the cost of the hot dogs they eat.

Write Verbal Expressions  An algebraic expression consists of sums and/or products of numbers and variables. In the algebraic expression \( 0.10d \), the letter \( d \) is called a variable. In algebra, variables are symbols used to represent unspecified numbers or values. Any letter may be used as a variable.

\[
0.10d \\
2x + 4 \\
3 + \frac{z}{6} \\
p \cdot q \\
4cd \div 3mn
\]

A term of an expression may be a number, a variable, or a product or quotient of numbers and variables. For example, \( 0.10d \), \( 2x \) and \( 4 \) are each terms.

In a multiplication expression, the quantities being multiplied are factors, and the result is the product. A raised dot or set of parentheses are often used to indicate a product. Here are several ways to represent the product of \( x \) and \( y \).

\[
xy \\
x \cdot y \\
x(y) \\
(x)y \\
(x)(y)
\]

An expression like \( x^n \) is called a power. The word power can also refer to the exponent. The exponent indicates the number of times the base is used as a factor. In an expression of the form \( x^n \), the base is \( x \). The expression \( x^n \) is read “\( x \) to the \( n \)th power.” When no exponent is shown, it is understood to be 1. For example, \( a = a^1 \).

**EXAMPLE 1** Write Verbal Expressions

Write a verbal expression for each algebraic expression.

a. \( 3x^4 \)  three times \( x \) to the fourth power
b. \( 5z^2 + 16 \)  5 times \( z \) to the second power plus sixteen

1A. \( 16n^2 - 3 \)
1B. \( \frac{1}{2}u + \frac{6b}{7} \)
Write Algebraic Expressions  Another important skill is translating verbal expressions into algebraic expressions.

### Key Concept
#### Translating Verbal to Algebraic Expressions

<table>
<thead>
<tr>
<th>Operation</th>
<th>Verbal Phrases</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition</td>
<td>more than, sum, plus, increased by, added to</td>
</tr>
<tr>
<td>Subtraction</td>
<td>less than, subtracted from, difference, decreased by, minus</td>
</tr>
<tr>
<td>Multiplication</td>
<td>product of, multiplied by, times, of</td>
</tr>
<tr>
<td>Division</td>
<td>quotient of, divided by</td>
</tr>
</tbody>
</table>

### EXAMPLE 2  Write Algebraic Expressions

Write an algebraic expression for each verbal expression.

a. a number \( t \) more than 6
   
   The words *more than* suggest addition. Thus, the algebraic expression is \( 6 + t \) or \( t + 6 \).

b. 10 less than the product of 7 and \( f \)
   
   *Less than* implies subtraction, and *product* suggests multiplication. So the expression is written as \( 7f - 10 \).

c. two thirds of the volume \( v \)
   
   The word *of* with a fraction implies that you should multiply. The expression could be written as \( \frac{2}{3}v \) or \( \frac{2v}{3} \).

### Check Your Progress

2A. the product of \( p \) and 6  2B. one third of the area \( a \)

Variables can represent quantities that are known and quantities that are unknown. They are also used in formulas to write expressions and equations.

### Real-World Career

**Sports Marketing**

Sports marketers promote and manage athletes, teams, facilities and sports-related businesses and organizations. A minimum of a bachelor’s degree in sports management or business administration is preferred.
Lesson 1-1 Variables and Expressions

Check Your Understanding

Example 1
Write a verbal expression for each algebraic expression.

1. \(2m\)
2. \(\frac{2}{3}r^4\)
3. \(a^2 - 18b\)

Example 2
Write an algebraic expression for each verbal expression.

4. the sum of a number and 14
5. 6 less a number \(t\)
6. 7 more than 11 times a number
7. 1 minus the quotient of \(r\) and 7
8. two-fifths of a number \(j\) squared
9. \(n\) cubed increased by 5

Example 3
10. GROCERIES Mr. Bailey purchased some groceries that cost \(d\) dollars. He paid with a $50 bill. Write an expression for the amount of change he will receive.

Example 1
Write a verbal expression for each algebraic expression.

11. \(4q\)
12. \(\frac{1}{8}y\)
13. \(15 + r\)
14. \(w - 24\)
15. \(3x^2\)
16. \(\frac{r^4}{9}\)
17. \(2a + 6\)
18. \(r^4 \cdot t^3\)

Example 2
Write an algebraic expression for each verbal expression.

19. \(x\) more than 7
20. a number less 35
21. 5 times a number
22. one third of a number
23. \(f\) divided by 10
24. the quotient of 45 and \(r\)
25. three times a number plus 16
26. 18 decreased by 3 times \(d\)
27. \(k\) squared minus 11
28. 20 divided by \(t\) to the fifth power

Example 3
29. GEOMETRY The volume of a cylinder is \(\pi\) times the radius \(r\) squared multiplied by the height \(h\) of the cylinder. Write an expression for the volume of a cylinder.

30. EARNINGS Jocelyn makes \(x\) dollars per hour working at the grocery store and \(n\) dollars per hour babysitting. Write an expression that describes her earnings if she babysat for 25 hours and worked at the grocery store for 15 hours.

Write a verbal expression for each algebraic expression.

31. \(25 + 6x^2\)
32. \(6f^2 + 5f\)
33. \(\frac{3a^6}{2}\)

34. HEALTH If the body mass index (BMI) is 25 or higher, then you are at a higher risk for heart disease. The BMI is the product of 703 and the quotient of the weight in pounds and the square of the height in inches.
   a. Write an expression that describes how to calculate the BMI.
   b. Calculate the BMI for a person that is 65 inches tall who weighs 140 pounds.
   c. Calculate the BMI for a person that is 5 feet 8 inches tall who weighs 155 pounds.
DREAMS Refer to the information at the left.

a. Write an expression to describe the number of dreams that feature people you know if you have \( d \) dreams.

b. Use the expression you wrote to predict the number of dreams that include people you know out of 28 dreams.

SPORTS In football, a touchdown is awarded 6 points and the team can then try for a point after a touchdown.

a. Write an expression that describes the number of points scored on touchdowns and points after touchdowns by one team in a game.

b. If a team wins a football game 27-0, write an equation to represent the possible number of touchdowns and points after touchdowns by the winning team.

c. If a team wins a football game 7-21, how many possible number of touchdowns and points after touchdowns were scored during the game by both teams?

MULTIPLE REPRESENTATIONS In this problem, you will explore the multiplication of powers with like bases.

a. TABULAR Copy and complete the table.

<table>
<thead>
<tr>
<th>( 10^2 \times 10^n )</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^2 \times 10^1 )</td>
<td>( 10 \times 10 \times 10 )</td>
</tr>
<tr>
<td>( 10^2 \times 10^2 )</td>
<td>( 10 \times 10 \times 10 \times 10 )</td>
</tr>
<tr>
<td>( 10^2 \times 10^3 )</td>
<td>( 10 \times 10 \times 10 \times 10 \times 10 )</td>
</tr>
<tr>
<td>( 10^2 \times 10^4 )</td>
<td>?</td>
</tr>
</tbody>
</table>

b. ALGEBRAIC Write an expression for the pattern in the table.

c. VERBAL Make a conjecture about the exponent of the product of two powers.

H.O.T. Problems Use Higher-Order Thinking Skills

38. REASONING Explain the differences between an algebraic expression and a verbal expression.

39. OPEN ENDED Define a variable to represent a real-life quantity, such as time in minutes or distance in feet. Then use the variable to write an algebraic expression to represent one of your daily activities. Describe in words what your expression represents, and explain your reasoning.

40. FIND THE ERROR Consuelo and James are writing an algebraic expression for the verbal expression \( \text{three times the sum of } n \text{ squared and } 3 \). Is either of them correct? Explain your reasoning.

Consuelo \[ 3(n^2 + 3) \]

James \[ 3n^3 + 3 \]

41. CHALLENGE For the cube, \( x \) represents a positive whole number. Find the value of \( x \) such that the volume of the cube and 6 times the area of one of its faces have the same value.

42. WRITING IN MATH Describe how to write an algebraic expression from a real-world situation. Include a definition of algebraic expression in your own words.
43. Which expression best represents the volume of the cube?
   A  the product of three and five
   B  three to the fifth power
   C  three squared
   D  three cubed

44. Which expression best represents the perimeter of the rectangle?
   F  \(2ℓw\)
   G  \(ℓ + w\)
   H  \(2ℓ + 2w\)
   J  \(4(ℓ + w)\)

45. **SHORT RESPONSE** The yards of fabric needed to make curtains is 3 times the length of a window in inches, divided by 36. Write an expression that represents the yards of fabric needed in terms of the length of the window \(ℓ\).

46. **GEOMETRY** Find the area of the rectangle.
   A  14 square meters
   B  16 square meters
   C  50 square meters
   D  60 square meters

47. **AMUSEMENT PARKS** A rollercoaster enthusiast club took a poll to see what each member’s favorite ride was. The results are shown in the table. Make a bar graph of the results. (Lesson 0–12)

<table>
<thead>
<tr>
<th>Ride</th>
<th>Big Plunge</th>
<th>Twisting Time</th>
<th>The Shiner</th>
<th>Raging Bull</th>
<th>The Bat</th>
<th>Teaser</th>
<th>The Adventure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Votes</td>
<td>5</td>
<td>22</td>
<td>16</td>
<td>9</td>
<td>25</td>
<td>6</td>
<td>12</td>
</tr>
</tbody>
</table>

Find the mean, median, and mode for each set of data. (Lesson 0–11)

48. \{7, 6, 5, 7, 4, 8, 2, 2, 7, 8\}  
49. \{-1, 0, 5, 2, -2, 0, -1, 2, -1, 0\}  
50. \{17, 24, 16, 3, 12, 11, 24, 15\}

51. **SPORTS** The results for an annual 5K race are shown at the right. Make a box-and-whisker plot for the data. Write a sentence describing what the length of the box-and-whisker plot tells about the times for the race. (Lesson 0–12)

<table>
<thead>
<tr>
<th>Annual 5-K Race Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joe 14:48</td>
</tr>
<tr>
<td>Jessica 19:27</td>
</tr>
<tr>
<td>Lupe 15:06</td>
</tr>
<tr>
<td>Dante 20:39</td>
</tr>
<tr>
<td>Tia 15:54</td>
</tr>
<tr>
<td>Amber 20:49</td>
</tr>
<tr>
<td>Amanda 16:30</td>
</tr>
</tbody>
</table>

Find each product or quotient. (Lesson 0–4)

52. \(\frac{3}{5} \cdot \frac{7}{11}\)  
53. \(\frac{4}{3} \div \frac{7}{6}\)  
54. \(\frac{5}{6} \cdot \frac{8}{3}\)

55. **SPORTS** Lisa has a new rectangular trampoline that has a width of 6 feet and length of 12 feet. What is the area of her trampoline in square feet? (Lesson 0–7)

56. \(\frac{3}{5} + \frac{4}{9}\)  
57. \(5.67 - 4.21\)  
58. \(\frac{5}{6} - \frac{8}{3}\)

59. \(10.34 + 14.27\)  
60. \(\frac{11}{12} + \frac{5}{36}\)  
61. \(37.02 - 15.86\)
Order of Operations

**Why?**

The admission prices for SeaWorld Adventure Park in Orlando, Florida, are shown in the table. If four adults and three children go to the park, the expression below represents the cost of admission for the group.

\[ 4(64.95) + 3(53.95) \]

**Evaluate Numerical Expressions** To find the cost of admission, the expression \( 4(64.95) + 3(53.95) \) must be evaluated. To **evaluate** an expression means to find its value.

**EXAMPLE 1** Evaluate Expressions

Evaluate \( 3^5 \).

\[
3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243
\]

**Check Your Progress**

1A. \( 2^4 \)

1B. \( 4^5 \)

1C. \( 7^3 \)

The numerical expression that represents the cost of admission contains more than one operation. The rule that lets you know which operation to perform first is called the **order of operations**.

**Key Concept**

**Order of Operations**

1. Evaluate expressions inside grouping symbols.
2. Evaluate all powers.
3. Multiply and/or divide from left to right.
4. Add and/or subtract from left to right.

**EXAMPLE 2** Use Order of Operations

Evaluate \( 16 - 8 \div 2^2 + 14 \).

\[
16 - 8 \div 2^2 + 14 = 16 - 8 \div 4 + 14 = 16 - 2 + 14 = 14 + 14 = 28
\]

**Check Your Progress** Evaluate each expression

2A. \( 3 + 42 \cdot 2 - 5 \)

2B. \( 20 - 7 + 8^2 - 7 \cdot 11 \)
When one or more grouping symbols are used, start evaluating within the innermost grouping symbols.

**EXAMPLE 3**

**Expressions with Grouping Symbols**

Evaluate each expression.

a. \(4 \div 2 + 5(10 - 6)\)

\[
4 \div 2 + 5(10 - 6) = 4 \div 2 + 5(4)
\]

= 2 + 20

= 22


b. \(6[32 - (2 + 3)^2]\)

\[
6[32 - (2 + 3)^2] = 6[32 - (5)^2]
\]

= 6[32 - 25]

= 6[7]

= 42


c. \(\frac{2^3 - 5}{15 + 9}\)

\[
\frac{2^3 - 5}{15 + 9} = \frac{8 - 5}{15 + 9}
\]

= \(\frac{3}{15 + 9}\)

= \(\frac{3}{24}\) or \(\frac{1}{8}\)

Evaluate the power in the numerator. Subtract 5 from 8 in the numerator. Add 15 and 9 in denominator, and simplify.

**Check Your Progress**

3A. \(5 \cdot 4(10 - 8) + 20\)  
3B. \(15 - [10 + (3 - 2)^2] + 6\)  
3C. \(\frac{(4 + 5)^2}{3(7 - 4)}\)

**Evalulate Algebraic Expressions** To evaluate an algebraic expression, replace the variables with their values. Then find the value of the numerical expression using the order of operations.

**EXAMPLE 4**

**Evaluate an Algebraic Expression**

Evaluate \(3x^2 + (2y + z^3)\) if \(x = 4, y = 5, z = 3\).

\[
3x^2 + (2y + z^3) = 3(4)^2 + (2 \cdot 5 + 3^3)
\]

Replace \(x\) with 4, \(y\) with 5, and \(z\) with 3.

Evaluate \(3^3\).

Multiply 2 by 10.

Add 10 to 27.

Evaluate \(4^2\).

Multiply 3 by 16.

Add 48 to 37.

= 85

**Check Your Progress**

Evaluate each expression.

4A. \(a^2(3b + 5) \div c\) if \(a = 2, b = 6, c = 4\)  
4B. \(5d + (6f - g)\) if \(d = 4, f = 3, g = 12\)
EXAMPLE 5 Write and Evaluate an Expression

ENVIRONMENTAL STUDIES Science on a Sphere (SOS)® demonstrates the effects of atmospheric storms, climate changes, and ocean temperature on the environment. The volume of a sphere is four thirds of \( \pi \) multiplied by the radius \( r \) to the third power.

a. Write an expression that represents the volume of a sphere.

<table>
<thead>
<tr>
<th>Words</th>
<th>four thirds of ( \pi ) multiplied by radius to the third power</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
<td>Let ( r ) = radius.</td>
</tr>
<tr>
<td>Expression</td>
<td>( \frac{4}{3} \pi r^3 ) or ( \frac{4}{3} \pi r^3 )</td>
</tr>
</tbody>
</table>

b. Find the volume of the 3-foot radius sphere used for SOS.

\[
V = \frac{4}{3} \pi r^3 \\
= \frac{4}{3} \pi (3)^3 \\
= \left( \frac{4}{3} \pi \right)(27) \\
= 36\pi \\
\]

The volume of the sphere is \( 36\pi \) cubic feet.

Check Your Progress

5. FOREST FIRES According to the California Department of Forestry, an average of 539.2 fires each year are started by burning debris, while campfires are responsible for 129.1 each year.

A. Write an algebraic expression that represents the number of fires, on average, in \( d \) years of debris burning and \( c \) years of campfires.

B. How many fires would there be in 5 years?

Check Your Understanding

Evaluate each expression.

1. \( 9^2 \)
2. \( 4^4 \)
3. \( 3^5 \)
4. \( 30 - 14 \div 2 \)
5. \( 5 \cdot 5 - 1 \cdot 3 \)
6. \( (2 + 5)^4 \)
7. \( [8(2) - 4^2] + 7(4) \)
8. \( \frac{11 - 8}{1 + 7 \cdot 2} \)
9. \( \frac{(4 \cdot 3)^2}{9 + 3} \)

Evaluate each expression if \( a = 4 \), \( b = 6 \), and \( c = 8 \).

10. \( 8b - a \)
11. \( 2a + (b^2 \div 3) \)
12. \( \frac{b(9 - c)}{a^2} \)

13. BOOKS Akira bought one new book for $20 and three used books for $4.95 each. Write and evaluate an expression to find how much money the books cost.

14. FOOD Koto purchased food at a fast food restaurant for herself and her friends. She bought 4 cheeseburgers for $2.25 each, 3 French fries for $1.25 each, and 4 drinks for $4.00. Write and evaluate an expression to find how much the food cost.
Evaluate each expression.

15. \(7^2\)  
16. \(14^3\)  
17. \(2^6\)

18. \(35 - 3 \cdot 8\)  
19. \(18 \div 9 + 2 \cdot 6\)  
20. \(10 + 8^3 \div 16\)

21. \(24 \div 6 + 2^3 \cdot 4\)  
22. \((11 \cdot 7) - 9 \cdot 8\)  
23. \(29 - 3(9 - 4)\)

24. \((12 - 6) \cdot 5^2\)  
25. \(3^5 - (1 + 10^2)\)  
26. \(108 \div [3(9 + 3^2)]\)

27. \([(6^3 - 9) \div 23]4\)  
28. \(\frac{8 + 3^3}{12 - 7}\)  
29. \(\frac{(1 + 6)^9}{5^2 - 4}\)

Evaluate each expression if \(g = 2\), \(r = 3\), and \(t = 11\).

30. \(g + 6t\)  
31. \(7 - gr\)  
32. \(r^2 + (g^3 - 8)^5\)

33. \((2t + 3g) \div 4\)  
34. \(t^2 + 8rt + r^2\)  
35. \(3g(g + r)^2 - 1\)

36. **GEOMETRY** Write an algebraic expression to represent the area of the triangle. Then evaluate it to find the area when \(h = 12\) inches.

37. **AMUSEMENT PARKS** In 1997, there were 3344 million amusement parks and arcades. This decreased by 148 million by 2002. Write and evaluate an expression to find the number of amusement parks and arcades in 2002.

38. **SPORTS** Kamilah works at the Duke University Athletic Ticket Office. One week she sold 15 preferred season tickets, 45 blue zone tickets, and 55 general admission tickets. Write and evaluate an expression to find the amount of money Kamilah processed.

Evaluate each expression.

39. \(4^2\)  
40. \(12^3\)  
41. \(3^6\)  
42. \(11^5\)

43. \((3 - 4^2)^2 + 8\)  
44. \(23 - 2(17 + 3^3)\)  
45. \(3[4 - 8 + 4^2(2 + 5)]\)  
46. \(\frac{2 \cdot 8^2 - 2^2 \cdot 8}{2 \cdot 8}\)

47. \(25 + \left[(16 - 3 \cdot 5) + \frac{12 + 3}{5}\right]\)  
48. \(7^3 - \frac{2}{3}(13 \cdot 6 + 9)4\)

Evaluate each expression if \(a = 8\), \(b = 4\), and \(c = 16\).

49. \(a^2bc - b^2\)  
50. \(\frac{c^2}{b^2} + \frac{b^2}{a^2}\)  
51. \(\frac{2b + 3c^2}{4a^2 - 2b}\)

52. \(\frac{3ab + c^2}{a}\)  
53. \(\left(\frac{a}{b}\right)^2 - \frac{c}{a - b}\)  
54. \(\frac{2a - b^2}{ab} + \frac{c - a}{b^2}\)

55. **SALES** At the Flea Market, merchants can rent a small space for $7.00 and a large space for $9.75. One time, 28 small and 12 large spaces were rented. Another time, 30 small and 15 large spaces were rented. Write and evaluate an expression to show the total amount of money collected.
56. **SHOPPING** Evelina is shopping for back to school clothes. She bought 3 skirts, 2 pairs of jeans, and 4 sweaters. Write and evaluate an expression to find out how much money Evelina spent on clothes, without including sales tax.

57. **BUSINESS** A sales representative receives an annual salary \( s \), an average commission each month \( c \), and a bonus \( b \) for each sales goal that she reaches.

   a. Write an algebraic expression to represent her total earnings in one year if she receives four equal bonuses.
   
   b. Suppose her annual salary is $52,000 and her average commission is $1225 per month. If each of the four bonuses equals $1150, how much does she earn in a year?

58. **PYRAMIDS** The glass pyramid at the Louvre in Paris has a base of 35.42 meters and a height of 21.64 meters. The Great Pyramid in Egypt has a base of 230 meters and a height of 146.5 meters. The expression for the volume of a pyramid is \( \frac{1}{3}bh \), where \( b \) is the base and \( h \) is the height.

   a. Draw both pyramids and label the dimensions.
   
   b. Write a verbal expression for the difference in volume of the two pyramids.
   
   c. Write an algebraic expression for the difference in volume of the two pyramids. Find the difference in volume.

---

59. **FIND THE ERROR** Tara and Curtis are simplifying \([4(10) - 3^2] + 6(4)\). Is either of them correct? Explain your reasoning.

   **Tara**
   
   \[
   = [4(10) - 9] + 6(4) \\
   = 4(1) + 6(4) \\
   = 4 + 24 \\
   = 28
   \]

   **Curtis**
   
   \[
   = [4(10) - 9] + 6(4) \\
   = (40 - 9) + 6(4) \\
   = 31 + 24 \\
   = 55
   \]

60. **REASONING** Explain how to evaluate \( a[(b - c) \div d] - f \) if you were given values for \( a, b, c, d, \) and \( f \). How would you evaluate the expression differently if the parentheses and brackets were removed and the expression was \( a \cdot b - c \div d - f \)?

61. **CHALLENGE** Write an expression using the whole numbers 1 to 5 using all five digits and addition, and/or subtraction to create a numeric expression with a value of 3.

62. **OPEN ENDED** Write an expression that uses exponents, at least three different operations and two sets of parentheses. Explain in words the steps you would take to evaluate the expression.

63. **WRITING IN MATH** Choose a geometric formula and explain how the order of operations applies when solving the formula.

64. **WRITING IN MATH** Explain why the expressions \((30 + 17) \times 10\) and \(10 \times 30 + 10 \times 17\) are equivalent.
64. Let \( m \) represent the number of miles. Which algebraic expression represents the number of feet in one mile?
   A \( 5280m \)
   B \( \frac{5280}{m} \)
   C \( m + 5280 \)
   D \( 5280 - m \)

65. SHORT RESPONSE
   Simplify: \( \left[ 10 + 15(2^3) \right] \div \left[ 7(2^2) - 2 \right] \)
   Step 1 \( \left[ 10 + 15(8) \right] \div \left[ 7(4) - 2 \right] \)
   Step 2 \( \left[ 10 + 120 \right] \div \left[ 28 - 2 \right] \)
   Step 3 \( 130 \div 26 \)
   Step 4 \( \frac{130}{26} \)
   Which is the first incorrect step? Explain the error.

66. EXTENDED RESPONSE A local movie theater has advertised that one out of every four customers will receive a free popcorn with the purchase of a movie ticket. So far, 25 of the first 80 customers have won.
   Part A Based on the results so far, what is the experimental probability that a customer will win?
   Part B What is the theoretical probability that a customer will win?
   Part C Explain the difference between theoretical and experimental probabilities.

67. GEOMETRY What is the perimeter of the triangle if \( a = 9 \) and \( b = 10 \)?
   F 164 mm  
   H 28 mm  
   G 118 mm  
   J 4 mm

---

Spiral Review

Write a verbal expression for each algebraic expression. (Lesson 1-1)

68. \( 14 - 9c \)
69. \( k^3 + 13 \)
70. \( \frac{4 - v}{w} \)

71. MONEY Destiny earns $8 per hour babysitting and $15 for each lawn she mows. Write an expression to show the amount of money she earns babysitting \( h \) hours and mowing \( m \) lawns. (Lesson 1-1)

Find the area of each figure. (Lesson 0-7)

72. \( \frac{6}{2} \)
73. \( \frac{9}{4} \)
74. \( \frac{b}{12} \)

75. SCHOOL Aaron correctly answered 27 out of 30 questions on his last biology test. What percent of the questions did he answer correctly? (Lesson 0-5)

Skills Review

Find the value of each expression. (Lessons 0-3 and 0-4)

76. \( 5.65 - 3.08 \)
77. \( 6 \div \frac{4}{5} \)
78. \( 4.85(2.72) \)
79. \( \frac{1}{12} + \frac{3}{3} \)
80. \( \frac{4}{9} \cdot \frac{3}{2} \)
81. \( \frac{7^2}{4} - \frac{7}{10} \)
Then
You used the order of operations to simplify expressions. (Lesson 1-2)

Now
- Recognize the properties of equality and identity.
- Recognize the Commutative and Associative Properties.

Properties of Equality and Identity
The expressions $4k + 8k$ and $12k$ are called equivalent expressions because they represent the same number. The properties below allow you to write an equivalent expression for a given expression.

### Properties of Equality

<table>
<thead>
<tr>
<th>Property</th>
<th>Words</th>
<th>Symbols</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflexive</td>
<td>Any quantity is equal to itself.</td>
<td>$a = a$</td>
<td>$5 = 5$</td>
</tr>
<tr>
<td>Symmetric</td>
<td>If one quantity equals a second quantity, then the second quantity equals the first.</td>
<td>$a = b$</td>
<td>If $8 = 2 + 6$, then $2 + 6 = 8$.</td>
</tr>
<tr>
<td>Transitive</td>
<td>If one quantity equals a second quantity and the second quantity equals a third quantity, then the first quantity equals the third quantity.</td>
<td>$a = b$ and $b = c$, then $a = c$.</td>
<td>If $6 + 9 = 3 + 12$ and $3 + 12 = 15$, then $6 + 9 = 15$.</td>
</tr>
<tr>
<td>Substitution</td>
<td>A quantity may be substituted for its equal in any expression.</td>
<td>If $a = b$, then $a$ may be replaced by $b$ in any expression.</td>
<td>If $n = 11$, then $4n = 4 \cdot 11$</td>
</tr>
</tbody>
</table>

The sum of any number and 0 is equal to the number. Thus, 0 is called the additive identity.

### Additive Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Words</th>
<th>Symbols</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Additive</td>
<td>For any number $a$, the sum of $a$ and 0 is $a$.</td>
<td>$a + 0 = 0 + a = a$</td>
<td>2 + 0 = 2</td>
</tr>
<tr>
<td>Identity</td>
<td>A number and its opposite are additive inverses of each other.</td>
<td>$a + (-a) = 0$</td>
<td>3 + (-3) = 0</td>
</tr>
<tr>
<td>Additive</td>
<td></td>
<td></td>
<td>4 - 4 = 0</td>
</tr>
<tr>
<td>Inverse</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
There are also special properties associated with multiplication. Consider the following equations.

4 \cdot n = 4

The solution of the equation is 1. Since the product of any number and 1 is equal to the number, 1 is called the **multiplicative identity**.

6 \cdot m = 0

The solution of the equation is 0. The product of any number and 0 is equal to 0. This is called the **Multiplicative Property of Zero**.

Two numbers whose product is 1 are called **multiplicative inverses** or **reciprocals**. Zero has no reciprocal because any number times 0 is 0.

### Key Concept

**Multiplication Properties**

<table>
<thead>
<tr>
<th>Property</th>
<th>Words</th>
<th>Symbols</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Multiplicative Identity</strong></td>
<td>For any number ( a ), the product of ( a ) and 1 is ( a ).</td>
<td>( a \cdot 1 = 1 )</td>
<td>( 14 \cdot 1 = 14 )</td>
</tr>
<tr>
<td></td>
<td>( 1 \cdot a = a )</td>
<td></td>
<td>( 1 \cdot 1 = 1 )</td>
</tr>
<tr>
<td><strong>Multiplicative Property of Zero</strong></td>
<td>For any number ( a ), the product of ( a ) and 0 is 0.</td>
<td>( a \cdot 0 = 0 )</td>
<td>( 9 \cdot 0 = 0 )</td>
</tr>
<tr>
<td></td>
<td>( 0 \cdot a = 0 )</td>
<td></td>
<td>( 0 \cdot 9 = 0 )</td>
</tr>
<tr>
<td><strong>Multiplicative Inverse</strong></td>
<td>For every number ( \frac{a}{b} ), where ( a, b \neq 0 ), there is exactly one number ( \frac{b}{a} ) such that the product of ( \frac{a}{b} ) and ( \frac{b}{a} ) is 1.</td>
<td>( \frac{a}{b} \cdot \frac{b}{a} = 1 )</td>
<td>( \frac{4}{5} \cdot \frac{5}{4} = \frac{20}{20} ) or 1</td>
</tr>
</tbody>
</table>

### EXAMPLE 1

**Evaluate Using Properties**

Evaluate \( 7(4 - 3) - 1 + 5 \cdot \frac{1}{5} \). Name the property used in each step.

\[
7(4 - 3) - 1 + 5 \cdot \frac{1}{5} = 7(1) - 1 + 5 \cdot \frac{1}{5}
\]

\[
= 7 - 1 + 5 \cdot \frac{1}{5}
\]

\[
= 7 - 1 + 1
\]

\[
= 6 + 1
\]

\[
= 7
\]

**Substitution: \( 4 - 3 = 1 \)**

**Multiplicative Identity: \( 7 \cdot 1 = 7 \)**

**Multiplicative Inverse: \( 5 \cdot \frac{1}{5} = 1 \)**

**Substitution: \( 7 - 1 = 6 \)**

**Substitution: \( 6 + 1 = 7 \)**

### Check Your Progress

Name the property used in each step.

1A. \( 2 \cdot 3 + (4 \cdot 2 - 8) \)

\[
= 2 \cdot 3 + (8 - 8)
\]

\[
= 2 \cdot 3 + (0)
\]

\[
= 6 + 0
\]

\[
= 6
\]

1B. \( \frac{1}{7} + 6(15 \div 3 - 5) \)

\[
= \frac{1}{7} + 6(5 - 5)
\]

\[
= \frac{1}{7} + 6(0)
\]

\[
= 1 + 0
\]

\[
= 1
\]
Use Commutative and Associate Properties  Nikki walks 2 blocks to her friend Sierra’s house. They walk another 4 blocks to school. At the end of the day, Nikki and Sierra walk back to Sierra’s house, and then Nikki walks home.

\[
\begin{align*}
\text{The distance from Nikki's house to school} & \quad \text{equals} \quad \text{the distance from the school to Nikki's house.} \\
2 + 4 & \quad = \quad 4 + 2
\end{align*}
\]

This is an example of the **Commutative Property** for addition.

**Key Concept**

**Commutative Property**

**Words**  The order in which you add or multiply numbers does not change their sum or product.

**Symbols**  For any numbers \(a\) and \(b\), \(a + b = b + a\) and \(a \cdot b = b \cdot a\).

**Examples**  
- \(4 + 8 = 8 + 4\)
- \(7 \cdot 11 = 11 \cdot 7\)

An easy way to find the sum or product of numbers is to group, or associate, the numbers using the **Associative Property**.

**Key Concept**

**Associative Property**

**Words**  The way you group three or more numbers when adding or multiplying does not change their sum or product.

**Symbols**  For any numbers \(a\), \(b\), and \(c\), 
\[(a + b) + c = a + (b + c)\] and \[(ab)c = a(bc)\].

**Examples**  
- \((3 + 5) + 7 = 3 + (5 + 7)\)
- \((2 \cdot 6) \cdot 9 = 2 \cdot (6 \cdot 9)\)

**Real-World EXAMPLE 2**

**Apply Properties of Numbers**

**PARTY PLANNING**  Eric makes a list of items that he needs to buy for a party and their prices to find his total costs. Find the total cost for these items for Eric’s party.

<table>
<thead>
<tr>
<th>Party Supplies</th>
<th>Item</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balloons</td>
<td>6.75</td>
<td>6.75</td>
</tr>
<tr>
<td>Decorations</td>
<td>14.00</td>
<td>14.00</td>
</tr>
<tr>
<td>Food</td>
<td>23.25</td>
<td>23.25</td>
</tr>
<tr>
<td>Beverages</td>
<td>20.50</td>
<td>20.50</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\text{Balloons} + \text{Decorations} + \text{Food} + \text{Beverages} & = 6.75 + 14.00 + 23.25 + 20.50 \\
& = (6.75 + 23.25) + (14.00 + 20.50) \\
& = 30.00 + 34.50 \\
& = 64.50
\end{align*}
\]

The total cost is $64.50.

**Check Your Progress**

2. **FURNITURE**  Rafael is buying furnishings for his first apartment. He buys a couch for $300, lamps for $30.50, a rug for $25.50, and a table for $50. Find the total cost of these items.
EXAMPLE 3  Use Multiplication Properties

Evaluate 5 · 7 · 4 · 2 using the properties of numbers. Name the property used in each step.

\[
5 \cdot 7 \cdot 4 \cdot 2 = 5 \cdot 2 \cdot 7 \cdot 4 \\
= (5 \cdot 2) \cdot (7 \cdot 4) \\
= 10 \cdot 28 \\
= 280
\]

Commutative (×)
Associative (×)
Multiply the numbers in parentheses.
Multiply.

Check Your Progress

Evaluate each expression using the properties of numbers. Name the property used in each step.

3A. 2.9 · 4 · 10
3B. \( \frac{5}{3} \cdot 25 \cdot 3 \cdot 2 \)

Check Your Understanding

Evaluate each expression. Name the property used in each step.

1. \( (1 \div 5)5 \cdot 14 \)
2. \( 6 + 4(19 - 15) \)
3. \( 5(14 - 5) + 6(3 + 7) \)
4. MONEY Carolyn has 9 quarters, 4 dimes, 7 nickels, and 2 pennies. This amount of money in cents can be represented as \( 9(25) + 4(10) + 7(5) + 2 \). Evaluate the expression to find how much money she has. Name the property used in each step.

Examples 2 and 3  pp. 18–19

Evaluate each expression using the properties of numbers. Name the property used in each step.

5. \( 23 + 42 + 37 \)
6. \( 2.75 + 3.5 + 4.25 + 1.5 \)
7. \( 3 \cdot 7 \cdot 10 \cdot 2 \)
8. \( \frac{1}{4} \cdot 24 \cdot \frac{2}{3} \)

Practice and Problem Solving

Evaluate each expression. Name the property used in each step.

9. \( 3(22 - 3 \cdot 7) \)
10. \( 7 + (9 - 3^2) \)
11. \( \frac{3}{4} \left[ 4 \div (7 - 4) \right] \)
12. \( [3 \div (2 \cdot 1)] \frac{2}{3} \)
13. \( 2(3 \cdot 2 - 5) + 3 \cdot \frac{1}{3} \)
14. \( 6 \cdot \frac{1}{6} + 5(12 \div 4 - 3) \)

Example 2  p. 18

15. GEOMETRY The expression \( 2 \cdot \frac{22}{7} \cdot 14^2 + 2 \cdot \frac{22}{7} \cdot 14 \cdot 7 \) represents the surface area of the cylinder at the right. Evaluate this expression to find the surface area. Name the property used in each step.

16. HOTEL RATES A traveler checks into a hotel on Friday and checks out the following Tuesday morning. Use the table to find the total cost of the room including tax.

<table>
<thead>
<tr>
<th>Hotel Rates Per Day</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Day</strong></td>
</tr>
<tr>
<td>Monday–Friday</td>
</tr>
<tr>
<td>Saturday–Sunday</td>
</tr>
</tbody>
</table>

Lesson 1-3 Properties of Numbers  19
Evaluate each expression using properties of numbers. Name the property used in each step.

17. \(25 + 14 + 15 + 36\)  
18. \(11 + 7 + 5 + 13\)  
19. \(3 \frac{2}{3} + 4 + 5 \frac{1}{3}\)  
20. \(4 \frac{4}{9} + 7 \frac{2}{9}\)  
21. \(4.3 + 2.4 + 3.6 + 9.7\)  
22. \(3.25 + 2.2 + 5.4 + 10.75\)  
23. \(12 \cdot 2 \cdot 6 \cdot 5\)  
24. \(2 \cdot 8 \cdot 10 \cdot 2\)  
25. \(0.2 \cdot 4.6 \cdot 5\)  
26. \(3.5 \cdot 3 \cdot 6\)  
27. \(1 \frac{5}{6} \cdot 24 \cdot 3 \frac{1}{11}\)  
28. \(2 \frac{3}{4} \cdot 1 \frac{1}{8} \cdot 32\)  

29. **SCUBA DIVING** A scuba diving store rents air tanks for $7.50, dive flags for $5.00, and wet suits for $10.95. They also sell disposable underwater cameras for $18.99.
   a. Write two expressions to represent the total sales after renting 2 wet suits, 3 air tanks, 2 dive flags, and selling 5 underwater cameras.
   b. What are the total sales?

30. **COOKIES** Bobby baked 2 dozen chocolate chip cookies, 3 dozen sugar cookies, and a dozen oatmeal raisin cookies. How many total cookies did he bake?

Evaluate each expression if \(a = -1, b = 4,\) and \(c = 6.\)

31. \(4a + 9b - 2c\)  
32. \(-10c + 3a + a\)  
33. \(a - b + 5a - 2b\)  
34. \(8a + 5b - 11a - 7b\)  
35. \(3c^2 + 2c + 2c^2\)  
36. \(3a - 4a^2 + 2a\)

37. **FOOTBALL** A football team is on the 35-yard line. The quarterback is sacked at the line of scrimmage. The team gains 0 yards. Which identity or property does this represent? Explain.

Find the value of \(x\). Then name the property used.

38. \(8 = 8 + x\)  
39. \(3.2 + x = 3.2\)  
40. \(10x = 10\)  
41. \(\frac{1}{2} \cdot x = \frac{1}{2} \cdot 7\)  
42. \(x + 0 = 5\)  
43. \(1 \cdot x = 3\)  
44. \(5 \cdot \frac{1}{5} = x\)  
45. \(2 + 8 = 8 + x\)  
46. \(x + \frac{3}{4} = 3 + \frac{3}{4}\)  
47. \(\frac{1}{3} \cdot x = 1\)

48. **GEOMETRY** Write an expression to represent the perimeter of the triangle. Then find the perimeter of the triangle if \(x = 2\) and \(y = 7.\)

49. **SPORTS** Tickets to a baseball game cost $25 each plus a $4.50 handling charge per ticket. If Sharon has a coupon for $10 off and orders 4 tickets, how much will she be charged?

50. **RETAIL** A store is having a sale on children’s clothing. The table shows the prices.
   a. Write three different expressions that represent 8 pairs of shorts and 8 tops.
   b. Evaluate the three expressions in part a to find the costs of the 16 items. What do you notice about all the total costs?
   c. If you buy 8 shorts and 8 tops, you receive a discount of 15%. Find the greatest and least amount of money you can spend on the 16 items at the sale.
51. GEOMETRY A regular octagon measures \((3x + 5)\) units on each side. What is the perimeter if \(x = 2\) ?

52. MULTIPLE REPRESENTATIONS You can use algebra tiles to model and explore algebraic expressions. The rectangular tile has an area of \(x\), with dimensions 1 by \(x\). The small square tile has an area of 1, with dimensions 1 by 1.

a. CONCRETE Make a rectangle with algebra tiles to model the expression \(4(x + 2)\) as shown above. What are the dimensions of this rectangle? What is the area of this rectangle?

b. ANALYTICAL What is the area of the green region and of the yellow region?

c. VERBAL Complete this statement: \(4(x + 2) = \_\). Write a convincing argument to justify your statement.

53. GEOMETRY Pedro wants to prove \(\triangle ADB \cong \triangle ADC\). To do this, he must show that \(\overline{AD} \cong \overline{AD}\), \(\overline{AB} \cong \overline{DC}\) and \(\overline{BD} \cong \overline{AC}\).

a. Copy the figure and show that \(\overline{AB} \cong \overline{CD}\), \(\overline{AB} \cong \overline{BD}\), and \(\overline{AB} \cong \overline{AC}\).

b. Explain how he can use the Reflexive and Transitive Properties to prove \(\triangle ADB \cong \triangle ADC\).

c. If the length of \(\overline{AC}\) is \(x\) cm, write an equation for the perimeter of the quadrilateral \(ACDB\).

H.O.T. Problems

54. OPEN ENDED Write two equations showing the Transitive Property of Equality. Justify your reasoning.

55. REASONING Explain why 0 has no multiplicative inverse.

56. REASONING The sum of any two whole numbers is always a whole number. So, the set of whole numbers \(\{0, 1, 2, 3, 4, \ldots\}\) is said to be closed under addition. This is an example of the Closure Property. State whether each of the following statements is true or false. If false, justify your reasoning.

a. The set of whole numbers is closed under subtraction.

b. The set of whole numbers is closed under multiplication.

c. The set of whole numbers is closed under division.

57. CHALLENGE Does the Commutative Property sometimes, always or never hold for subtraction? Explain your reasoning.

58. REASONING Explain whether 1 can be an additive identity. Give an example to justify your answer.

59. WHICH ONE DOESN'T BELONG? Identify the sentence that does not belong with the other three. Explain your reasoning.

\[
\begin{align*}
    x + 12 &= 12 + x \\
    7h &= h \cdot 7 \\
    1 + a &= a + 1 \\
    (2)k &= 2(jk)
\end{align*}
\]

60. WRITING IN MATH Determine whether the Commutative Property applies to division. Justify your answer.
61. A deck is shaped like a rectangle with a width of 12 feet and a length of 15 feet. What is the area of the deck?
   A. 3 ft^2  
   B. 27 ft^2  
   C. 108 ft^2  
   D. 180 ft^2

62. GEOMETRY A box in the shape of a rectangular prism has a volume of 56 cubic inches. If the length of each side is multiplied by 2, what will be the approximate volume of the box?
   F. 112 in^3  
   G. 224 in^3  
   H. 336 in^3  
   J. 448 in^3

63. \[27 \div 3 + (12 - 4) = \]
   A. \(-\frac{11}{5}\)  
   B. \(\frac{27}{11}\)  
   C. 17  
   D. 25

64. GRIDDED RESPONSE Ms. Beal had 1 bran muffin, 16 ounces of orange juice, 3 ounces of sunflower seeds, 2 slices of turkey, and half a cup of spinach. Find the total number of grams of protein she consumed.

<table>
<thead>
<tr>
<th>Protein Content</th>
<th>Protein (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>bran muffin (1)</td>
<td>3</td>
</tr>
<tr>
<td>orange juice (8 oz)</td>
<td>2</td>
</tr>
<tr>
<td>sunflower seeds (1 oz)</td>
<td>2</td>
</tr>
<tr>
<td>turkey (1 slice)</td>
<td>12</td>
</tr>
<tr>
<td>spinach (1 c)</td>
<td>5</td>
</tr>
</tbody>
</table>

65. \[3 \cdot 5 + 1 - 2 = \]
66. \[14 \div 2 \cdot 6 - 5^2 = \]
67. \[3 \cdot \frac{9^2 - 3^2 \cdot 9}{3 \cdot 9} = \]

68. GEOMETRY Write an expression for the perimeter of the figure.

69. \(\frac{6}{17}\)  
70. \(\frac{2}{23}\)  
71. \(\frac{3}{5}\)

72. SURVEY Andrew took a survey of his friends to find out their favorite type of music. Of the 34 friends surveyed, 22 said they liked rock music the best. What percent like rock music the best?

73. a rectangle with length 5 feet and width 8 feet
74. a square with length 4.5 inches

75. \[\frac{12}{15} \cdot \frac{3}{14} = \]
76. \[\frac{5}{7} \cdot \left(\frac{-4}{5}\right) = \]
77. \[\frac{10}{11} \cdot \frac{21}{35} = \]
78. \[\frac{63}{65} \cdot \frac{120}{126} = \]
79. \[\frac{-4}{3} \cdot \left(\frac{-9}{2}\right) = \]
80. \[\frac{1}{3} \cdot \frac{2}{5} = \]
The Distributive Property

Why?

John burns approximately 420 Calories per hour by inline skating. The chart below shows the time he spent inline skating in one week.

<table>
<thead>
<tr>
<th>Day</th>
<th>Mon</th>
<th>Tue</th>
<th>Wed</th>
<th>Thu</th>
<th>Fri</th>
<th>Sat</th>
<th>Sun</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (h)</td>
<td>1</td>
<td>1/2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>2 1/2</td>
</tr>
</tbody>
</table>

To determine the total number of Calories that he burned inline skating that week, you can use the Distributive Property.

Evaluate Expressions There are two methods you could use to calculate the number of Calories John burned inline skating. You could find the total time spent inline skating and then multiply by the Calories burned per hour. Or you could find the number of Calories burned each day and then add to find the total.

Method 1 Rate Times Total Time

\[420 \left(1 + \frac{1}{2} + 1 + 2 + \frac{21}{2}\right)\]

\[= 420 \times 7\]

\[= 2940\]

Method 2 Sum of Daily Calories Burned

\[420(1) + 420 \left(\frac{1}{2}\right) + 420(1) + 420(2) + 420 \left(\frac{21}{2}\right)\]

\[= 420 + 210 + 420 + 840 + 1050\]

\[= 2940\]

Either method gives the same total of 2940 Calories burned. This is an example of the Distributive Property.

Key Concept

Distributive Property

Symbol

For any numbers \(a\), \(b\), and \(c\),

\[a(b + c) = ab + ac\] and \((b + c)a = ba + ca\]

\[a(b - c) = ab - ac\] and \((b - c)a = ba - ca\].

Examples

\[3(2 + 5) = 3 \times 2 + 3 \times 5 \quad 4(9 - 7) = 4 \times 9 - 4 \times 7\]

\[3(7) = 6 + 15 \quad 4(2) = 36 - 28\]

\[21 = 21 \quad 8 = 8\]

The Symmetric Property of Equality allows the Distributive Property to be written as follows.

If \(a(b + c) = ab + ac\), then \(ab + ac = a(b + c)\).
**EXAMPLE 1**

**Distribute Over Addition**

**SPORTS** A group of 7 adults and 6 children are going to a University of South Florida Bulls baseball game. Use the Distributive Property to write and evaluate an expression that determines the total cost for tickets for the group.

**Understand** You need to gather data from the table to find out the cost of each ticket and then find the total cost.

**Plan** 7 + 6 or 13 people are going to the game, so the tickets are $2 each.

**Solve** Write an expression that shows the product of the cost of each ticket and the sum of adult tickets and children’s tickets.

\[2(7 + 6) = 2(7) + 2(6)\]  
Distributive Property

\[= 14 + 12\]  
Multiply.

\[= 26\]  
Add.

The total cost is $26.

**Examine** Use estimation to check your answer. The total number of tickets needed is 13 and they cost $2 each. Multiply 13 by 2 to get 26. Therefore, the total cost of tickets is $26.

**Check Your Progress**

1. **SPORTS** A group of 3 adults, an 11-year old, and 2 children who are both under 10 years old are going to the baseball game. Write and evaluate an expression to determine the cost of tickets for the group.

You can use the Distributive Property to make mental math easier.

**EXAMPLE 2**

**Mental Math**

Use the Distributive Property to rewrite 7 \cdot 49. Then evaluate.

\[7 \cdot 49 = 7(50 - 1)\]  
Think: 49 = 50 − 1

\[= 7(50) - 7(1)\]  
Distributive Property

\[= 350 - 7\]  
Multiply.

\[= 343\]  
Subtract.

**Check Your Progress**

Use the Distributive Property to rewrite each expression. Then evaluate.

2A. 304(15)  
2B. 44 \cdot 2\frac{1}{2}

2C. 210(5)  
2D. 52(17)

**Study Tip**

Using mental math in Example 2 can help you find the answers quickly. Compare your mental math answer to the written solution.

**Simplify Expressions** You can use algebra tiles to investigate how the Distributive Property relates to algebraic expressions.
Lesson 1-4  Distributive Property

Consider a rectangle with algebra tiles that is 3 units wide and \( x + 2 \) units long. The rectangle has 3 \( x \)-tiles and 6 1-tiles. The area of the rectangle is \( x + 1 + 1 + x + 1 + 1 + x + 1 + 1 \) or \( 3x + 6 \). Therefore, \( 3(x + 2) = 3x + 6 \).

**EXAMPLE 3  Algebraic Expressions**

Rewrite each expression using the Distributive Property. Then simplify.

**a.** \( 7(3w - 5) \)

\[
7(3w - 5) = 7 \cdot 3w - 7 \cdot 5 \\
= 21w - 35
\]

**b.** \( 4(6v^2 + v - 3) \)

\[
4(6v^2 + v - 3) = 4(6v^2) + 4(v) - 4(3) \\
= 24v^2 + 4v - 12
\]

**Check Your Progress**

3A. \((8 + 4n)2\)

3B. \(-6(r + 3g - t)\)

3C. \((2 - 5q)-3\)

3D. \(-4(-8 - 3m)\)

**Like terms** are terms that contain the same variables, with corresponding variables having the same power.

The Distributive Property and the properties of equality can be used to show that \( 4k + 8k = 12k \). In this expression, \( 4k \) and \( 8k \) are like terms.

\[
4k + 8k = (4 + 8)k \\
= 12k
\]

An expression is in **simplest form** when it contains no like terms or parentheses.

**EXAMPLE 4  Combine Like Terms**

a. Simplify \( 17u + 25u \).

\[
17u + 25u = (17 + 25)u \\
= 42u
\]

b. Simplify \( 6t^2 + 3t - t \).

\[
6t^2 + 3t - t = 6t^2 + (3 - 1)t \\
= 6t^2 + 2t
\]

**Check Your Progress**

Simplify each expression. If not possible, write simplified.

4A. \( 6n - 4n \)

4B. \( b^2 + 13b + 13 \)

4C. \( 4y^3 + 2y - 8y + 5 \)

4D. \( 7a + 4 - 6a^2 - 2a \)

**Problem-Solving Tip**

Make a Model

It can be helpful to visualize a problem using algebra tiles or folded paper.
EXAMPLE 5 Write and Simplify Expressions

Use the expression twice the difference of 3x and y increased by five times the sum of x and 2y.

a. Write an algebraic expression for the verbal expression.

<table>
<thead>
<tr>
<th>Words</th>
<th>Variables</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>twice the difference</td>
<td>Let x and y represent the numbers.</td>
<td></td>
</tr>
<tr>
<td>of 3x and y</td>
<td></td>
<td>2(3x − y)</td>
</tr>
<tr>
<td>increased by</td>
<td></td>
<td>+ 5(x + 2y)</td>
</tr>
</tbody>
</table>

b. Simplify the expression, and indicate the properties used.

\[
2(3x − y) + 5(x + 2y) = 2(3x) − 2(y) + 5(x) + 5(2y) \quad \text{Distributive Property}
\]

\[
= 6x − 2y + 5x + 10y \quad \text{Multiply.}
\]

\[
= 6x + 5x − 2y + 10y \quad \text{Commutative (+)}
\]

\[
= (6 + 5)x + (-2 + 10)y \quad \text{Distributive Property}
\]

\[
= 11x + 8y \quad \text{Substitution}
\]

Check Your Progress

5. Write an algebraic expression 5 times the difference of q squared and r plus 8 times the sum of 3q and 2r.

A. Write an algebraic expression for the verbal expression.

B. Simplify the expression, and indicate the properties used.

The coefficient of a term is the numerical factor. For example, in \(6ab\), the coefficient is 6, and in \(\frac{x^2}{3}\), the coefficient is \(\frac{1}{3}\). In the term \(y\), the coefficient is 1 since \(1 \cdot y = y\) by the Multiplicative Identity Property.

Study Tip

Like Terms

Like terms could be defined as terms with the same variable, but different coefficients.

Concept Summary

Properties of Numbers

The following properties are true for any numbers \(a, b,\) and \(c\).

<table>
<thead>
<tr>
<th>Property</th>
<th>Add ((\text{Addition}))</th>
<th>Mult ((\text{Multiplication}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commutative</td>
<td>(a + b = b + a)</td>
<td>(ab = ba)</td>
</tr>
<tr>
<td>Associative</td>
<td>((a + b) + c = a + (b + c))</td>
<td>((ab)c = a(bc))</td>
</tr>
<tr>
<td>Identity</td>
<td>(0 + a = a)</td>
<td>(1 \cdot a = a)</td>
</tr>
<tr>
<td>Zero</td>
<td></td>
<td>(0 \cdot a = 0)</td>
</tr>
<tr>
<td>Distributive</td>
<td>(a(b + c) = ab + ac)</td>
<td>((b + c)a = ba + ca)</td>
</tr>
<tr>
<td>Substitution</td>
<td>If (a = b), then (a) may be substituted for (b).</td>
<td></td>
</tr>
</tbody>
</table>
**Check Your Understanding**

1. **PILOT** A pilot at an air show charges $25 per passenger to take people for rides over the city. If she took 12 adults and 15 children for rides in one day, write and evaluate an expression to describe the situation.

2. Use the Distributive Property to rewrite each expression. Then evaluate.
   - 2. \(14(51)\)
   - 3. \(6\frac{1}{9}(9)\)

3. Use the Distributive Property to rewrite each expression. Then simplify.
   - 4. \(2(4 + t)\)
   - 5. \((g - 9)5\)

4. Simplify each expression. If not possible, write simplified.
   - 6. \(15m + m\)
   - 7. \(3x^3 + 5y^3 + 14\)
   - 8. \((5m + 2m)10\)

5. Write an algebraic expression for each verbal expression. Then simplify, indicating the properties used.
   - 9. 4 times the sum of 2 times \(x\) and six
   - 10. one half of 4 times \(y\) plus the quantity of \(y\) and \(3\)

**Practice and Problem Solving**

11. **TIME MANAGEMENT** Margo uses color dots to track her activities on a calendar. Red dots represent homework time, yellow dots represent work, and green dots represent track practice. In a typical week, she uses 5 red dots, 3 yellow dots, and 4 green dots. How many activities does Margo do in 4 weeks?

12. **BLOOD SUPPLY** The Red Cross is holding blood drives in two locations. In one day, Center 1 collected 715 pints and Center 2 collected 1035 pints. Write and evaluate an expression to estimate the total number of pints of blood donated over a 3-day period.

13. Use the Distributive Property to rewrite each expression. Then evaluate.
   - 13. \((4 + 5)6\)
   - 14. \(7(13 + 12)\)
   - 15. \(6(6 - 1)\)

16. \((3 + 8)15\)

17. \(14(8 - 5)\)

18. \((9 - 4)19\)

19. \(4(7 - 2)\)

20. \(7(2 + 1)\)

21. \(7 \cdot 497\)

22. \(6(525)\)

23. \(36 \cdot 3\frac{1}{4}\)

24. \(\left(4\frac{3}{7}\right)21\)

25. Use the Distributive Property to rewrite each expression. Then simplify.
   - 25. \(2(x + 4)\)
   - 26. \((5 + n)3\)

27. \((4 - 3m)8\)

28. \(-3(2x - 6)\)

29. Simplify each expression. If not possible, write simplified.
   - 29. \(13r + 5r\)
   - 30. \(3x^3 - 2x^2\)
   - 31. \(7m + 7 - 5m\)

32. \(5z^2 + 3z + 8z^2\)

33. \((2 - 4n)17\)

34. \(11(4d + 6)\)

35. \(7m + 2m + 5p + 4m\)

36. \(3x + 7(3x + 4)\)

37. \(4(fg + 3g) + 5g\)

38. Write an algebraic expression for each verbal expression. Then simplify, indicating the properties used.
   - 38. the product of 5 and \(m\) squared, increased by the sum of the square of \(m\) and 5
   - 39. 7 times the sum of \(a\) squared and \(b\) minus 4 times the sum of \(a\) squared and \(b\)
40. **GEOMETRY** Find the perimeter of an isosceles triangle with side lengths of $5 + x$, $5 + x$, and $xy$. Write in simplest form.

41. **GEOMETRY** A regular hexagon measures $3x + 5$ units on each side. What is the perimeter in simplest form?

**Simplify each expression.**

42. $6x + 4y + 5x$  
43. $3m + 5g + 6g + 11m$  
44. $4a + 5a^2 + 2a^2 + a^2$

45. $5k + 3k^3 + 7k + 9k^3$  
46. $6d + 4(3d + 5)$  
47. $2(6x + 4) + 7x$

48. **FOOD** Kenji is picking up take-out food for his study group.

   a. Write and evaluate an expression to find the total cost of four sandwiches, three soups, three salads, and five drinks.

   b. How much would it cost if Kenji bought four of each item on the menu?

**Use the Distributive Property to rewrite each expression. Then simplify.**

49. $\left(\frac{1}{3} - 2b\right)27$  
50. $4(8p + 4q - 7r)$  
51. $6(2c - cd^2 + d)$

**Simplify each expression. If not possible, write simplified.**

52. $6x^2 + 14x - 9x$  
53. $4y^3 + 3y^3 + y^4$  
54. $a + \frac{a}{5} + \frac{2}{5}a$

55. **MULTIPLE REPRESENTATIONS** The area of the model at the right is $2(x - 4)$ or $2x - 8$. The expression $2(x - 4)$ is in factored form.

   a. **GEOMETRIC** Use algebra tiles to form a rectangle with area $2x + 6$. Use the result to write $2x + 6$ in factored form.

   b. **TABULAR** Use algebra tiles to form rectangles to represent each area in the table. Record the factored form of each expression.

   c. **VERBAL** Explain how you could find the factored form of an expression.

**Area Factored Form**

<table>
<thead>
<tr>
<th>Area</th>
<th>Factored Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2x + 6$</td>
<td></td>
</tr>
<tr>
<td>$3x + 3$</td>
<td></td>
</tr>
<tr>
<td>$3x - 12$</td>
<td></td>
</tr>
<tr>
<td>$5x + 10$</td>
<td></td>
</tr>
</tbody>
</table>

**H.O.T. Problems** **Use Higher-Order Thinking Skills**

56. **CHALLENGE** Use the Distributive Property to simplify $6x^2[(3x - 4) + (4x + 2)]$.

57. **REASONING** Should the Distributive Property be a property of multiplication, addition, or both? Explain your answer.

58. **OPEN ENDED** Write a real-life example in which the Distributive Property would be useful. Write an expression that demonstrates the example.

59. **WRITING IN MATH** Use the data about skating on page 23 to explain how the Distributive Property can be used to calculate quickly. Also, compare the two methods of finding the total Calories burned.
60. Which illustrates the Symmetric Property of Equality?
   A  If $a = b$, then $b = a$.
   B  If $a = b$, and $b = c$, then $a = c$.
   C  If $a = b$, then $b = c$.
   D  If $a = a$, then $a + 0 = a$.

61. Anna is three years younger than her sister Emily. Which expression represents Anna’s age if we express Emily’s age as $y$ years?
   F  $y + 3$
   G  $y - 3$
   H  $3y$
   J  $\frac{3}{y}$

62. Which property is used below? If $4xy^2 = 8y^2$ and $8y^2 = 72$, then $4xy^2 = 72$.
   A  Reflexive Property
   B  Substitution Property
   C  Symmetric Property
   D  Transitive Property

63. SHORT RESPONSE A drawer contains 16 white socks, 12 blue socks, and 8 black socks. One sock is randomly chosen. What is the probability the sock is blue?

Spiral Review

Evaluate each expression. Name the property used in each step. (Lesson 1-3)

64. $14 + 23 + 8 + 15$  
65. $0.24 \cdot 8 \cdot 7.05$  
66. $\frac{1}{4} \cdot 9 \cdot \frac{5}{6}$

67. SPORTS Braden runs 6 times a week for 30 minutes and lifts weights 3 times a week for 20 minutes. Write and evaluate an expression for number of hours Braden works out in 4 weeks. (Lesson 1-2)

SPORTS Refer to the table showing Blanca’s cross-country times for the first 8 meets of the season. Round answers to the nearest second. (Lesson 0-11)

68. Find the mean of the data.
69. Find the median of the data.
70. Find the mode of the data.

71. SURFACE AREA What is the surface area of the cube? (Lesson 0-9)

Skills Review

Evaluate each expression. (Lesson 1-2)

72. $12(7 + 2)$  
73. $11(5) - 8(5)$  
74. $(13 - 9) \cdot 4$

75. $3(6) + 7(6)$  
76. $(1 + 19) \cdot 8$  
77. $16(5 + 7)$
Mid-Chapter Quiz
Lessons 1-1 through 1-4

Write a verbal expression for each algebraic expression.  (Lesson 1-1)
1. 21 – x³
2. 3m⁵ + 9

Write an algebraic expression for each verbal expression.  (Lesson 1-1)
3. five more than s squared
4. four times y to the fourth power

5. CAR RENTAL  The XYZ Car Rental Agency charges a flat rate of $29 per day plus $0.32 per mile driven. Write an algebraic expression for the rental cost of a car for x days that is driven y miles.  (Lesson 1-1)

Evaluate each expression.  (Lesson 1-2)
6. 24 ÷ 3 – 2 • 3
7. 5 + 2²
8. 4(3 + 9)
9. 36 – 2(1 + 3)²
10. \(\frac{40 - 2³}{4 + 3(2²)}\)

11. AMUSEMENT PARK  The cost of an adult ticket and a child’s ticket to a local amusement park is shown below. Write and evaluate an expression to find the total cost of 5 adults and 8 children.

12. MULTIPLE CHOICE  Write an algebraic expression to represent the perimeter of the rectangle shown below. Then evaluate it to find the perimeter when w = 8 cm.  (Lesson 1-2)

13. \((8 - 2³) + 21\)
14. \(3(1 ÷ 3) \cdot 9\)
15. \([5 ÷ (3 \cdot 1)]\frac{3}{5}\)

Evaluate each expression using properties of numbers. Name the property used in each step. (Lesson 1-3)
16. 18 + 35 + 32 + 15
17. 0.25 • 7 • 4

Use the Distributive Property to rewrite each expression. Then evaluate.  (Lesson 1-4)
18. 3(5 + 2)
19. (9 – 6)12
20. 8(7 – 4)

Use the Distributive Property to rewrite each expression. Then simplify.  (Lesson 1-4)
21. 4(x + 3)
22. (6 – 2y)7
23. –5(3m – 2)

24. DVD SALES  A video store chain has three locations. Use the information in the table below to write and evaluate an expression to estimate the total number of DVD’s sold over a 4-day period.  (Lesson 1-4)

<table>
<thead>
<tr>
<th>Location</th>
<th>Daily Sales Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location 1</td>
<td>145</td>
</tr>
<tr>
<td>Location 2</td>
<td>211</td>
</tr>
<tr>
<td>Location 3</td>
<td>184</td>
</tr>
</tbody>
</table>

25. MULTIPLE CHOICE  Rewrite the expression \((8 – 3p)(-2)\) using the Distributive Property. (Lesson 1-4)

A 37 cm  C 74 cm  B 232 cm  D 45 cm  

F 16 – 6p  H –16 + 6p  G –10p  J 10p
**Then**
You simplified expressions.
(Lesson 1-1 through 1-4)

**Now**
- Solve equations with one variable.
- Solve equations with two variables.

**KY Program of Studies**
HS-NPO-S-PNO4 Students will justify the solution steps in simplifying expressions or solving an equation.
HS-AT-S-VEO1 Students will write expressions, equations, inequalities and relations in equivalent forms. Also addresses HS-AT-S-VEO2.

**New Vocabulary**
- open sentence
- equation
- solving
- solution
- replacement set
- set
- element
- solution set
- identity

**KY Math Online**
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- Personal Tutor
- Self-Check Quiz
- Homework Help
- Math in Motion

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**Equations**

**Why?**
Mark’s baseball team scored 3 runs in the first inning. At the top of the third inning, their score was 4. The open sentence below represents the change in their score from the first inning to the top of the third inning.

\[3 + r = 4\]

The solution is 1. The team got 1 run in the second inning.

**Solve Equations** A mathematical statement that contains two algebraic expressions and a symbol to compare them is an **open sentence**. A sentence that contains an equals sign, \(=\), is an **equation**.

Finding a value for a variable that makes a sentence true is called **solving** the open sentence. This replacement value is a **solution**.

A set of numbers from which replacements for a variable may be chosen is called a **replacement set**. A set is a collection of objects or numbers that is often shown using braces. Each object or number in the set is called an **element**, or member. A **solution set** is the set of elements from the replacement set that make an open sentence true.

**EXAMPLE 1** **Use a Replacement Set**
Find the solution set of the equation \(2q + 5 = 13\) if the replacement set is \(\{2, 3, 4, 5, 6\}\).

Use a table to solve. Replace \(q\) in \(2q + 5 = 13\) with each value in the replacement set.

Since the equation is true when \(q = 4\), the solution of \(2q + 5 = 13\) is \(q = 4\).

The solution set is \(\{4\}\).

**Check Your Progress**
Find the solution set for each equation if the replacement set is \(\{0, 1, 2, 3\}\).

**1A.** \(8m - 7 = 17\)

**1B.** \(28 = 4(1 + 3d)\)
You can often solve an equation by applying the order of operations.

**STANDARDIZED TEST EXAMPLE 2**

Solve $6 + (5^2 - 5) ÷ 2 = p$.

A 3  B 6  C 13  D 16

**Read the Test Item**

You need to apply the order of operations to the expression in order to solve for $p$.

**Solve the Test Item**

$6 + (5^2 - 5) ÷ 2 = p$  
$6 + (25 - 5) ÷ 2 = p$  
$6 + 20 ÷ 2 = p$  
$6 + 10 = p$  
$16 = p$  

Add. The correct answer is D.

**Check Your Progress**

2. Solve $t = 9^2 ÷ (5 - 2)$.

F 3  G 6  H 14.2  J 27

Some equations have a unique solution. Other equations do not have a solution.

**EXAMPLE 3**

**Solutions of Equations**

Solve each equation.

a. $7 - (4^2 - 10) + n = 10$

Simplify the equation first and then look for a solution.

$7 - (4^2 - 10) + n = 10$  
$7 - (16 - 10) + n = 10$  
$7 - 6 + n = 10$  
$1 + n = 10$  

The only value for $n$ that makes the equation true is 9. Therefore, this equation has a unique solution of 9.

b. $n(3 + 2) + 6 = 5n + (10 - 3)$

$n(3 + 2) + 6 = 5n + (10 - 3)$  
$n(5) + 6 = 5n + (10 - 3)$  
$n(5) + 6 = 5n + 7$  
$5n + 6 = 5n + 7$  

No matter what real value is substituted for $n$, the left side of the equation will always be one less than the right side of the equation. So, the equation will never be true. Therefore, there is no solution of this equation.

**Check Your Progress**

3A. $(18 + 4) + m = (5 - 3)m$  
3B. $8 \cdot 4 \cdot k + 9 \cdot 5 = (36 - 4)k - (2 \cdot 5)$
An equation that is true for any value of the variable is called an **identity**.

### EXAMPLE 4
**Identities**

Solve \((2 \cdot 5 - 8)(3h + 6) = [(2h + h) + 6]^2\).

\[
(2 \cdot 5 - 8)(3h + 6) = [(2h + h) + 6]^2
\]

- **Original Equation**
- **Multiply** \(2 \cdot 5\).
- **Subtract** 8 from 10.
- **Distributive Property**
- **Add** \(2h + h\).

\[
10 - 8)(3h + 6) = [(2h + h) + 6]^2
\]

\[
2(3h + 6) = [(2h + h) + 6]^2
\]

\[
6h + 12 = [(2h + h) + 6]^2
\]

\[
6h + 12 = [3h + 6]^2
\]

\[
6h + 12 = 6h + 12
\]

No matter what value is substituted for \(h\), the left side of the equation will always be equal to the right side of the equation. So, the equation will always be true. Therefore, the solution of this equation could be any real number.

### Check Your Progress

Solve each equation.

4A. \(12(10 - 7) + 9g = g(2^2 + 5) + 36\)

4B. \(2d + (2^3 - 5) = 10(5 - 2) + d(12 \div 6)\)

4C. \(3(b + 1) - 5 = 3b - 2\)

4D. \(5 - \frac{1}{2}(c - 6) = 4\)

### Solve Equations with Two Variables

Some equations contain two variables. It is often useful to make a table of values in which you can use substitution to find the corresponding values of the second variable.

### EXAMPLE 5
**Equations Involving Two Variables**

**MOVIE RENTALS** Mr. Hernandez pays $10 each month for movie rentals that are delivered in the mail. He can also rent movies in the store for $1.50 per title. Write and solve an equation to find the total amount Mr. Hernandez spends this month if he rents 3 movies from the store.

The cost of the movie plan is a flat rate. The variable is the number of movies he rents from the store. The total cost is the price of the plan plus $1.50 times the number of movies from the store.

\[C = 1.50m + 10\]

To find the total cost for the month, substitute 3 for \(m\) in the equation.

\[
C = 1.50m + 10 \quad \text{Original equation}
\]

\[
= 1.50(3) + 10 \quad \text{Substitute 3 for } m.
\]

\[
= 4.50 + 10 \quad \text{Multiply.}
\]

\[
= 14.50
\]

Mr. Hernandez spends $14.50 on movie rentals in one month.

### Check Your Progress

5. **TRAVEL** Amelia drives an average of 65 miles per hour. Write and solve an equation to find the time it will take her to drive 36 miles.
Check Your Understanding

Example 1  p. 31
Find the solution of each equation if the replacement set is \{11, 12, 13, 14, 15\}.

1. \( n + 10 = 23 \)
2. \( 7 = \frac{c}{2} \)
3. \( 29 = 3x - 7 \)
4. \( (k - 8)12 = 84 \)

Example 2  p. 32
MULTIPLE CHOICE Solve \( \frac{d + 5}{10} = 2 \).

A 10  B 15  C 20  D 25

Examples 3 and 4  pp. 32–33

Solve each equation.

6. \( x = 4(6) + 3 \)  
7. \( 14 - 82 = w \)  
8. \( 5 + 22a = 2 + 10 \div 2 \)  
9. \( (2 \cdot 5) + \frac{c^3}{3} = c^3 \div (15 + 2) + 10 \)

Example 5  p. 33

10. RECYCLING  San Francisco has a recycling facility that accepts unused paint from residents. Volunteers blend and mix the paint into different colors and give it away in 5-gallon buckets. Write and solve an equation to find the number of buckets of paint given away from the 30,000 gallons that are donated to the center.

Practice and Problem Solving

Example 1  p. 31
Find the solution of each equation if the replacement sets are \( y = \{1, 3, 5, 7, 9\} \) and \( z = \{10, 12, 14, 16, 18\} \).

11. \( z + 10 = 22 \)  
12. \( 52 = 4z \)  
13. \( \frac{15}{y} = 3 \)  
14. \( 17 = 24 - y \)  
15. \( 2z - 5 = 27 \)  
16. \( 4(y + 1) = 40 \)  
17. \( 22 = \frac{60}{y} + 2 \)  
18. \( 111 = z^2 + 11 \)

Examples 2–4  pp. 32–33

Solve each equation.

19. \( a = 32 - 9(2) \)  
20. \( w = 56 \div (2^2 + 3) \)  
21. \( \frac{27 + 5}{16} = 8 \)  
22. \( \frac{12 \cdot 5}{15 - 3} = y \)  
23. \( r = \frac{9(6)}{(8 + 1)3} \)  
24. \( a = \frac{4(14 - 1)}{3(6) - 5} + 7 \)  
25. \( (4 - 2^2 + 5)w = 25 \)  
26. \( 7 + x - (3 + 32 \div 8) = 3 \)  
27. \( 3^2 - 2 \cdot 3 + u = (3^3 - 3 \cdot 8)(2) + u \)  
28. \( (3 \cdot 6 \div 2)v + 10 = 3^2v + 9 \)  
29. \( 6k + (3 \cdot 10 - 8) = (2 \cdot 3)k + 22 \)  
30. \( (3 \cdot 5)t + (21 - 12) = 15t + 3^2 \)  
31. \( (2^4 - 3 \cdot 5)q + 13 = (2 \cdot 9 - 4^2)q + \left(\frac{3 \cdot 4}{12} - 1\right) \)  
32. \( \frac{3 \cdot 22}{18 + 4} - \left(\frac{4^2}{9 + 7} - 1\right) = r + \left(\frac{8 \cdot 9}{3} + 3\right) \)

33. SCHOOL  A conference room can seat a maximum of 85 people. The principal and two counselors need to meet with the school’s juniors to discuss college admissions. If each student must bring a parent with them, how many students can attend each meeting?

34. GEOMETRY  The perimeter of a regular octagon is 128 inches. Find the length of each side.
Example 5

**SPORTS** A 200-pound athlete who trains for four hours per day requires 2836 Calories for basic energy requirements. During training, the same athlete requires 3091 Calories for extra energy requirements. Write an equation to find \( C \), the total daily Calorie requirement for this athlete. Then solve the equation.

**36. ENERGY** A small electric generator can power 3550 watts of electricity. Write and solve an equation to find how many 75-watt light bulbs one small generator could power.

Make a table of values for each equation if the replacement set is \{-2, -1, 0, 1, 2\}.

**37.** \( y = 3x - 2 \)

**38.** \( 3.25x + 0.75 = y \)

**39.** \( t - 13 = 7; \{10, 13, 17, 20\} \)

**40.** \( 14(x + 5) = 126; \{3, 4, 5, 6, 7\} \)

**41.** \( 22 = \frac{n}{3}; \{62, 64, 66, 68, 70\} \)

**42.** \( 35 = \frac{g - 8}{2}; \{78, 79, 80, 81\} \)

**43.** \( 3(9) - 2 \div 1 + 4 = d \)

**44.** \( j = 15 \div 3 \cdot 5 - 4^2 \)

**45.** \( c + (3^2 - 3) = 21 \)

**46.** \((3^3 - 3 \cdot 9) + (7 - 2^2)b = 24b \)

**47. HEALTH** Blood flow rate can be expressed as \( F = \frac{p_1 - p_2}{r} \) where \( F \) is the flow rate, \( p_1 \) and \( p_2 \) are the initial pressure and final pressure exerted against the blood vessel’s walls, respectively, and \( r \) is the resistance created by the size of the blood vessel.

a. Write and solve an equation to determine the resistance of the blood vessel for an initial pressure 100 mm Hg, a final pressure of 0 mm Hg, and a flow rate of 5 liters per minute.

b. Use the equation to complete the table below.

<table>
<thead>
<tr>
<th>Initial Pressure ( p_1 ) (mm Hg)</th>
<th>Final Pressure ( p_2 ) (mm Hg)</th>
<th>Resistance ( r ) (mm Hg/L/min)</th>
<th>Blood Flow Rate ( F ) (L/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0</td>
<td>30</td>
<td>10</td>
</tr>
<tr>
<td>100</td>
<td>0</td>
<td>40</td>
<td>4</td>
</tr>
<tr>
<td>90</td>
<td>0</td>
<td>10</td>
<td>12</td>
</tr>
</tbody>
</table>

Determine whether the given number is a solution of the equation.

**48.** \( x + 6 = 15; \ 9 \)

**49.** \( 12 + y = 26; \ 14 \)

**50.** \( 2t - 10 = 4; \ 3 \)

**51.** \( 3r + 7 = -5; \ 2 \)

**52.** \( 6 + 4m = 18; \ 3 \)

**53.** \( -5 + 2p = -11; \ -3 \)

**54.** \( \frac{g}{2} = 20; \ 10 \)

**55.** \( \frac{w - 4}{5} = -3; \ -11 \)

**56.** \( \frac{g}{3} - 4 = 12; \ 48 \)

Make a table of values for each equation if the replacement set is \{-2, -1, 0, 1, 2\}.

**57.** \( y = 3x + 5 \)

**58.** \( -2x - 3 = y \)

**59.** \( y = \frac{1}{2}x + 2 \)

**60.** \( 4.2x - 1.6 = y \)

**61. GEOMETRY** The length of a rectangle is 2 inches greater than the width. The length of the base of an isosceles triangle is 12 inches, and the lengths of the other two sides are 1 inch greater than the width of the rectangle.

a. Draw a picture of each figure and label the dimensions.

b. Write two expressions to find the perimeters of the rectangle and triangle.

c. Find the width of the rectangle if the perimeter of each figure is equal.
62. **CONSTRUCTION** The construction of a building requires 10 tons of steel for each story of the building.
   a. Define a variable and write an equation for the number of tons of steel required if the building has 15 stories.
   b. How many tons of steel are needed?

63. **MULTIPLE REPRESENTATIONS** In this problem, you will further explore writing equations.
   a. **CONCRETE** Use centimeter cubes to build a tower similar to the one, shown at the right.
   b. **TABULAR** Copy and complete the table shown below. Record the number of layers in the tower and the number of cubes used in the table.

<table>
<thead>
<tr>
<th>Layers</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
</table>
   c. **ANALYTICAL** As the number of layers in the tower increases, how does the number of cubes in the tower change?
   d. **ALGEBRAIC** Write a rule that gives the number of cubes in terms of the number of layers in the tower.

---

**Real-World Link**

In 2007, Chicago had three supertall skyscrapers (over 1,000 feet) under construction that used a new method of steel construction. A web of supports stretch from the center to the outside wall supports. This method allows buildings to be built taller and with more features than ever before.

---

**H.O.T. Problems**

Use **Higher-Order Thinking Skills**

64. **REASONING** Compare and contrast an expression and an equation.

65. **OPEN ENDED** Write an equation that when solved results in an identity.

66. **REASONING** Explain why an open sentence always has at least one variable.

67. **FIND THE ERROR** Tom and Li-Cheng are solving the equation $x = 4(3 - 2) + 6 ÷ 8$. Is either of them correct? Explain your reasoning.

**Tom**

\[
\begin{align*}
x &= 4(3 - 2) + 6 ÷ 8 \\
&= 4(1) + 6 ÷ 8 \\
&= 4 + 6 ÷ 8 \\
&= 4 + \frac{6}{8} \\
&= 4 + \frac{3}{4} \\
&= 4 \frac{3}{4}
\end{align*}
\]

**Li-Cheng**

\[
\begin{align*}
x &= 4(3 - 2) + 6 ÷ 8 \\
&= 4(1) + 6 ÷ 8 \\
&= 4 + 6 ÷ 8 \\
&= 4 + \frac{6}{8} \\
&= 4 + \frac{3}{4} \\
&= 4 \frac{3}{4}
\end{align*}
\]

68. **CHALLENGE** Find all of the solutions of $x^2 + 5 = 30$.

69. **OPEN ENDED** Write a two-step equation with a solution of $-7$.

70. **WRITING IN MATH** Explain how you can determine that an equation has no real numbers as a solution. How can you determine that an equation has all real numbers as solutions?
71. **STATISTICS** A researcher wants to find out how often teenagers in her town exercise. Which sample group should she survey to get results that best represent all the teenagers in the town?

A a summer baseball league  
B her nieces and nephews  
C high school students chosen at random  
D the teenagers at the mall one Saturday afternoon

72. **SHORT RESPONSE** The expected attendance for the Drama Club production is 65% of the student body. If the student body consists of 300 students, how many students are expected to attend?

\[ \text{Attendance} = 0.65 \times 300 = 195 \]

73. **GEOMETRY** A speedboat and a sailboat both take off from the same port. The speedboat heads north 6 miles, and then turns west 8 miles. The sailboat heads south 3 miles and east 4 miles. What is the distance between the boats?

F 12 mi  
G 15 mi  
H 18 mi  
J 24 mi

74. Michelle can read 1.5 pages per minute. How many pages can she read in two hours?

\[ \text{Pages} = 1.5 \times 2 \times 60 = 180 \]

75. **ZOO** A zoo has about 500 children and 750 adults visit each day. Write an expression to represent about how many of each type of visitor they will have over a month. (Lesson 1-4)

\[ \text{Children} = 500 \times 30 \]
\[ \text{Adults} = 750 \times 30 \]

Find the value of \( p \) in each equation. Then name the property that is used. (Lesson 1-3)

76. \( 7.3 + p = 7.3 \)  
77. \( 12p = 1 \)  
78. \( 1p = 4 \)

Express each percent as a fraction. (Lesson 0-5)

79. 35%  
80. 15%  
81. 28%

82. **MOVING BOXES** Steve is packing his books to move. The figure shows the dimensions of the boxes he is using. How many cubic inches can each box hold? (Lesson 0-8)

83. **TRAVEL** The distance from Raleigh, North Carolina, to Philadelphia, Pennsylvania, is approximately 428 miles. The average gas mileage of José’s car is 45 miles per gallon. About how many gallons of gas will be needed to make the trip?

\[ \text{Gallons} = \frac{428}{45} \approx 9.51 \]

84. **PART-TIME JOB** An employer pays $8.50 per hour. If 20% of pay is withheld for taxes, what are the take-home earnings from 28 hours of work?

\[ \text{Earnings} = 8.50 \times 28 \times (1 - 0.20) = 153.80 \]

**Skills Review**

Find each sum or difference. (Lesson 0-3)

85. \( 1.14 + 5.6 \)  
86. \( 4.28 - 2.4 \)  
87. \( 8 - 6.35 \)
88. \( \frac{4}{5} + \frac{1}{6} \)  
89. \( \frac{2}{7} + \frac{3}{4} \)  
90. \( \frac{6}{8} - \frac{1}{2} \)
Relations

Why?
The deeper in the ocean you are, the greater pressure is on your body. This is because there is more water over you. The force of gravity pulls the water weight down, creating a greater pressure.

The equation that relates the total pressure of the water to the depth of the water is \( P = rh \), where
- \( P \) = the pressure,
- \( r \) = the density of water,
- \( g \) = the acceleration of gravity, and
- \( h \) = the height of water above you.

Represent a Relation
This relationship between the depth of the water and the pressure exerted can be represented by a line on a coordinate grid.

A coordinate system is formed by the intersection of two number lines, the horizontal axis and the vertical axis.

A point is represented on a graph using ordered pairs.
- An ordered pair is a set of numbers, or coordinates, written in the form \((x, y)\).
- The \( x \) value, called the \( x \)-coordinate, represents the horizontal placement of the point.
- The \( y \) value, or \( y \)-coordinate, represents the vertical placement of the point.

A set of ordered pairs is called a relation. A relation can be depicted in several different ways. An equation can represent a relation as well as graphs, tables, and mappings.

A mapping illustrates how each element of the domain is paired with an element in the range. The set of the first numbers of the ordered pairs is the domain. The set of second numbers of the ordered pairs is the range of the relation. This mapping represents the ordered pairs \((-2, 4), (-1, 4), (0, 6) \)(1, 8), and \((2, 10)\).
Study the different representations of the same relation below.

<table>
<thead>
<tr>
<th>Ordered Pairs</th>
<th>Table</th>
<th>Graph</th>
<th>Mapping</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1, 2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(−2, 4)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0, −3)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The x-values of a relation are members of the domain and the y-values of a relation are members of the range. In the relation above, the domain is {−2, 1, 0} and the range is {−3, 2, 4}.

**EXAMPLE 1** Representations of a Relation

**a.** Express {(2, 5), (−2, 3), (5, −2), (−1, −2)} as a table, a graph, and a mapping.

**Table**
Place the x-coordinates into the first column of the table. Place the corresponding y-coordinates in the second column of the table.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>−2</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>−2</td>
</tr>
<tr>
<td>−1</td>
<td>−2</td>
</tr>
</tbody>
</table>

**Graph**
Graph each ordered pair on a coordinate plane.

**Mapping**
List the x-values in the domain and the y-values in the range. Draw arrows from the x-values in the domain to the corresponding y-values in the range.

**b.** Determine the domain and the range of the relation.
The domain of the relation is {2, −2, 5, −1}. The range of the relation is {5, 3, −2}.

**Check Your Progress**

1A. Express {(4, −3), (3, 2), (−4, 1), (0, −3)} as a table, graph and mapping.

1B. Determine the domain and range.
In a relation, the value of the variable that is subject to choice is called the **independent variable**. The variable with a value that is dependent on the value of the independent variable is called the **dependent variable**. The domain contains values of the independent variable. The range contains the values of the dependent variable.

### Real-World Example 2: Independent and Dependent Variables

Identify the independent and dependent variables for each relation.

**a. DANCE** The dance committee is selling tickets to the Fall Ball. The more tickets that they sell increases the amount of money they can spend for decorations.

The number of tickets sold is the independent variable because it is unaffected by the money spent on decorations. The money spent on decorations is the dependent variable because it depends on the number of tickets sold.

**b. MOVIES** Generally, the average price of going to the movies has steadily increased over time.

Time is the independent variable because it is unaffected by the cost of attending the movies. The price of going to the movies is the dependent variable because it is affected by time.

### Check Your Progress

Identify the independent and dependent variables for each relation.

2A. The air pressure inside a tire increases with the temperature.

2B. As the amount of rain decreases, so does the water level of the river.

### Graphs of a Relation

A relation can be graphed without a scale on either axis. These graphs can be interpreted by analyzing their shape.

### Example 3: Analyze Graphs

The graph represents the distance Francesca has ridden on her bike. Describe what happens in the graph.

As time increases, the distance increases until the graph becomes a horizontal line.

At this section of the graph, time is increasing but the distance remains constant. At this section Francesca stopped biking and rested. Then she continued to ride her bike.

### Check Your Progress

Describe what is happening in each graph.

3A. Driving to School

3B. Change in Income

In 1948, a movie ticket cost $0.36. In 2006, the average ticket price in the United States was $6.50.

Source: National Association of Theatre Owners
Check Your Understanding

Example 1
Express each relation as a table, a graph, and a mapping. Then determine the domain and range.

1. \{ (4, 3), (2, 2), (5, 6) \}
2. \{ (5, 7), (1, 4), (0, 5), (2, 3) \}

Example 2
Identify the independent and dependent variables for each relation.

3. Increasing the temperature of a compound increases the pressure.
4. Mike’s cell phone is part of a family plan. If he uses more minutes than his share, then there are fewer minutes available for the rest of his family.
5. Julian is buying concert tickets for him and his friends. The more concert tickets he buys the greater the cost.
6. A store is having a sale over Labor Day weekend. The more customers that purchase something from the store, the greater the profits will be.

Example 3
Describe what is happening in each graph.

7. The graph represents the distance the track team runs during a practice.
8. The graph represents revenues generated through an online store.

Practice and Problem Solving

Express each relation as a table, a graph, and a mapping. Then determine the domain and range.

9. \{ (0, 0), (3, 2), (6, 4), (1, 1) \}
10. \{ (5, 2), (5, 6), (3, 2), (0, 2) \}
11. \{ (6, 1), (7, 3), (2, 2), (2, 5) \}
12. \{ (6, 7), (3, 2), (8, 8), (6, 2), (2, 6) \}
13. \{ (5, 2), (5, 6), (3, 2), (0, 2) \}
14. \{ (4, 3), (1, 3), (7, 2), (2, 2), (1, 5) \}

Identify the independent and dependent variables for each relation.

15. The Spanish classes are having a fiesta lunch. Each student that attends is to bring a Spanish side dish or dessert. The more students that attend, the more food there will be.
16. The faster you drive your car, the longer it will take to come to a complete stop.

Describe what is happening in each graph.

17. The graph represents the height of a bungee jumper.
18. The graph represents the sales of lawn mowers.
Describe what is happening in each graph.

19. The graph represents the value of a rare baseball card.

20. The graph represents the distance covered on an extended car ride.

For Exercises 21–23, use the graph at the right.

21. Name the ordered pair at point A and explain what it represents.

22. Name the ordered pair at point B and explain what it represents.

23. Identify the independent and dependent variables for the function.

For Exercises 24–26, use the graph at the right.

24. Name the ordered pair at point C and explain what it represents.

25. Name the ordered pair at point D and explain what it represents.

26. Identify the independent and dependent variables.

Express each relation as a set of ordered pairs. Describe the domain and range.

27. Buying Aquarium Fish

<table>
<thead>
<tr>
<th>Number of Fish</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2.50</td>
</tr>
<tr>
<td>2</td>
<td>$5.50</td>
</tr>
<tr>
<td>5</td>
<td>$10.00</td>
</tr>
<tr>
<td>8</td>
<td>$18.75</td>
</tr>
</tbody>
</table>

28. Express the relation in each table, mapping, or graph as a set of ordered pairs.

29. 

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>-1</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>-2</td>
<td>-6</td>
</tr>
<tr>
<td>7</td>
<td>-3</td>
</tr>
</tbody>
</table>

30. Domain | Range

31. 

Source: Tim's Tropics
32. **COMPETITIVE SPORTS** Refer to the information at the left. Which of the following graphs best represents a participant in a triathlon? Explain.

![Graph A](image1.png)  
**Graph A**  
![Graph B](image2.png)  
**Graph B**  
![Graph C](image3.png)  
**Graph C**

Draw a graph to represent each situation.

33. **ANTIQUES** A grandfather clock that is over 100 years old has increased in value rapidly from when it was first purchased.

34. **CAR** A car depreciates in value. The value decreases quickly in the first few years.

35. **REAL ESTATE** A house typically increases in value over time.

36. **EXERCISE** An athlete alternates between running and walking during a workout.

37. **PHYSIOLOGY** A typical adult has about 2 pounds of water for each 3 pounds of body weight. This can be represented by the equation $w = 2\left(\frac{b}{3}\right)$, where $w$ is the weight of water in pounds and $b$ is the body weight in pounds.

   a. Make a table to show the relation between body and water weight for people weighing 100, 105, 110, 115, 120, 125, and 130 pounds. Round to the nearest tenth if necessary.
   
   b. What are the independent and dependent variables?
   
   c. State the domain and range, and then graph the relation.
   
   d. Reverse the independent and dependent variables. Graph this relation. Explain what the graph indicates in this circumstance.

### H.O.T. Problems

38. **OPEN ENDED** Describe a real-life situation that can be represented using a relation and discuss how one of the quantities in the relation depends on the other. Then give an example of such a relation in three different ways.

39. **CHALLENGE** Describe a real-world situation where it is reasonable to have a negative number included in the domain or range.

40. **REASONING** Compare and contrast dependent and independent variables.

41. **CHALLENGE** The table presents a relation. Graph the ordered pairs. Then reverse the $y$-coordinate and the $x$-coordinate in each ordered pair. Graph these ordered pairs on the same coordinate plane. Graph the line $y = x$. Describe the relationship between the two sets of ordered pairs.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

42. **WRITING MATH** Use the data about the pressure of water at the beginning of the lesson to explain the difference between a dependent and an independent variable.
43. A school’s cafeteria employees surveyed 250 students asking what beverage they drank with lunch. They used the data to create the table below.

<table>
<thead>
<tr>
<th>Beverage</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>milk</td>
<td>38</td>
</tr>
<tr>
<td>chocolate milk</td>
<td>112</td>
</tr>
<tr>
<td>juice</td>
<td>75</td>
</tr>
<tr>
<td>water</td>
<td>25</td>
</tr>
</tbody>
</table>

What percent of the students surveyed preferred drinking juice with lunch?

A 25%  
B 35%  
C 30%  
D 40%

44. Which of the following is equivalent to $6(3-g) + 2(11-g)$?

F $2(20-g)$  
G $8(14-g)$  
H $8(5-g)$  
J $40-g$

45. SHORT RESPONSE  Grant and Hector want to build a clubhouse at the midpoint between their houses. If Grant’s house is at point $G$ and Hector’s house is at point $H$, what will be the coordinates of the clubhouse?

46. If $3b = 2b$, which of the following is true?

A $b = \frac{3}{2}$  
B $b = \frac{2}{3}$  
C $b = 1$  
D $b = 0$

Spiral Review

Solve each equation. (Lesson 1-5)

47. $6(a + 5) = 42$  
48. $92 = k + 11$  
49. $17 = \frac{45}{w} + 2$

50. HOT-AIR BALLOON A hot-air balloon owner charges $150 for a one-hour ride in his hot-air balloon. If he gave 6 rides on Saturday and 5 rides on Sunday, write and evaluate an expression to describe his total income for the weekend. (Lesson 1-4)

Find the perimeter of each figure. (Lesson 0-6)

51.  
52.  
53.  

54. LOLLIPOPS A bag of lollipops contains 19 cherry, 13 grape, 8 sour apple, 15 strawberry, and 9 orange flavored lollipops. What is the probability of drawing a sour apple flavored lollipop? (Lesson 0-10)

Skills Review

Evaluate each expression. (Lesson 1-2)

55. $8^2$  
56. $(-6)^2$  
57. $(2.5)^2$

58. $(-1.8)^2$  
59. $(3 + 4)^2$  
60. $(1 - 4)^2$
Functions

Why?

The stopping distance of a car is the distance covered between the time a driver first applies the brakes and the time the car comes to a complete stop. This includes time for the driver to react. The faster a car is traveling, the longer the stopping distance. The stopping distance is a function of the speed of the car.

Identify Functions

A function is a relationship between input and output. In a function, there is exactly one output for each input.

Key Concept

For Your Words

A function is a relation in which each element of the domain is paired with exactly one element of the range.

Examples

<table>
<thead>
<tr>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>-1</td>
</tr>
</tbody>
</table>

EXAMPLE 1

Identify Functions

Determine whether each relation is a function. Explain.

a. Domain | Range
-2         | -3
0           | 6
3           | 9
4

For each member of the domain, there is only one member of the range. So this mapping represents a function. It does not matter if more than one element of the domain is paired with one element of the range.

b. Domain | Range
1            | 4
3            | 2
5            | 4
1

The element 1 in the domain is paired with both 4 and -4 in the range. So, when x equals 1 there is more than one possible value for y. The relation in this table is not a function.

Check Your Progress

1. {(2, 1), (3, -2), (3, 1), (2, -2)}

For More Practice

- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

KY Math Online

glencoe.com
A graph that consists of points that are not connected is a **discrete function**. A function graphed with a line or smooth curve is a **continuous function**.

**EXAMPLE 2** Draw Graphs

**ICE SCULPTING** At an ice sculpting competition, each sculpture’s height was measured to make sure that it was within the regulated height range of 0 to 6 feet. The measurements were as follows: Team 1, 4 feet; Team 2, 4.5 feet; Team 3, 3.2 feet; Team 4, 5.1 feet; Team 5, 4.8 feet.

a. Make a table of values showing the relation between the ice sculpting team and the height of their sculpture.

<table>
<thead>
<tr>
<th>Team Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (ft)</td>
<td>4</td>
<td>4.5</td>
<td>3.2</td>
<td>5.1</td>
<td>4.8</td>
</tr>
</tbody>
</table>

b. Determine the domain and range of the function.

The domain of the function is \{1, 2, 3, 4, 5\} because this value is the independent variable. It is unaffected by the heights.

The range of the function is \{4, 4.5, 3.2, 5.1, 4.8\} because this value is the dependent variable. This value depends on the team number.

c. Write the data as a set of ordered pairs. Then graph the data.

Use the table. The team number is the independent variable and the height of the sculpture is the dependent variable. Therefore, the ordered pairs are (1, 4), (2, 4.5), (3, 3.2), (4, 5.1), (5, 4.8).

Because the team numbers and their corresponding heights cannot be between the points given, the points should not be connected.

d. State whether the function is **discrete** or **continuous**. Explain your reasoning.

Because the points are not connected, the function is discrete.

**Check Your Progress**

2. Suppose you are filling a bird feeder with seeds. The bird feeder will hold up to 3 quarts of seed. The feeder weighs 2.3 pounds when it is empty and 13.5 pounds when it is full.

A. Make a table that shows the bird feeder with 0, 1, 2, and 3 quarts of seed in it weighing 2.3, 6, 9.8, 13.5 pounds respectively.

B. Determine the domain and range of the function.

C. Write the data as a set of ordered pairs. Then graph the data.

D. State whether the function is **discrete** or **continuous**. Explain your reasoning.
You can use the **vertical line test** to see if a graph represents a function. If no vertical line can be drawn so that it intersects the graph more than once, then the graph is a function. If a vertical line can be drawn so that it intersects the graph at two or more points, the relation is not a function.

Recall from Lesson 1-6 that an equation is a representation of a relation. If the relation is a function, then the equation represents a function.

**EXAMPLE 3**  
**Equations as Functions**

Determine whether $-3x + y = 8$ represents a function.

First make a table of values. Then graph the equation.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
</tr>
</tbody>
</table>

The graph is a line. Place a pencil at the left of the graph to represent a vertical line. Slowly move the pencil across the graph.

For any value of $x$, the vertical line passes through no more than one point on the graph. So, the graph and the equation represent a function.

**Check Your Progress**

Determine if each of the equations represents a function.

3A. $4x = 8$  
3B. $4x = y + 8$

A function can be represented in different ways.

**Concept Summary**  
**Representations of a Function**
Find Function Values  Equations that are functions can be written in a form called **function notation**. For example, consider \( y = 3x - 8 \).

<table>
<thead>
<tr>
<th>Equation</th>
<th>Function Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = 3x - 8 )</td>
<td>( f(x) = 3x - 8 )</td>
</tr>
</tbody>
</table>

In a function, \( x \) represents the elements of the domain, and \( f(x) \) represents the elements of the range. Suppose you want to find the value in the range that corresponds to the element 5 in the domain. This is written \( f(5) \) and is read “\( f \) of 5.” The value \( f(5) \) is found by substituting 5 for \( x \) in the equation.

### EXAMPLE 4  Function Values

For \( f(x) = -4x + 7 \), find each value.

a. \( f(2) \)
   \[
   f(2) = -4(2) + 7
   = -8 + 7
   = -1
   \]
   \( x = 2 \)
   Multiply.
   Add.

b. \( f(-3) + 1 \)
   \[
   f(-3) + 1 = [-4(-3) + 7] + 1
   = 19 + 1
   = 20
   \]
   \( x = -3 \)
   Simplify.
   Add.

### Check Your Progress

For \( f(x) = 2x - 3 \), find each value.

4A. \( f(1) \)
4B. \( 6 - f(5) \)
4C. \( f(-2) \)
4D. \( f(-1) + f(2) \)

A function with a variable term that has an exponent other than 1 forms a **nonlinear function** and the graph is not a line.

### EXAMPLE 5  Nonlinear Function Values

If \( h(t) = -16t^2 + 68t + 2 \), find each value.

a. \( h(4) \)
   \[
   h(4) = -16(4)^2 + 68(4) + 2
   = -256 + 272 + 2
   = 18
   \]
   \( t = 4 \)
   Replace \( t \) with 4.
   Multiply.
   Add.

b. \( 2[h(g)] \)
   \[
   2[h(g)] = 2[-16(g)^2 + 68(g) + 2]
   = 2(-16g^2 + 68g + 2)
   = -32g^2 + 136g + 4
   \]
   \( g = \) \( g \)
   Replace \( t \) with \( g \).
   Simplify.
   Distributive Property

### Check Your Progress

If \( f(t) = 2t^3 \), find each value.

5A. \( f(4) \)
5B. \( 3[f(t)] + 2 \)
5C. \( f(-5) \)
5D. \( f(-3) - f(1) \)
Check Your Understanding

Determine whether each relation is a function. Explain.

1. Domain | Range
-4 | -1
-2 | 1
0 | 2
2 | -4
4 |

2. Domain | Range
2 | 6
5 | 7
6 | 9
6 | 10

3. \{(2, 2), (-1, 5), (5, 2), (2, -4)\}

4. \(y = \frac{1}{2}x - 6\)

5. 

6. 

7. 

8. 

Example 2  

9. SCHOOL ENROLLMENT  
The table shows the total enrollment in U.S. public schools.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Enrollment (in thousands)</td>
<td>48,560</td>
<td>48,710</td>
<td>48,948</td>
<td>49,091</td>
</tr>
</tbody>
</table>

Source: The World Almanac

a. Write a set of ordered pairs representing the data in the table if \(x\) is the number of school years since 2004–2005.

b. Draw a graph showing the relationship between the year and enrollment.

c. Describe the domain and range of the data.

10. CELL PHONE PICTURES  
The cost of sending cell phone pictures is given by \(y = 0.25x\), where \(x\) is the number of pictures that you send. Write the equation in function notation and then find \(f(5)\) and \(f(12)\). What do these values represent? Determine the domain and range of this function.

Examples 4 and 5  
If \(f(x) = 6x + 7\) and \(g(x) = x^2 - 4\), find each value.

11. \(f(-3)\)
12. \(f(m)\)
13. \(f(r - 2)\)
14. \(g(5)\)
15. \(g(a) + 9\)
16. \(g(-4f)\)
17. \(f(q + 1)\)
18. \(f(2) + g(-2)\)
19. \(g(-b)\)
Example 1

Determine whether each relation is a function. Explain.

20. Domain | Range
   -4      | 5
   -6      | 4
   -3      | 3

21. Domain | Range
   1      | 5
   4      | -6
   -8     | 7
   3      | 8

22. Domain | Range
   4      | 6
   -5      | 3
   6      | -3
   -5      | 5

Example 2

26. HOME VALUE The table shows the median home prices in Raleigh, North Carolina, from 2004 to 2006.

<table>
<thead>
<tr>
<th>Year</th>
<th>Median Home Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>$169,900</td>
</tr>
<tr>
<td>2005</td>
<td>$194,900</td>
</tr>
<tr>
<td>2006</td>
<td>$207,700</td>
</tr>
</tbody>
</table>

a. Write a set of ordered pairs representing the data in the table.
b. Draw a graph showing the relationship between the year and enrollment.
c. What is the domain and range for this data?

Example 3

Determine whether each relation is a function.

27. \{\(5, -7\), \(6, -7\), \(-8, -1\), \(0, -1\)\}
28. \{\(4, 5\), \(3, -2\), \(-2, 5\), \(4, 7\)\}
29. \(y = -8\)
30. \(x = 15\)
31. \(y = 3x - 2\)
32. \(y = 3x + 2y\)

Examples 4 and 5

If \(f(x) = -2x - 3\) and \(g(x) = x^2 + 5x\), find each value.

33. \(f(-1)\)
34. \(f(6)\)
35. \(g(2)\)
36. \(g(-3)\)
37. \(g(-2) + 2\)
38. \(f(0) - 7\)
39. \(f(4y)\)
40. \(g(-6m)\)
41. \(f(c - 5)\)
42. \(f(r + 2)\)
43. \(5[f(d)]\)
44. \(3[g(n)]\)

45 EDUCATION The average national math test scores \(f(t)\) for 17-year-olds can be represented as a function of the national science scores \(t\) by \(f(t) = 0.8t + 72\).

a. Graph this function.
b. What is the science score that corresponds to a math score of 308?
c. What is the domain and range of this function?
Lesson 1-7 Functions

Determine whether each relation is a function.

46. 

47. 

48. **BABYSITTING** Christina earns $7.50 an hour babysitting.

   a. Write an algebraic expression to represent the money Christina will earn if she works \( h \) hours.

   b. Choose five values for the number of hours Christina can baby sit and create a table with \( h \) and the value for the amount of money she will make during that time.

   c. Create a graph showing the relationship between the numbers of hours worked and the amount earned for the values in your table.

   d. Does it make sense to connect the points in your graph with a line? Why or why not?

**H.O.T. Problems** Use Higher-Order Thinking Skills

49. **OPEN ENDED** Write a set of three ordered pairs that represent a function. Choose another display that represents this function.

50. **REASONING** The ordered pairs \((0, 1), (3, 2), (3, -5), \) and \((5, 4)\) represent a relation between \( x \) and \( y \). Graph the ordered pairs. Determine whether the relation is a function. Explain.

51. **CHALLENGE** Consider \( f(x) = -4.3x - 2 \). Write \( f(g + 3.5) \) and simplify by combining like terms.

52. **WRITE A QUESTION** A classmate graphed a set of ordered pairs and used the vertical line test to determine whether it was a function. Write a question to help her decide if the same strategy can be applied to a mapping.

53. **CHALLENGE** If \( f(3b - 1) = 9b - 1 \), find one possible expression for \( f(x) \).

54. **FIND THE ERROR** Corazon and Maggie are analyzing the relation to determine whether it is a function. Is either of them correct? Explain your reasoning.

<table>
<thead>
<tr>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-5</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

   **Corazon**
   No, one member of the range is matched with two members of the domain.

   **Maggie**
   No, each member of the domain is matched with one member of the range.

55. **WRITING IN MATH** Use the data about stopping distances of cars on p. 45 to explain how graphs and functions can be used to model real-world situations.

Real-World Link

Babysitters earn an average of $6.04 per hour.

Source: Runzheimer International
56. Which point on the number line represents a number whose square is less than itself?

\[ \begin{array}{cccc}
A & B & C & D \\
-2 & -1 & 0 & 1 & 2 & 3 \\
\end{array} \]

A A C C
B B D D

57. Determine which of the following relations is a function.

\[ \begin{align*}
F & \quad \{(-3, 2), (4, 1), (-3, 5)\} \\
G & \quad \{(2, -1), (4, -1), (2, 6)\} \\
H & \quad \{(-3, -4), (-3, 6), (8, -2)\} \\
J & \quad \{(5, -1), (3, -2), (-2, -2)\} \\
\end{align*} \]

58. GEOMETRY What is the length of \( x \)?

\[ \text{A} \ 3 \text{ in.} \quad \text{B} \ 4 \text{ in.} \quad \text{C} \ 5 \text{ in.} \quad \text{D} \ 6 \text{ in.} \]

59. SHORT RESPONSE Camille made 16 out of 19 of her serves during her first volleyball game. She made 13 out of 16 of her serves during her second game. During which game did she make a greater percent of her serves?

Spiral Review

Solve each equation. (Lesson 1-5)

60. \( x = \frac{27 + 3}{10} \)
61. \( m = \frac{3^2 + 4}{7 - 5} \)
62. \( z = 32 + 4(-3) \)

63. SCHOOL SUPPLIES Tom is buying school supplies. The table shows the prices of some items he will need. If he needs 4 glue sticks, 10 pencils, and 4 notebooks, write and solve an equation to determine whether Tom can get them for under $10. Describe what the variables represent. (Lesson 1-6)

<table>
<thead>
<tr>
<th>School Supplies</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glue stick</td>
<td>$1.99</td>
</tr>
<tr>
<td>Pencil</td>
<td>$0.25</td>
</tr>
<tr>
<td>Notebook</td>
<td>$1.85</td>
</tr>
</tbody>
</table>

Write a verbal expression for each algebraic expression. (Lesson 1-1)

64. \( 4y + 2 \)
65. \( \frac{2}{3}x \)
66. \( a^2b + 5 \)

Find the volume of each rectangular prism. (Lesson 0-8)

67. \( \text{Volume} = 5.4 \text{ cm} \times 3.2 \text{ cm} \times 2.2 \text{ cm} \)
68. \( \text{Volume} = 1 \frac{1}{2} \text{ in.} \times 1 \frac{1}{2} \text{ in.} \times 1 \frac{1}{2} \text{ in.} \)
69. \( \text{Volume} = 40 \text{ mm} \times 40 \text{ mm} \times 40 \text{ mm} \)

Skills Review

Evaluate each expression. (Lesson 1-2)

70. If \( x = 3 \), then \( 6x - 5 = \) ______.
71. If \( n = -1 \), then \( 2n + 1 = \) ______.
72. If \( p = 4 \), then \( 3p + 4 = \) ______.
73. If \( q = 7 \), then \( 7q - 9 = \) ______.
74. If \( k = -11 \), then \( 4k + 6 = \) ______.
75. If \( y = 10 \), then \( 8y - 15 = \) ______.
You can use a TI-nspire graphing calculator to explore the different ways to represent a function.

**ACTIVITY**

Graph \( f(x) = 2x + 3 \) on the TI-nspire graphing calculator.

**Step 1** From the Home screen, select Graphs & Geometry.

**Step 2** Type \( 2x + 3 \) in the entry line.

Represent the function as a table.

**Step 3** Press \( \text{F2} \). Choose View, then Add Function Table. Then press \( \text{F2} \) or the click button.

**Step 4** Press \( \text{F3} + \text{F1} \) to toggle from the table to the graph. Press \( \text{F1} \) until an arrow appears on the graph. Use the click button to grab the line and move it. Notice how the values in the table change.

**Analyze the Results**

Graph each function. Make a table of five ordered pairs that also represents the function.

1. \( g(x) = -x - 3 \)
2. \( h(x) = \frac{1}{3}x + 3 \)
3. \( f(x) = -\frac{1}{2}x - 5 \)
4. \( f(x) = 3x - \frac{1}{2} \)
5. \( g(x) = -2x + 5 \)
6. \( h(x) = \frac{1}{5}x + 4 \)
Logical Reasoning and Counterexamples

**Why?**

The Butterfly Gardens is a conservatory in British Columbia, Canada, with over 50 species of butterflies. There is also an Emerging Room where you can see caterpillars change into butterflies.

**Conditional Statements** The statement *If an insect is a butterfly, then it was a caterpillar* is called a conditional statement. A conditional statement can be written in the form *If A, then B*. Statements in this form are called **if-then statements**.

\[
\begin{align*}
\text{If } & A, \\
\text{then } & B.
\end{align*}
\]

*If an insect is a butterfly, then it was a caterpillar.*

The part of the statement immediately following the word *if* is called the **hypothesis**.

The part of the statement that immediately follows *then* is called the **conclusion**.

**EXAMPLE 1**  
Identify Hypothesis and Conclusion

Identify the hypothesis and conclusion of each statement.

a. **CELEBRATION**  
   If it is the Fourth of July, then we will see fireworks.
   The hypothesis follows the word *if* and the conclusion follows the *then*.
   
   **Hypothesis:** it is the 4\(^{th}\) of July
   **Conclusion:** we will see fireworks

b. **If**  
   \[2x - 10 = 0\], then \[x = 5\].
   **Hypothesis:** \[2x - 10 = 0\]
   **Conclusion:** \[x = 5\]

**Check Your Progress**

1A. If we have enough sugar, then we will make cookies.
1B. If \[16z - 5 = 43\], then \[z = 3\].

Sometimes a conditional statement does not contain the words *if* and *then*. But a conditional statement can always be rewritten in if-then form.
EXAMPLE 2  Write a Conditional in If-Then Form

Identify the hypothesis and conclusion of each statement. Then write each statement in if-then form.

a. Chen gets chocolate chip ice cream when she is at the ice cream parlor.
   Hypothesis: Chen is at the ice cream parlor
   Conclusion: she will get chocolate chip ice cream
   If-Then Form: If Chen is at the ice cream parlor, then she will get chocolate chip ice cream.

b. For the equation \(3y + 4 = 25\), \(y = 7\).
   Hypothesis: \(3y + 4 = 25\)
   Conclusion: \(y = 7\)
   If-Then Form: If \(3y + 4 = 25\), then \(y = 7\).

Check Your Progress

2A. The neon light is on when the store is open.
2B. A circle with a radius of \(w - 4\) has a circumference of \(2\pi(w - 4)\).

Deductive Reasoning and Counterexamples  The process of using facts, rules, definitions, or properties to reach a valid conclusion is called **deductive reasoning**. If you know that the hypothesis of a true conditional is true for a given case, deductive reasoning allows you to say that the conclusion is true for that case.

EXAMPLE 3  Deductive Reasoning

Determine a valid conclusion that follows from the statement below for each condition. If a valid conclusion does not follow, write no valid conclusion and explain why.

*If one number is odd and another is even, then their product must be even.*

a. The numbers are 5 and 8.
   5 is odd and 8 is even, so the hypothesis is true.
   Their product is 40, which is even, so the conclusion is also true.

b. The product is 24.
   The product is part of the conclusion. The product is even, so the conclusion is true.
   The hypothesis is also true for numbers such as 3 and 8. However, for numbers such as 4 and 6 the hypothesis is not true. So, there is no valid conclusion.

Check Your Progress

3. Determine a valid conclusion that follows from the statement *There is a baseball game at the stadium*. If a valid conclusion does not follow, write no valid conclusion and explain why.
   A. We are at the stadium.
   B. We are at the mall.
To show that a conditional is false, we can use a counterexample. A **counterexample** is a specific case in which the hypothesis is true and the conclusion is false.

**EXAMPLE 4**

**Counterexamples**

Find a counterexample for each conditional statement.

**a.** If \( a + b > c \), then \( b > c \).

One counterexample is when \( a = 7, b = 3, \) and \( c = 9 \). The hypothesis is true, \( 7 + 3 > 9 \). However, the conclusion \( 3 > 9 \) is false.

**b.** If the leaves on the tree are brown, then it is fall.

If the leaves are brown then the tree could have died. So, the conclusion is not necessarily true.

**Check Your Progress**

4A. If \( ab > 0 \), then \( a \) and \( b \) are greater than 0.

4B. If a clothing store is selling wool coats, then it must be December.

**Check Your Understanding**

**Example 1**

Identify the hypothesis and conclusion of each statement.

1. If the game is on Saturday, then Eduardo will play.

2. If the chicken burns, then it was left in the oven too long.

3. If \( 52 - 4x = 28 \), then \( x = 6 \).

**Example 2**

Identify the hypothesis and conclusion of each statement. Then write each statement in if-then form.

4. Alisa plays with her dog in the yard when the weather is nice.

5. Two lines that are perpendicular form right angles.

6. A prime number is only divisible by one and itself.

**Example 3**

Determine a valid conclusion that follows from the statement below for each given condition. If a valid conclusion does not follow, write no valid conclusion and explain why.

*If a number is a multiple of 10, then the number is divisible by 5.*

7. The number is divisible by 5.

8. The number is 5010.

9. The number is 955.

**Example 4**

Find a counterexample for each conditional statement.

10. If Jack is at the park, then he is flying a kite.

11. If a teacher assigns a writing project, then it must be more than two pages long.

12. If \( |x| = 7 \), then \( x = 7 \).

13. If a number \( y \) is multiplied by \( \frac{1}{3} \), then \( \frac{1}{3}y < y \).
Example 1
Identify the hypothesis and conclusion of each statement.

14. If a team is playing at home, then they wear their white uniforms.
15. If you are in a grocery store, then you will buy food.
16. If $2n - 7 > 25$, then $n > 16$.
17. If $x$ equals $y$ and $y$ equals $z$, then $x$ equals $z$.
18. If it is not raining outside, we will walk the dogs.
19. If you play basketball, then you are tall.

Example 2
Identify the hypothesis and conclusion of each statement.

Then write each statement in if-then form.

20. Lamar’s third-period class is art. 
21. Joe will go to the mall after class.
22. For $x = 4$, $6x - 10 = 14$.
23. $5m - 8 < 52$ when $m < 12$.
24. A rectangle with sides of equal length is a square.
25. The sum of two even numbers is an even number.
26. August has 31 days.
27. Science teachers like to conduct experiments.

Example 3
Determine whether a valid conclusion follows from the statement below for each given condition. If a valid conclusion does not follow, write no valid conclusion and explain why.

If Belinda scores higher than 90% on the exam, then she will receive an A for the course.

28. Belinda scores a 91% on the exam.
29. Belinda scores an 89% on the exam.
30. Belinda receives an A for the course.
31. Belinda receives a B for the course.

Example 4
Find a counterexample for each conditional statement.

33. If you attend the banquet, then you will eat the food.
34. If the four sides of a quadrilateral are equal, then the shape is a square.
35. If a number is divisible by 3, then the number is odd.
36. If $3x + 17 \leq 53$, then $x < 12$.
37. If $x^2 = 1$, then $x$ must equal 1.
38. If an animal has spots, then it is a Dalmatian.
39. If a number is prime, then it is an odd number.
40. If an animal cannot fly, then the animal is not a bird.

41. RESEARCH  The Old Farmer’s Almanac uses a formula devised by the founder, Robert B. Thomas, in 1792 to predict weather patterns. Use the Internet or some other resource to research the weather predictions and actual weather for your region for the past five years. Summarize your data as examples and counterexamples.
42. Determine whether a valid conclusion follows from the statement below for each given condition. If a valid conclusion does not follow, write no valid conclusion and explain why.

If the dimensions of rectangle ABCD are doubled, then the perimeter is doubled.

a. The new rectangle measures 16 inches by 10 inches.
b. The perimeter of the new rectangle is 52 inches.

43. GEOMETRY Use the following statement.

If there are three line segments \( \overline{AB}, \overline{BC}, \) and \( \overline{CD} \), then they form a triangle.

a. Draw a diagram to provide an example for the conditional statement.
b. Draw a diagram to provide a counterexample for the conditional statement.

44. GROUNDHOG DAY On Groundhog Day (February 2), some people say that if a groundhog sees its shadow, then there will be 6 more weeks of winter. If it does not see its shadow, then there will be an early spring.

a. The most famous groundhog, Punxsutawney Phil in Pennsylvania, sees his shadow 85% of the time. Write an algebraic expression to represent how many times he sees his shadow in \( y \) years.
b. The table lists each possible situation if Phil sees his shadow or not and whether there are actually 6 more weeks of winter or an early spring. From the given conditional statement, determine whether each situation is true or false.

<table>
<thead>
<tr>
<th>Sees His Shadow or Not</th>
<th>6 More Weeks of Winter or an Early Spring</th>
<th>True or False</th>
</tr>
</thead>
<tbody>
<tr>
<td>shadow</td>
<td>Winter</td>
<td>true</td>
</tr>
<tr>
<td>shadow</td>
<td>Spring</td>
<td>?</td>
</tr>
<tr>
<td>no shadow</td>
<td>Winter</td>
<td>?</td>
</tr>
<tr>
<td>no shadow</td>
<td>Spring</td>
<td>?</td>
</tr>
</tbody>
</table>

c. Of the situations listed in the table, explain which situation could be considered a counterexample to the original statement.

45. CHALLENGE Determine whether the following statement is always true. If not, provide a counterexample.

If \( 2(b + c) = 2b + 2c \), then \( 2 + (b \cdot c) = (2 + b)(2 + c) \).

46. CHALLENGE For what values of \( n \) is the opposite of \( n \) greater than \( n \)? For what values of \( n \) is the opposite of \( n \) less than \( n \)? For what values is \( n \) equal to its opposite?

47. OPEN ENDED Write a conditional statement and label the hypothesis and conclusion.

48. REASONING Determine whether this statement is true or false. If the length of a rectangle is doubled, then the area of the rectangle is doubled. Justify your answer.

49. OPEN ENDED Write a conditional statement. Write a counterexample to the statement. Explain your reasoning.

50. WRITING IN MATH Explain how deductive reasoning is used to show that a conditional is true or false.

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Ground Hog’s Day began with Pennsylvania’s early settlers. They based it on the legend of Candlemas Day which states, “For as the sun shines on Candlemas Day, so far will the snow swirl in May…”.
The first official trek to Gobbler’s Knob in Punxsutawney, Pennsylvania was on February 2, 1887.
Source: Punxsutawney Groundhog Club

---

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---

---
51. Which value of \( b \) serves as a counterexample to the statement \( 2b < 3b \)?
   
<table>
<thead>
<tr>
<th>A</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>B</td>
<td>D</td>
</tr>
<tr>
<td>( \frac{1}{4} )</td>
<td>4</td>
</tr>
</tbody>
</table>

52. **SHORT RESPONSE** A deli serves boxed lunches with a sandwich, fruit, and a dessert. The sandwich choices are turkey, roast beef, or ham. The fruit choices are an orange or an apple. The dessert choices are a cookie or a brownie. How many different boxed lunches does the deli serve?

53. Which illustrates the Transitive Property of Equality?

   A. If \( c = 1 \), then \( c \cdot \frac{1}{c} = 1 \).
   B. If \( c = d \) and \( d = f \), then \( c = f \).
   C. If \( c = d \), then \( d = c \).
   D. If \( c = d \) and \( d = c \), then \( c = 1 \).

54. Simplify the expression \( 5d(7 - 3) - 16 + 3 \cdot 2d \).

<table>
<thead>
<tr>
<th>A</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>10d</td>
<td>21d</td>
</tr>
<tr>
<td>B</td>
<td>D</td>
</tr>
<tr>
<td>14d</td>
<td>25d</td>
</tr>
</tbody>
</table>

55. Determine whether each relation is a function. (Lesson 1-7)

<table>
<thead>
<tr>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-10</td>
</tr>
<tr>
<td>-1</td>
<td>-12</td>
</tr>
<tr>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>42</td>
</tr>
</tbody>
</table>

56. \( ((0, 2), (3, 5), (0, -1), (-2, 4)) \)

57. \[
\begin{array}{c|c}
 x & y \\
-\hline
17 & 6 \\
18 & 6 \\
19 & 5 \\
20 & 4 \\
\end{array}
\]

58. **GEOMETRY** The graph shows the relation between the length of one side of an equilateral triangle and the perimeter of the triangle. Express the relation as a set of ordered pairs and describe the domain and range. (Lesson 1-6)

59. Find the perimeter of each figure. (Lesson 0-6)

60. Find the perimeter of each figure. (Lesson 0-6)

61. **CLOTHING** Robert has 30 socks in his sock drawer. 16 of the socks are white, 6 are black, 2 are red, and 6 are yellow. If Robert pulls out a sock randomly, what is the probability that he pulls out a black sock? (Lesson 0-10)

62. \( 7^2 \)

63. \( (-9)^2 \)

64. \( 2.7^2 \)

65. \( (-12.25)^2 \)

66. \( 5^2 \)

67. \( 25^2 \)
A **set** is any collection of objects. The set that contains all objects is called the **universal set**, or the **universe**, usually labeled $U$. Each object is called a **member** or **element** of the set.

**ACTIVITY 1**

**Step 1** Cut 18 small slips of paper. Draw the shapes shown below on the slips.

![Shapes](image)

**Step 2** Place the shapes inside a loop of string. Use a slip of paper to label it $U$.

**Step 3** Make an arrangement as shown. Call the set of squares $A$.

The set of squares is a **subset** of $U$. The **empty set**, denoted by $\emptyset$ or $\varnothing$, is a set with no objects. It is a subset of any set. A set is also a subset of itself. In math notation, we can write $A \subseteq U$, $A \subseteq A$, and $\emptyset \subseteq U$.

**Step 4** We can identify a set by writing a description in brackets, such as $\{\text{squares}\}$. Put a loop around $B = \{\text{circles}\}$. Label it $B$. Notice that $B \subseteq U$.

**Step 5** If $A = \{\text{squares}\}$, then the **complement** of $A$, written $\overline{A}$, is every object in $U$ that is not in $A$. $\overline{A} = \{\text{circles and triangles}\}$, or $\{\text{non-squares}\}$. Draw the elements in $\overline{B}$. Write a description of $\overline{B}$ in brackets.

**Model and Analyze**

1. Let $C = \{\text{triangles}\}$. Write a description of the set in brackets.
2. Let $R = \{\text{yellow shapes}\}$. Write a description of the set in brackets.
3. Let $U = \{\text{squares}\}$. Subsets of $U$ can have 0, 1, 2, 3, 4, 5, or 6 elements. How many subsets of $U$ have exactly two elements? How many subsets are there total?
ACTIVITY 2

You can perform operations on two or more numbers, such as addition, subtraction, multiplication, and division. Finding the complement of a set is an operation on one set. You can also perform operations on two or more sets at a time.

Step 1 Use $U$ from Step 2 in Activity 1. Arrange the shapes as shown below. Label the sets. Write a description of $L$ in brackets. Write a description of $Q$ in brackets.

Step 2 In the diagram in Step 1, the region where $L$ and $Q$ overlap is shaded. Describe the shapes in the shaded region.

Step 3 The **intersection** of two sets is the set of elements common to both. The symbol for this operation is $\cap$. Intersection means that an element is in $L$ and $Q$. Draw the elements in $L \cap Q$.

Step 4 The **union** of two sets is the set of elements in one set or the other set. The symbol for this operation is $\cup$. You might think of this operation as **adding up** or combining all elements in two or more sets. Draw the elements in the set $L \cup Q$.

Step 5 Recall that finding the complement is an operation on only one set. Draw the elements in $\overline{L \cap Q}$.

Step 6 Draw the elements in $\overline{L \cup Q}$.

Exercises

Refer to the Venn diagram shown at the right. Write a description of the shapes in each set.

5. $M$
6. $P$
7. $T$
8. $M \cap P$
9. $M \cap T$
10. $P \cap T$
11. $M \cup P$
12. $M \cup T$
13. $P \cup T$
14. $M \cap P \cap T$
15. $M \cup P \cup T$
16. $\overline{M \cup P \cup T}$

17. **CHALLENGE** Use $U$ from Step 2, Activity 1. Find two sets $W$ and $Z$ such that $W \cap Z = \emptyset$. Draw a diagram with $W$, $Z$, and $U$ labeled and all shapes shown. Write a description of $W$, $Z$, and $W \cup Z$ in brackets.
Chapter Summary

Key Concepts
Order of Operations  (Lesson 1-2)
- Evaluate expressions inside grouping symbols.
- Evaluate all powers.
- Multiply and/or divide in order from left to right.
- Add or subtract in order from left to right.
Properties of Equality  (Lessons 1-3 and 1-4)
- For any numbers $a, b,$ and $c$:
  - Reflexive:  $a = a$
  - Symmetric: If $a = b$, then $b = a$.
  - Transitive:  If $a = b$ and $b = c$, then $a = c$.
  - Substitution: If $a = b$, then $a$ may be replaced by $b$ in any expression.
  - Distributive:  $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$
  - Commutative:  $a + b = b + a$ and $ab = ba$
  - Associative:  $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$

Solving Equations  (Lesson 1-5)
- Apply order of operations and the properties of real numbers to solve equations.
Relations  (Lesson 1-6)
- Relations can be represented by ordered pairs, a table, a mapping, or a graph.
Functions  (Lesson 1-7)
- Use the Vertical Line Test to determine if a relation is a function.
Conditional Statements  (Lesson 1-8)
- An if-then statement has a hypothesis and a conclusion.

Vocabulary Check
State whether each sentence is true or false. If false, replace the underlined term to make a true sentence.

1. A coordinate system is formed by the intersection of two number lines.
2. An exponent indicates the number the base is to be multiplied by.
3. An expression is in simplest form when it contains like terms and parentheses.
4. In an expression involving multiplication, the quantities being multiplied are called factors.
5. In a function, there is exactly one output for each input.
6. Order of operations tells us to always perform multiplication before subtraction.
7. Since the product of any number and 1 is equal to the number, 1 is called the multiplicative inverse.
Lesson-by-Lesson Review

1-1 Variables and Expressions (pp. 5–9)

Write a verbal expression for each algebraic expression.

8. \( h - 7 \)  
9. \( 3x^2 \)  
10. \( 5 + 6m^3 \)

Write an algebraic expression for each verbal expression.

11. a number increased by 9  
12. two-thirds of a number \( d \) to the third power  
13. 5 less than four times a number

Evaluate each expression.

14. \( 2^5 \)  
15. \( 6^3 \)  
16. \( 4^4 \)

17. BOWLING  Fantastic Pins Bowling Alley charges $2.50 for shoe rental plus $3.25 for each game. Write an expression representing the cost to rent shoes and bowl \( g \) games.

EXAMPLE 1

Write a verbal expression for \( 4x + 9 \).

Nine more than four times a number \( x \)

EXAMPLE 2

Write an algebraic expression for the difference of twelve and two times a number cubed.

Variable  Let \( x \) represent the number.

Expression  \( 12 - 2x^3 \)

EXAMPLE 3

Evaluate \( 3^4 \).

The base is 3 and the exponent is 4.

\[ 3^4 = 3 \cdot 3 \cdot 3 \cdot 3 \]

Use 3 as a factor 4 times.

\[ = 81 \]

Multiply.

EXAMPLE 4

Evaluate the expression \( 3(9 - 5)^2 \div 8 \).

\[ 3(9 - 5)^2 \div 8 = 3(4)^2 \div 8 \]

Work inside parentheses.

\[ = 3(16) \div 8 \]

Evaluate \( 4^2 \).

\[ = 48 \div 8 \]

Multiply.

\[ = 6 \]

Divide.

EXAMPLE 5

Evaluate the expression \( (5m - 2n) \div p^2 \) if \( m = 8, n = 4, p = 2 \).

\[ (5m - 2n) \div p^2 \]

Replace \( m \) with 8, \( n \) with 4, and \( p \) with 2.

\[ = (5 \cdot 8 - 2 \cdot 4) \div 2^2 \]

Multiply.

\[ = (40 - 8) \div 2^2 \]

Divide.

\[ = 32 \div 4 \]

Evaluate \( 2^2 \).

\[ = 8 \]

Divide.

1-2 Order of Operations (pp. 10–15)

Evaluate each expression.

18. \( 24 - 4 \cdot 5 \)  
19. \( 15 + 3^2 - 6 \)  
20. \( 7 + 2(9 - 3) \)  
21. \( 8 \cdot 4 - 6 \cdot 5 \)  
22. \( \left[ \left( \frac{2^5 - 5}{9} \right) \right] \)  
23. \( \frac{11 + 4^2}{5^2 - 4^2} \)

Evaluate each expression if \( a = 4, b = 3, \) and \( c = 9 \).

24. \( c + 3a \)  
25. \( 5b^2 \div c \)  
26. \( \left( a^2 + 2bc \right) \div 7 \)

27. ICE CREAM  The cost of a one-scoop sundae is $2.75, and the cost of a two-scoop sundae is $4.25. Write and evaluate an expression to find the total cost of 3 one-scoop sundaes and 2 two-scoop sundaes.
Evaluate each expression using properties of numbers. Name the property used in each step.

28. \( 18 \cdot 3(1 \div 3) \)
29. \( \left[ 5 \div (8 - 6) \right]^2 \)
30. \( (16 - 4^2) + 9 \)
31. \( 2 \cdot \frac{1}{2} + 4(4 \cdot 2 - 7) \)
32. \( 18 + 41 + 32 + 9 \)
33. \( 7^2 + 5 + \frac{3}{5} \)
34. \( 8 \cdot 0.5 \cdot 5 \)
35. \( 5.3 + 2.8 + 3.7 + 6.2 \)

36. **SCHOOL SUPPLIES** Monica needs to purchase a binder, a textbook, a calculator, and a workbook for her Algebra class. The binder costs $9.25, the textbook $32.50, the calculator $18.75, and the workbook $15.00. Find the total cost for Monica’s Algebra supplies.

EXAMPLE 6

Evaluate \( 6(4 \cdot 2 - 7) + 5 \cdot \frac{1}{5} \). Name the property used in each step.

\[
6(4 \cdot 2 - 7) + 5 \cdot \frac{1}{5} \\
= 6(8 - 7) + 5 \cdot \frac{1}{5} \\
= 6(1) + 5 \cdot \frac{1}{5} \\
= 6 + 5 \cdot \frac{1}{5} \\
= 6 + 1 \\
= 7
\]

EXAMPLE 8

Rewrite the expression \( 6(x + 4) \) using the Distributive Property. Then simplify.

\[
6(x + 4) = 6 \cdot x + 6 \cdot 4 \\
= 6x + 24
\]

EXAMPLE 9

Rewrite the expression \( (3x - 2)(-5) \) using the Distributive Property. Then simplify.

\[
(3x - 2)(-5) \\
= (3x)(-5) - (2)(-5) \\
= -15x + 10
\]
Find the solution of each equation if the replacement sets are \( x = \{1, 3, 5, 7, 9\} \) and \( y = \{6, 8, 10, 12, 14\} \).

50. \( y - 9 = 3 \)  
51. \( 14 + x = 21 \)  
52. \( 4y = 32 \)  
53. \( 3x - 11 = 16 \)  
54. \( \frac{42}{y} = 7 \)  
55. \( 2(x - 1) = 8 \)

Solve each equation.

56. \( a = 24 - 7(3) \)  
57. \( z = 63 ÷ (3^2 - 2) \)

58. **AGE** Shandra’s age is four more than three times Sherita’s age. Write an equation to find Shandra’s age. Then solve the equation if Sherita’s age is 3 years.

**EXAMPLE 10**

Solve the equation \( 5w - 19 = 11 \) if the replacement set is \( w = \{2, 4, 6, 8, 10\} \).

Replace \( w \) in \( 5w - 19 = 11 \) with each value in the replacement set.

<table>
<thead>
<tr>
<th>( w )</th>
<th>( 5w - 19 = 11 )</th>
<th>True or False?</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>( 5(2) - 19 = 11 )</td>
<td>False</td>
</tr>
<tr>
<td>4</td>
<td>( 5(4) - 19 = 11 )</td>
<td>False</td>
</tr>
<tr>
<td>6</td>
<td>( 5(6) - 19 = 11 )</td>
<td>True</td>
</tr>
<tr>
<td>8</td>
<td>( 5(8) - 19 = 11 )</td>
<td>False</td>
</tr>
<tr>
<td>10</td>
<td>( 5(10) - 19 = 11 )</td>
<td>False</td>
</tr>
</tbody>
</table>

Since the equation is true when \( w = 6 \), the solution of \( 5w - 19 = 11 \) is \( w = 6 \).

**EXAMPLE 11**

Express the relation \((-3, 4), (1, -2), (0, 1), (3, -1)\) as a table, a graph, and a mapping.

**Table**

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
</tr>
</tbody>
</table>

**Graph**

Graph each ordered pair on a coordinate plane.

**Mapping**

List the \( x \)-values in set \( X \) and the \( y \)-values in set \( Y \). Draw arrows from the \( x \)-values in set \( X \) to the corresponding \( y \)-values in set \( Y \).

**GARDENING** On average, 7 plants grow for every 10 seeds of a certain type planted. Make a table to show the relation between seeds planted and plants growing for 50, 100, 150, and 200 seeds. Then state the domain and range and graph the relation.
Determine whether each relation is a function.

65. 

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>3</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

66. 

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

67. 

![Graph of a relation]

68. \{(8, 4), (6, 3), (4, 2), (2, 1), (0, 0)\}

If $f(x) = 2x + 4$ and $g(x) = x^2 - 3$, find each value.

69. $f(-3)$

70. $g(2)$

71. $f(0)$

72. $g(-4)$

73. $f(m + 2)$

74. $g(3p)$

75. **GRADES** A teacher claims that the relationship between number of hours studied for a test and test score can be described by $g(x) = 45 + 9x$, where $x$ represents the number of hours studied. Graph this function.

EXAMPLE 12

Determine whether the relation shown below is a function.

For each member of the domain, there is only one member of the range that corresponds to it. So this mapping represents a function. It does not matter that more than one element of the domain is paired with one element of the range.

EXAMPLE 13

Determine whether $2x - y = 1$ represents a function.

First make a table of values. Then graph the equation.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>-3</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Using the vertical line test, it can be shown that $2x - y = 1$ does represent a function.

EXAMPLE 14

Identify the hypothesis and conclusion for the statement “If the football team wins their last game then they will win the championship.”

The hypothesis follows the word *if* and the conclusion follows the word *then*.

Hypothesis: the football team wins their last game

Conclusion: they will win the championship

Identify the hypothesis and conclusion of each statement.

76. If Orlando practices the piano then he will perform well at his recital.

77. If $2x + 7 > 31$, then $x > 12$.

Find a counterexample for each conditional statement.

78. If it is raining outside, then you will get wet.

79. If $4x - 11 = 53$, then $x < 16$. 
Write an algebraic expression for each verbal expression.
1. six more than a number
2. twelve less than the product of three and a number
3. four divided by the difference between a number and seven

Evaluate each expression.
4. \( \frac{32 \div 4 + 2^3 - 3}{7 + 3^2} \)
5. \( \frac{(2 \cdot 4)^2}{2 + 3} \)

6. MULTIPLE CHOICE Find the value of the expression \( a^2 + 2ab + b^2 \) if \( a = 6 \) and \( b = 4 \).
   A 68  
   B 92  
   C 100  
   D 121

Evaluate each expression. Name the property used in each step.
7. \( 13 + (16 - 4^2) \)
8. \( \frac{2}{9} \left[ 9 \div (7 - 5) \right] \)
9. \( 37 + 29 + 13 + 21 \)

Rewrite each expression using the Distributive Property. Then simplify.
10. \( 4(x + 3) \)
11. \( (5y - 2)(-3) \)

12. MOVIE TICKETS A company operates three movie theaters. The chart shows the typical number of tickets sold each week at the three locations. Write and evaluate an expression for the total typical number of tickets sold by all three locations in four weeks.

<table>
<thead>
<tr>
<th>Location</th>
<th>Tickets Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>438</td>
</tr>
<tr>
<td>B</td>
<td>374</td>
</tr>
<tr>
<td>C</td>
<td>512</td>
</tr>
</tbody>
</table>

Find the solution of each equation if the replacement sets are \( x = \{1, 3, 5, 7, 9\} \) and \( y = \{2, 4, 6, 8, 10\} \).
13. \( 3x - 9 = 12 \)
14. \( y^2 - 5y - 11 = 13 \)

15. CELL PHONES The ABC Cell Phone Company offers a plan that includes a flat fee of $29 per month plus a $0.12 charge per minute. Write an equation to find \( C \), the total monthly cost for \( m \) minutes. Then solve the equation for \( m = 50 \).

Express the relation shown in each table, mapping, or graph as a set of ordered pairs.
16. \[
\begin{array}{c|c}
 x & y \\
 \hline
 -2 & 4 \\
 1 & 2 \\
 3 & 0 \\
 4 & -2 \\
\end{array}
\]
17. Domain Range
   \[
   \begin{array}{c|c}
   -3 & -2 \\
   -1 & 0 \\
   1 & 2 \\
   3 & 4 \\
   \end{array}
   \]
18. MULTIPLE CHOICE Determine the domain and range for the relation \( \{(2, 5), (-1, 3), (0, -1), (3, 3), (-4, -2)\} \).
   \[
   \begin{array}{c|c}
   F & D: \{2, -1, 0, 3, -4\}, R: \{5, 3, -1, 3, -2\} \\
   G & D: \{5, 3, -1, 3, -2\}, R: \{2, -1, 0, 3, 4\} \\
   H & D: \{0, 1, 2, 3, 4\}, R: \{-4, -3, -2, -1, 0\} \\
   J & D: \{2, -1, 0, 3, -4\}, R: \{2, -1, 0, 3, 4\} \\
   \end{array}
   \]
19. Determine whether the relation \( \{(2, 3), (-1, 3), (0, 4), (3, 2), (-2, 3)\} \) is a function.

If \( f(x) = 5 - 2x \) and \( g(x) = x^2 + 7x \), find each value.
20. \( g(3) \)
21. \( f(-6y) \)

Identify the hypothesis and conclusion of each statement.
22. If the temperature goes below 32°F, it will snow outside.
23. If Ivan breaks his arm, he will need to go to the hospital.

Find a counterexample for each conditional statement.
24. If you go to the pool, you will get wet.
25. If a quadrilateral has one pair of sides that are parallel, then it is a square.
Eliminate Unreasonable Answers

You can eliminate unreasonable answers to help you find the correct one when solving multiple choice test items. Doing so will save you time by narrowing down the list of possible correct answers.

Strategies for Eliminating Unreasonable Answers

**Step 1**
Read the problem statement carefully to determine exactly what you are being asked to find.

Ask yourself:
- What am I being asked to solve?
- What format (i.e. fraction, number, decimal, percent, type of graph) will the correct answer be?
- What units (if any) will the correct answer have?

**Step 2**
Carefully look over each possible answer choice and evaluate for reasonableness.
- Identify any answer choices that are clearly incorrect and eliminate them.
- Eliminate any answer choices that are not in the proper format.
- Eliminate any answer choices that do not have the correct units.

**Step 3**
Solve the problem and choose the correct answer from those remaining. Check your answer.

**EXAMPLE**
Read each problem. Eliminate any unreasonable answers. Then use the information in the problem to solve.

Jason earns 8.5% commission his weekly sales at an electronics retail store. Last week he had $4,200 in sales. What was his commission for the week?

A $332  
B $357  
C $425  
D $441
Using mental math, you know that 10% of $4,200 is $420. Since 8.5% is less than 10%, you know that Jason earned less than $420 in commission for his weekly sales. So, answer choices C and D can be eliminated because they are greater than $420. The correct answer is either A or B.

\[ $4,200 \times 0.085 = $357 \]
So, the correct answer is B.

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### Exercises

Read each problem. Eliminate any unreasonable answers. Then use the information in the problem to solve.

1. Coach Roberts expects 35% of the student body to turn out for a pep rally. If there are 560 students, how many does Coach Roberts expect to attend the pep rally?
   - A 184
   - B 196
   - C 214
   - D 390

2. Jorge and Sally leave school at the same time. Jorge walks 300 yards north and then 400 yards east. Sally rides her bike 600 yards south and then 800 yards west. What is the distance between the two students?
   - F 500 yd
   - G 750 yd
   - H 1,200 yd
   - J 1,500 yd

3. What is the range of the relation below?
   \[ \{(1, 2), (3, 4), (5, 6), (7, 8)\} \]
   - A all real numbers
   - B all even numbers
   - C \{2, 4, 6, 8\}
   - D \{1, 3, 5, 7\}

4. The expression \( 3n + 1 \) gives the total number of squares needed to make each figure of the pattern where \( n \) is the figure number. How many squares will be needed to make Figure 9?
   - F 28 squares
   - G 32.5 squares
   - H 56 squares
   - J 88.5 squares

5. The expression \( 3x - (2x + 4x - 6) \) is equivalent to
   - A \(-3x - 6\)
   - B \(-3x + 6\)
   - C \(3x + 6\)
   - D \(3x - 6\)
Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Evaluate the expression $2^6$.
   A 12
   B 32
   C 64
   D 128

2. Monica claims: If you are in the drama club, then you are also on the academic team. Which student is a counterexample to this statement?

<table>
<thead>
<tr>
<th>Drama Club</th>
<th>Academic Team</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thomas</td>
<td>Kim</td>
</tr>
<tr>
<td></td>
<td>Ronnie</td>
</tr>
</tbody>
</table>

   F Beth
   G Kim
   J Thomas

3. Let $y$ represent the number of yards. Which algebraic expression represents the number of feet in $y$?
   A $y - 3$
   B $y + 3$
   C $3y$
   D $\frac{3}{y}$

4. What is the domain of the following relation?
   \{(1, 3), (-6, 4), (8, 5)\}
   F \{3, 4, 5\}
   G \{-6, 1, 8\}
   H \{-6, 1, 3, 4, 5, 8\}
   J \{1, 3, 4, 5, 8\}

5. The table shows the number of some of the items sold at the concession stand at the first day of a soccer tournament. Estimate how many items were sold from the concession stand throughout the four days of the tournament?

<p>| Concession Sales Day 1 Results |</p>
<table>
<thead>
<tr>
<th>Item</th>
<th>Number Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Popcorn</td>
<td>78</td>
</tr>
<tr>
<td>Hot Dogs</td>
<td>80</td>
</tr>
<tr>
<td>Chip</td>
<td>48</td>
</tr>
<tr>
<td>Sodas</td>
<td>51</td>
</tr>
<tr>
<td>Bottled Water</td>
<td>92</td>
</tr>
</tbody>
</table>

   A 1,350 items
   B 1,400 items
   C 1,450 items
   D 1,500 items

6. There are 24 more cars for sale at a dealership than twice the number of trucks. If there are 100 cars for sale, how many trucks are there for sale at the dealership?
   F 28
   H 34
   G 32
   J 38

7. Refer to the relation in the table below. Which of the following values would result in the relation not being a function?

<table>
<thead>
<tr>
<th>$x$</th>
<th>-6</th>
<th>-2</th>
<th>0</th>
<th>?</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>-1</td>
<td>8</td>
<td>3</td>
<td>-3</td>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

   A -1
   B 3
   C 7
   D 8

Test-TakingTip

Question 2 A counterexample is a specific case in which the hypothesis of a conditional statement is true, but the conclusion is false.
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

8. The edge of each box below is 1 unit long.

![Figures 1, 2, and 3]

a. Make a table showing the perimeters of the first 3 figures in the pattern.

b. Look for a pattern in the perimeters of the shapes. Write an algebraic expression for the perimeter of Figure \( n \).

c. What would be the perimeter of Figure 10 in the pattern?

9. The table shows the costs of certain items at a corner hardware store.

<table>
<thead>
<tr>
<th>Item</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>box of nails</td>
<td>$3.80</td>
</tr>
<tr>
<td>box of screws</td>
<td>$5.25</td>
</tr>
<tr>
<td>claw hammer</td>
<td>$12.95</td>
</tr>
<tr>
<td>electric drill</td>
<td>$42.50</td>
</tr>
</tbody>
</table>

a. Write two expressions to represent the total cost of 3 boxes of nails, 2 boxes of screws, 2 hammers, and 1 electric drill.

b. What is the total cost of the items purchased?

10. GRIDDED RESPONSE Evaluate the expression below.

\[
\frac{5^3 \cdot 4^2 - 5^2 \cdot 4^3}{5 \cdot 4}
\]

11. Use the equation \( y = 2(4 + x) \) to answer each question.

a. Complete the following table for the different values of \( x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

b. Plot the points from the table on a coordinate grid. What do you notice about the points?

c. Make a conjecture about the relationship between the change in \( x \) and the change in \( y \).

Extended Response

Record your answers on a sheet of paper. Show your work.

12. The volume of a sphere is four-thirds the product of \( \pi \) and the radius cubed.

![Sphere diagram]

a. Write an expression for the volume of a sphere with radius \( r \).

b. Find the volume of a sphere with a radius of 6 centimeters. Describe how you found your answer.

Need Extra Help?

<table>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
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<td>1-8</td>
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<td>1-4</td>
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