In Chapters 1 and 2, you learned about lines and angles and used deductive reasoning to write geometric proofs.

In Chapter 3, you will:
- Identify angle relationships that occur with parallel lines and a transversal and prove lines parallel from given angle relationships.
- Use slope to analyze a line and to write its equation.
- Find the distance between a point and a line and between two parallel lines.

Students will use the definitions, properties and theorems about congruent triangles and other figures to prove additional theorems and apply these to solve real-world problems.

Architects, carpenters, and engineers, use parallel and perpendicular lines to design buildings, make furniture, and make machines such as escalators.
Get Ready for Chapter 3

Diagnose Readiness You have two options for checking Prerequisite Skills.

Text Option Take the Quick Check below. Refer to the Quick Review for help.

Quick Check

Refer to the figure to identify each of the following.
(Lesson 1-1)

1. How many planes are shown in this figure?
2. Name three points that are collinear.
3. Are points C and D coplanar? Explain.
4. PHOTOGRAPHY Tina is taking a picture of her friends. If she sets a tripod level on the ground, will the bottom of each of the three legs of the tripod be coplanar? (Lesson 2-5)

EXAMPLE 1

Refer to the figure.

a. How many planes are shown in this figure?
Six: plane FGLK, plane JHMP, plane FKJ, plane GLMH, plane FGHJ, and plane KLMP
b. Name three points that are collinear.
Points M, O, and H are collinear.
Yes. Points F, K, and J all lie in plane FKJ.

EXAMPLE 2

Find each angle measure. (Lesson 1-5)

5. \( \angle 1 \)
6. \( \angle 2 \)
7. \( \angle 3 \)
8. \( \angle 4 \)

Find \( m \angle 1 \).

\[ m \angle 1 + 37 + 90 = 180 \]
\[ m \angle 1 = 53 \]

Add. Simplify.

EXAMPLE 3

Find \( x \) in \( a + 8 = b(x - 7) \) if \( a = 12 \) and \( b = 10 \).

\[ a + 8 = b(x - 7) \]
\[ 12 + 8 = 10(x - 7) \]
\[ 12 + 8 = 10x - 70 \]
\[ 90 = 10x \]
\[ x = 9 \]

Write the equation.
Simplify.
Add.
Divide.

For each equation, find the value of \( x \) for the given values of \( a \) and \( b \). (Lesson 0-4)

9. \( a + 8 = -4(x - b) \), for \( a = 8 \) and \( b = 3 \)
10. \( b = 3x + 4a \), for \( a = -9 \) and \( b = 12 \)
11. \[ \frac{a + 2}{b + 13} = 5x \], for \( a = 18 \) and \( b = -1 \)

12. MINIATURE GOLF A miniature golf course offers a $1 ice cream cone with each round of golf purchased. If five friends each had a cone after golfing and spend a total of $30, how much does one round of golf cost? (Lesson 0-5)

Online Option KY Math Online Take a self-check Chapter Readiness Quiz at glencoe.com.

Chapter 3 Parallel and Perpendicular Lines 169
Get Started on Chapter 3

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 3. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

Parallel and Perpendicular Lines Make this Foldable to help you organize your Chapter 3 notes about relationships between lines. Begin with a sheet of 11” × 17” paper and six index cards.

1. Fold lengthwise about 3” from the bottom.
2. Fold the paper in thirds.
3. Open and staple the edges on either side to form three pockets.
4. Label the pockets as shown. Place two index cards in each pocket.

New Vocabulary

<table>
<thead>
<tr>
<th>English</th>
<th>Español</th>
</tr>
</thead>
<tbody>
<tr>
<td>parallel lines</td>
<td>rectas paralelas</td>
</tr>
<tr>
<td>skew lines</td>
<td>rectas alabeadas</td>
</tr>
<tr>
<td>parallel planes</td>
<td>planos paralelos</td>
</tr>
<tr>
<td>transversal</td>
<td>transversal</td>
</tr>
<tr>
<td>interior angles</td>
<td>ángulos internos</td>
</tr>
<tr>
<td>exterior angles</td>
<td>ángulos externos</td>
</tr>
<tr>
<td>corresponding angles</td>
<td>ángulos correspondientes</td>
</tr>
<tr>
<td>slope</td>
<td>pendiente</td>
</tr>
<tr>
<td>rate of change</td>
<td>tasa de cambio</td>
</tr>
<tr>
<td>slope-intercept form</td>
<td>forma pendiente-intersección</td>
</tr>
<tr>
<td>point-slope form</td>
<td>forma punto-pendiente</td>
</tr>
<tr>
<td>equidistant</td>
<td>equidistante</td>
</tr>
</tbody>
</table>

Review Vocabulary

- congruent angles • p. 178 • ángulos congruentes two angles that have the same degree measure
- perpendicular • p. 180 • perpendicular two lines, segments, or rays that intersect to form right angles
- vertical angles • p. 46 • ángulos opuestos por el vértice two nonadjacent angles formed by intersecting lines

\[ \angle 3 \equiv \angle 4 \]
\[ r \text{ and } s \text{ are perpendicular.} \]
\[ \angle 1 \text{ and } \angle 2 \text{ are vertical angles.} \]
Parallel Lines and Transversals

**Why?**
An *Ames room* creates the illusion that a person standing in the right corner is much larger than a person standing in the left corner.

From a front viewing hole the front and back walls appear parallel, when in fact they are slanted. The ceiling and floor appear horizontal, but are actually tilted.

**Relationships Between Lines and Planes** The construction of the Ames room above makes use of intersecting, parallel, and skew lines, as well as intersecting and parallel planes, to create an optical illusion.

**Key Concepts**

<table>
<thead>
<tr>
<th>Parallel lines</th>
<th>are coplanar lines that do not intersect.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example</td>
<td><em>JK</em></td>
</tr>
</tbody>
</table>

**Skew lines** are lines that do not intersect and are not coplanar.

**Example** Lines *ℓ* and *m* are skew.

**Parallel planes** are planes that do not intersect.

**Example** Planes *A* and *B* are parallel.

*JK* || *LM* is read as *line JK is parallel to line LM*.

If segments or rays are contained within lines that are parallel or skew, then the segments or rays are parallel or skew.

**Real-World Example 1** Identify Parallel and Skew Relationships
Identify each of the following using the wedge of cheese below.

a. all segments parallel to *JP*
   - *KQ* and *LR*

b. a segment skew to *KL*
   - *JP*, *PQ*, or *PR*

c. a plane parallel to plane *PQR*
   - Plane *JKL* is the only plane parallel to plane *PQR*.
Check Your Progress

Identify each of the following using the cube shown.

1A. all segments skew to \( \overrightarrow{BC} \)
1B. a segment parallel to \( \overrightarrow{EH} \)
1C. all planes parallel to plane \( DCH \)

Transversal Angle Pair Relationships

A line that intersects two or more coplanar lines at two different points is called a transversal. In the diagram below, line \( t \) is a transversal of lines \( q \) and \( r \). Notice that line \( t \) forms a total of eight angles with lines \( q \) and \( r \). These angles, and specific pairings of these angels, are given special names.

Key Concepts

**Transversal Angle Pair Relationships**

- **Four interior angles** lie in the region between lines \( q \) and \( r \).
  - \( \angle 3, \angle 4, \angle 5, \angle 6 \)
- **Four exterior angles** lie in the two regions that are not between lines \( q \) and \( r \).
  - \( \angle 1, \angle 2, \angle 7, \angle 8 \)
- **Consecutive interior angles** are interior angles that lie on the same side of transversal \( t \).
  - \( \angle 4 \) and \( \angle 5, \angle 3 \) and \( \angle 6 \)
- **Alternate interior angles** are nonadjacent interior angles that lie on opposite sides of transversal \( t \).
  - \( \angle 3 \) and \( \angle 5, \angle 4 \) and \( \angle 6 \)
- **Alternate exterior angles** are nonadjacent exterior angles that lie on opposite sides of transversal \( t \).
  - \( \angle 1 \) and \( \angle 7, \angle 2 \) and \( \angle 8 \)
- **Corresponding angles** lie on the same side of transversal \( t \) and on the same side of lines \( q \) and \( r \).
  - \( \angle 1 \) and \( \angle 5, \angle 2 \) and \( \angle 6 \)
  - \( \angle 3 \) and \( \angle 7, \angle 4 \) and \( \angle 8 \)

EXAMPLE 2

Classify Angle Pair Relationships

Refer to the figure below. Classify the relationship between each pair of angles as alternate interior, alternate exterior, corresponding, or consecutive interior angles.

a. \( \angle 1 \) and \( \angle 5 \) alternate exterior
b. \( \angle 6 \) and \( \angle 7 \) consecutive interior
c. \( \angle 2 \) and \( \angle 4 \) corresponding
d. \( \angle 2 \) and \( \angle 6 \) alternate interior

Check Your Progress

2A. \( \angle 3 \) and \( \angle 7 \)  
2B. \( \angle 5 \) and \( \angle 7 \)  
2C. \( \angle 4 \) and \( \angle 8 \)  
2D. \( \angle 2 \) and \( \angle 3 \)
When more than one line can be considered a transversal, first identify the transversal for a given angle pair by locating the line that connects the vertices of the angles.

**Example 3** Identify Transversals and Classify Angle Pairs

Identify the transversal connecting each pair of angles in the photo. Then classify the relationship between each pair of angles.

a. \( \angle 1 \) and \( \angle 3 \)

The transversal connecting \( \angle 1 \) and \( \angle 3 \) is line \( h \). These are alternate exterior angles.

b. \( \angle 5 \) and \( \angle 6 \)

The transversal connecting \( \angle 5 \) and \( \angle 6 \) is line \( k \). These are consecutive interior angles.

c. \( \angle 2 \) and \( \angle 6 \)

The transversal connecting \( \angle 2 \) and \( \angle 6 \) is line \( \ell \). These are corresponding angles.

**Check Your Progress**

3A. \( \angle 3 \) and \( \angle 5 \)

3B. \( \angle 2 \) and \( \angle 8 \)

3C. \( \angle 5 \) and \( \angle 7 \)

3D. \( \angle 2 \) and \( \angle 9 \)

**Check Your Understanding**

**Example 1**

Refer to the figure at the right to identify each of the following.

1. a plane parallel to plane \( ZWX \)
2. a segment skew to \( \overline{TS} \) that contains point \( W \)
3. all segments parallel to \( \overline{SV} \)

4. **Construction**

   Use the diagram of the partially framed storage shed shown to identify each of the following.

   a. Name three pairs of parallel planes.
   b. Name three segments parallel to \( \overline{DE} \).
   c. Name two segments parallel to \( \overline{FE} \).
   d. Name two pairs of skew segments.

**Example 2**

Classify the relationship between each pair of angles as **alternate interior**, **alternate exterior**, **corresponding**, or **consecutive interior** angles.

5. \( \angle 1 \) and \( \angle 8 \)
6. \( \angle 2 \) and \( \angle 4 \)
7. \( \angle 3 \) and \( \angle 6 \)
8. \( \angle 6 \) and \( \angle 7 \)
Identify the transversal connecting each pair of angles. Then classify the relationship between each pair of angles.

9. $\angle 2$ and $\angle 4$
10. $\angle 5$ and $\angle 6$
11. $\angle 4$ and $\angle 7$
12. $\angle 2$ and $\angle 7$

Refer to the figure to identify each of the following.

13. all segments parallel to $\overline{DM}$
14. a plane parallel to plane $ACD$
15. a segment skew to $BC$
16. all planes intersecting plane $EDM$
17. all segments skew to $\overline{AE}$
18. a segment parallel to $\overline{EN}$
19. a segment parallel to $\overline{AB}$ through point $J$
20. a segment skew to $\overline{CL}$ through point $E$

Identify the transversal connecting each pair of angles. Then classify the relationship between each pair of angles as alternate interior, alternate exterior, corresponding, or consecutive interior angles.

21. $\angle 4$ and $\angle 9$
22. $\angle 5$ and $\angle 7$
23. $\angle 3$ and $\angle 5$
24. $\angle 10$ and $\angle 11$
25. $\angle 1$ and $\angle 6$
26. $\angle 6$ and $\angle 8$
27. $\angle 2$ and $\angle 3$
28. $\angle 9$ and $\angle 10$
29. $\angle 4$ and $\angle 11$
30. $\angle 7$ and $\angle 11$

SAFETY Identify the transversal connecting each pair of angles in the photo of a fire escape shown. Then classify the relationship between each pair of angles.

31. $\angle 1$ and $\angle 2$
32. $\angle 2$ and $\angle 4$
33. $\angle 4$ and $\angle 5$
34. $\angle 6$ and $\angle 7$
35. $\angle 7$ and $\angle 8$
36. $\angle 2$ and $\angle 3$

37. POWER Use the photo and information at the left.
   a. What must be the relationship between power lines $p$ and $m$? Explain your reasoning.
   b. What is the relationship between line $q$ and lines $p$ and $m$?
Describe the relationship between each pair of segments as parallel, skew, or intersecting.

38. $\overline{FG}$ and $\overline{BC}$
39. $\overline{AB}$ and $\overline{CG}$
40. $\overline{DH}$ and $\overline{HG}$
41. $\overline{DH}$ and $\overline{BF}$
42. $\overline{EF}$ and $\overline{BC}$
43. $\overline{CD}$ and $\overline{AD}$

44. **OPTICAL ILLUSION** The illusion at the right is created using squares and straight lines.

a. How are $\overline{AB}$ and $\overline{CD}$ related? Justify your reasoning.

b. How are $\overline{MN}$ and $\overline{QR}$ related? $\overline{AB}$, $\overline{CD}$, and $\overline{OP}$?

45. **ESCALATORS** Escalators consist of steps on a continuous loop that is driven by a motor. At the top and bottom of the platform, the steps collapse to provide a level surface for entrance and exit.

a. What is the relationship between the treads of the ascending stairs?

b. What is the relationship between the treads of the two steps at the top of the incline?

c. How do the treads of the steps on the incline of the escalator relate to the treads of the steps on the bottom of the conveyor?

46. **OPEN ENDED** Plane $\mathcal{P}$ contains lines $a$ and $b$. Line $c$ intersects plane $\mathcal{P}$ at point $J$. Lines $a$ and $b$ are parallel, lines $a$ and $c$ are skew, and lines $b$ and $c$ are not skew. Draw a figure based upon this description.

47. **CHALLENGE** Suppose points $A$, $B$, and $C$ lie in plane $\mathcal{P}$, and points $D$, $E$, and $F$ lie in plane $\mathcal{Q}$. Line $m$ contains points $D$ and $F$ and does not intersect plane $\mathcal{P}$. Line $n$ contains points $A$ and $E$.

a. Draw a diagram to represent the situation.

b. What is the relationship between planes $\mathcal{P}$ and $\mathcal{Q}$?

c. What is the relationship between lines $m$ and $n$?

**REASONING** Plane $\mathcal{X}$ and plane $\mathcal{Y}$ are parallel and plane $\mathcal{Z}$ intersects plane $\mathcal{X}$. Line $\overline{AB}$ is in plane $\mathcal{X}$, line $\overline{CD}$ is in plane $\mathcal{Y}$, and line $\overline{EF}$ is in plane $\mathcal{Z}$. Determine whether each statement is always, sometimes, or never true. Explain.

48. $\overline{AB}$ is skew to $\overline{CD}$.
49. $\overline{AB}$ intersects $\overline{EF}$.

50. **WRITING IN MATH** Explain why planes cannot be skew.

---

**Real-World Link**

Escalators are more efficient than elevators for moving people short distances because of their high loading capacity. Large escalators can transport 6000 passengers per hour.

Source: Otis Elevator Co.
51. Which of the following angle pairs are alternate exterior angles?

A $\angle 1$ and $\angle 5$  
B $\angle 2$ and $\angle 6$  
C $\angle 2$ and $\angle 10$  
D $\angle 5$ and $\angle 9$

52. What is the measure of $\angle XYZ$?

F $30^\circ$  
G $60^\circ$  
H $120^\circ$  
J $150^\circ$

53. SHORT RESPONSE  Name the coordinates of the points representing the $x$- and $y$-intercepts of the graph shown below.

54. SAT/ACT Of the following, the one that is not equivalent to 485 is:

A $(3 \times 100) + (4 \times 10) + 145$  
B $(3 \times 100) + (18 \times 10) + 5$  
C $(4 \times 100) + (8 \times 10) + 15$  
D $(4 \times 100) + (6 \times 10) + 25$

55. $m\angle 9 = 2x - 4$,  
56. $m\angle 11 = 4x$,  
57. $m\angle 19 = 100 + 20x$,  
$m\angle 10 = 2x + 4$  
$m\angle 12 = 2x - 6$  
$m\angle 20 = 20x$

58. PROOF Prove the following. (Lesson 2-7)

Given: $\overline{WY} \cong \overline{ZX}$  
A is the midpoint of $\overline{WY}$.  
A is the midpoint of $\overline{ZX}$.

Prove: $\overline{WA} \cong \overline{ZA}$

ALGEBRA Use the figure at the right. (Lesson 1-5)

59. If $m\angle CFD = 12a + 45$, find $a$ so that $\overrightarrow{FC} \perp \overrightarrow{FD}$.

60. If $m\angle AFB = 8x - 6$ and $m\angle BFC = 14x + 8$, find the value of $x$ so that $\angle AFC$ is a right angle.

61. Find $x$. (Lesson 3-2)

62.  

63.  

Skills Review
You can use The Geometer’s Sketchpad® to explore the angles formed by two parallel lines and a transversal.

### ACTIVITY Parallel Lines and a Transversal

**Step 1** Draw a line.
- Draw and label points $F$ and $G$. Then use the line tool to draw $FG$.

**Step 2** Draw a parallel line.
- Draw a point that is not on $FG$ and label it $J$. Select $FG$ and point $J$, and then chose Parallel Line from the Construct menu. Draw and label a point $K$ on this parallel line.

**Step 3** Draw a transversal.
- Draw and label point $A$ on $FG$ and point $B$ on $JK$. Select $A$ and $B$ and then choose Line from the Construct menu to draw transversal $AB$. Then draw and label points $C$ and $D$ on $AB$ as shown.

**Step 4** Measure each angle.
- Measure all eight angles formed by these lines. For example, select points $F$, $A$, then $C$, and choose Angle from the Measure menu to find $m\angle FAC$.

### Analyze the Results

1. Record the measures from Step 4 in a table like this one. Which angles have the same measure?

<table>
<thead>
<tr>
<th>Angles</th>
<th>$\angle FAC$</th>
<th>$\angle CAG$</th>
<th>$\angle GAB$</th>
<th>$\angle FAB$</th>
<th>$\angle JBA$</th>
<th>$\angle ABK$</th>
<th>$\angle KBD$</th>
<th>$\angle JBD$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Measure</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Drag point $C$ or $F$ to move transversal $AB$ so that it intersects the two parallel lines at a different angle. Add a row 2nd Measure to your table and record the new measures. Repeat these steps until your table has 3rd, 4th, and 5th Measure rows of data.

3. Using the angles listed in the table, identify and describe the relationship between all angle pairs that have the following special names. Then write a conjecture in if-then form about each angle pair when formed by any two parallel lines cut by a transversal.
   - a. corresponding
   - b. alternate interior
   - c. alternate exterior
   - d. consecutive interior

4. Drag point $C$ or $F$ so that the measure of any of the angles is 90.
   - a. What do you notice about the measures of the other angles?
   - b. Make a conjecture about a transversal that is perpendicular to one of two parallel lines.

---

*Explore 3-2 Geometry Software Lab: Angles and Parallel Lines* 177
Angles and Parallel Lines

**Why?**

Those who work in construction and maintenance often use an access scaffold. This structure provides support and access to elevated areas. The transversal $t$ shown provides structural support to the two parallel working areas.

**Parallel Lines and Angle Pairs** In the photo, line $t$ is a transversal of lines $a$ and $b$, and $\angle 1$ and $\angle 2$ are corresponding angles. Since lines $a$ and $b$ are parallel, there is a special relationship between corresponding angle pairs.

**Postulate 3.1**

**Corresponding Angles Postulate**

If two parallel lines are cut by a transversal, then each pair of corresponding angles is congruent.

**Examples**

\[
\begin{align*}
\angle 1 & \cong \angle 3, \quad \angle 2 \cong \angle 4, \quad \angle 5 \cong \angle 7, \quad \angle 6 \cong \angle 8 \\
\end{align*}
\]

**EXAMPLE 1** Use Corresponding Angles Postulate

In the figure, $m\angle 5 = 72$. Find the measure of each angle. Tell which postulates (or theorems) you used.

- **a. $\angle 4$**
  
  $\angle 4 \cong \angle 5$  \hspace{1cm} **Corresponding Angles Postulate**
  
  $m\angle 4 = m\angle 5$  \hspace{1cm} **Definition of congruent angles**
  
  $m\angle 4 = 72$  \hspace{1cm} **Substitution**

- **b. $\angle 2$**
  
  $\angle 2 \cong \angle 4$  \hspace{1cm} **Vertical Angles Theorem**
  
  $\angle 4 \cong \angle 5$  \hspace{1cm} **Corresponding Angles Postulate**
  
  $\angle 2 \cong \angle 5$  \hspace{1cm} **Transitive Property of Congruence**
  
  $m\angle 2 = m\angle 5$  \hspace{1cm} **Definition of congruent angles**
  
  $m\angle 2 = 72$  \hspace{1cm} **Substitution**

**Check Your Progress**

In the figure, suppose that $m\angle 8 = 105$. Find the measure of each angle. Tell which postulate(s) or theorem(s) you used.

- **1A. $\angle 1$**
  - **1B. $\angle 2$**
  - **1C. $\angle 3$**

In Example 1, $\angle 2$ and $\angle 5$ are congruent alternate exterior angles. This and other examples suggest the following theorems about the other angle pairs formed by two parallel lines cut by a transversal.
Lesson 3-2 Angles and Parallel Lines

**StudyTip**

**Angle Relationships**

These theorems generalize the relationships between specific pairs of angles. If you get confused about the relationships, you can verify them with the methods you used in Example 1, using only corresponding, vertical, and supplementary angles.

---

**Parallel Lines and Angle Pairs**

**3.1 Alternate Interior Angles Theorem** If two parallel lines are cut by a transversal, then each pair of alternate interior angles is congruent.

Examples \( \angle 1 \cong \angle 3 \) and \( \angle 2 \cong \angle 4 \)

**3.2 Consecutive Interior Angles Theorem** If two parallel lines are cut by a transversal, then each pair of consecutive interior angles is supplementary.

Examples \( \angle 1 \) and \( \angle 2 \) are supplementary. \( \angle 3 \) and \( \angle 4 \) are supplementary.

**3.3 Alternate Exterior Angles Theorem** If two parallel lines are cut by a transversal, then each pair of alternate exterior angles is congruent.

Examples \( \angle 5 \cong \angle 7 \) and \( \angle 6 \cong \angle 8 \)

You will prove Theorems 3.2 and 3.3 in Exercises 30 and 35, respectively.

Since postulates are accepted without proof, you can use the Corresponding Angles Postulate to prove each of the theorems above.

**Proof**

**Alternate Interior Angles Theorem**

Given: \( a \parallel b \)

\( t \) is a transversal of \( a \) and \( b \).

Prove: \( \angle 4 \cong \angle 5, \angle 3 \cong \angle 6 \)

Paragraph Proof:

We are given that \( a \parallel b \) with a transversal \( t \). By the Corresponding Angles Postulate, corresponding angles are congruent. So, \( \angle 2 \cong \angle 4 \) and \( \angle 6 \cong \angle 8 \). Also, \( \angle 5 \cong \angle 2 \) and \( \angle 8 \cong \angle 3 \) because vertical angles are congruent. Therefore, \( \angle 5 \cong \angle 4 \) and \( \angle 3 \cong \angle 6 \) since congruence of angles is transitive.

**Real-World Link**

**COMMUNITY PLANNING** Redding Lane and Creek Road are parallel streets that intersect Park Road along the west side of Wendell Park. If \( m\angle 1 = 118 \), find \( m\angle 2 \).

 \( \angle 2 \cong \angle 1 \quad \text{Alternate Interior Angles Postulate} \\
 m\angle 2 = m\angle 1 \quad \text{Definition of congruent angles} \\
 m\angle 2 = 118 \quad \text{Substitution} \\

**Check Your Progress**

**COMMUNITY PLANNING** Refer to the diagram above to find each angle measure. Tell which postulate(s) or theorem(s) you used.

2A. If \( m\angle 1 = 100 \), find \( m\angle 4 \).

2B. If \( m\angle 3 = 70 \), find \( m\angle 4 \).
**Algebra and Angle Measures** The special relationships between the angles formed by two parallel lines and a transversal can be used to find unknown values.

**EXAMPLE 3** Find Values of Variables

**ALGEBRA** Use the figure at the right to find the indicated variable. Explain your reasoning.

**a.** If \( m \angle 4 = 2x - 17 \) and \( m \angle 1 = 85 \), find \( x \).

\[
\angle 3 \cong \angle 1 \quad \text{Vertical Angles Theorem}
\]

\[
m \angle 3 = m \angle 1 \quad \text{Definition of congruent angles}
\]

\[
m \angle 3 = 85 \quad \text{Substitution}
\]

Since lines \( r \) and \( s \) are parallel, \( \angle 4 \) and \( \angle 3 \) are supplementary by the Consecutive Interior Angles Theorem.

\[
m \angle 3 + m \angle 4 = 180 \quad \text{Definition of supplementary angles}
\]

\[
85 + 2x - 17 = 180 \quad \text{Substitution}
\]

\[
2x + 68 = 180 \quad \text{Simplify.}
\]

\[
2x = 112 \quad \text{Subtract 68 from each side.}
\]

\[
x = 56 \quad \text{Divide each side by 2.}
\]

**b.** Find \( y \) if \( m \angle 3 = 4y + 30 \) and \( m \angle 7 = 7y + 6 \).

\[
\angle 3 \cong \angle 7 \quad \text{Alternate Interior Angles Theorem}
\]

\[
m \angle 3 = m \angle 7 \quad \text{Definition of congruent angles}
\]

\[
4y + 30 = 7y + 6 \quad \text{Substitution}
\]

\[
30 = 3y + 6 \quad \text{Subtract 4y from each side.}
\]

\[
24 = 3y \quad \text{Subtract 6 from each side.}
\]

\[
y = 8 \quad \text{Divide each side by 3.}
\]

**Check Your Progress**

**3A.** If \( m \angle 2 = 4x + 7 \) and \( m \angle 7 = 5x - 13 \), find \( x \).

**3B.** Find \( y \) if \( m \angle 5 = 68 \) and \( m \angle 3 = 3y - 2 \).

A special relationship occurs when the transversal of two parallel lines is a perpendicular line.

**Theorem 3.4** Perpendicular Transversal Theorem

In a plane, if a line is perpendicular to one of two parallel lines, then it is perpendicular to the other.

**Example** If line \( a \parallel b \) and line \( a \perp t \), then line \( b \perp t \).

You will prove Theorem 3.4 in Exercise 37.
Lesson 3-2 Angles and Parallel Lines

Check Your Understanding

Example 1  
In the figure, $m\angle 1 = 94$. Find the measure of each angle. Tell which postulate(s) or theorem(s) you used.

1. $\angle 3$  
2. $\angle 5$  
3. $\angle 4$

Example 2  
In the figure, $m\angle 4 = 101$. Find the measure of each angle. Tell which postulate(s) or theorem(s) you used.

4. $\angle 6$  
5. $\angle 7$  
6. $\angle 5$

7. ROADS In the diagram, the guard rail is parallel to the surface of the roadway and the vertical supports are parallel to each other. Find the measures of angles 2, 3, and 4.

Example 3  
Find the value of the variable(s) in each figure. Explain your reasoning.

8. $y$°  
9. $x$°

10. $(x + 55)^\circ$  

Practice and Problem Solving

Examples 1 and 2  
In the figure, $m\angle 11 = 22$ and $m\angle 14 = 18$. Find the measure of each angle. Tell which postulate(s) or theorem(s) you used.

11. $\angle 4$  
12. $\angle 3$  
13. $\angle 12$

14. $\angle 8$  
15. $\angle 6$  
16. $\angle 2$

17. $\angle 10$  
18. $\angle 5$  
19. $\angle 1$

Example 3  
SOLAR ENERGY A solar dish collects energy by directing radiation from the sun to a receiver located at the focal point of the dish. Assume that the radiation rays are parallel. Determine the relationship between each pair of angles and explain your reasoning.

20. $\angle 1$ and $\angle 2$  
21. $\angle 1$ and $\angle 3$  
22. $\angle 4$ and $\angle 5$  
23. $\angle 3$ and $\angle 4$
Find the value of the variable(s) in each figure. Explain your reasoning.

24. \((x + 12)° - 114° = y°

25. \((3x - 15)° - 105° = (y + 25)°

26. \((2x)° - 54° = 25°

27. \(96° - (2x)° - 94° = (3y + 44)°

28. \((5y)° - 120° = (2x)° - 108°

29. \((3y + 40)° - 120° = x°

30. PROOF Copy and complete the proof of Theorem 3.2.

Given: \(m \parallel n\); \(\ell\) is a transversal.

Prove: \(\angle 1\) and \(\angle 2\) are supplementary; \(\angle 3\) and \(\angle 4\) are supplementary.

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ?</td>
<td>a. Given</td>
</tr>
<tr>
<td>b. (\angle 1) and (\angle 3) form a linear pair; (\angle 3) and (\angle 4) form a linear pair.</td>
<td>b. ?</td>
</tr>
<tr>
<td>c. ?</td>
<td>c. If two angles form a linear pair, then they are supplementary.</td>
</tr>
<tr>
<td>d. (\angle 1 \cong \angle 4), (\angle 2 \cong \angle 3)</td>
<td>d. ?</td>
</tr>
<tr>
<td>e. (m\angle 1 = m\angle 4), (m\angle 2 = m\angle 3)</td>
<td>e. Definition of Congruence</td>
</tr>
<tr>
<td>f. ?</td>
<td>f. ?</td>
</tr>
</tbody>
</table>

31. \(\angle 1\) and \(\angle 8\)

32. \(\angle 1\) and \(\angle 5\)

33. \(\angle 3\) and \(\angle 6\)

34. \(\angle 1\) and \(\angle 2\)

35. PROOF Write a two-column proof of the Alternate Exterior Angles Theorem. (Theorem 3.3)

36. BRIDGES Refer to the diagram of the double decker Michigan Avenue Bridge in Chicago, Illinois. The two levels of the bridge are parallel.

a. How are the measures of the odd-numbered angles related? Explain.

b. How are the measures of the even-numbered angles related? Explain.

c. How are any pair of angles in which one is odd and the other is even related?

d. What geometric term(s) can be used to relate the two roadways contained by the bridge?
37. **PROOF** In a plane, prove that if a line is perpendicular to one of two parallel lines, then it is perpendicular to the other. (Theorem 3.4)

Find \(x\). *(Hint: Draw an auxiliary line.)*

38.

39.

40. **PROBABILITY** Suppose you were to pick any two angles in the figure below.

   a. How many possible angle pairings are there? Explain.

   b. Describe the possible relationships between the measures of the angles in each pair. Explain.

   c. Describe the likelihood of randomly selecting a pair of congruent angles. Explain your reasoning.

41. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate the relationship between same-side exterior angles.

   a. **GEOMETRY** Draw five pairs of parallel lines, \(m\) and \(n\), \(a\) and \(b\), \(r\) and \(s\), \(j\) and \(k\), and \(x\) and \(y\), cut by a transversal \(t\), and measure each angle pair.

   b. **TABULAR** Record your data in a table.

   c. **VERBAL** Make a conjecture about the relationship between the pair of angles formed on the exterior of parallel lines and on the same side of the transversal.

   d. **LOGICAL** What type of reasoning did you use to form your conjecture? Explain.

   e. **PROOF** Write a proof of your conjecture.

**H.O.T. Problems**

42. **WRITING IN MATH** If line \(a\) is parallel to line \(b\) and \(\angle 1 \cong \angle 2\), describe the relationship between lines \(b\) and \(c\). Explain your reasoning.

43. **WRITING IN MATH** Compare and contrast the Alternate Interior Angles Theorem and the Consecutive Interior Angles Theorem.

44. **OPEN ENDED** Draw a pair of parallel lines cut by a transversal and measure the two exterior angles on the same side of the transversal. Include the measures on your drawing. Based on the pattern you have seen for naming other pairs of angles, what do you think the name of the pair you measured would be?

45. **CHALLENGE** Find \(x\) and \(y\).

46. **REASONING** Determine the minimum number of angle measures you would have to know to find the measures of all the angles formed by two parallel lines cut by a transversal. Explain.
47. Suppose \( \angle 4 \) and \( \angle 5 \) form a linear pair. If \( m\angle 1 = 2x \), \( m\angle 2 = 3x - 20 \), and \( m\angle 3 = x - 4 \), what is \( m\angle 3 \)?

\[ m\angle 1 = 2x, \quad m\angle 2 = 3x - 20, \quad m\angle 3 = x - 4 \]

\[ \text{A} \ 26^\circ \quad \text{B} \ 28^\circ \quad \text{C} \ 30^\circ \quad \text{D} \ 32^\circ \]

48. SAT/ACT A farmer raises chickens and pigs. If his animals have a total of 120 heads and a total of 300 feet, how many chickens does the farmer have?

\[ \begin{align*}
F & \quad 60 \\
G & \quad 70 \\
H & \quad 80 \\
J & \quad 90
\end{align*} \]

49. SHORT RESPONSE If \( m \parallel n \), then which of the following statements must be true?

\[ \text{I. } \angle 3 \text{ and } \angle 6 \text{ are Alternate Interior Angles.} \\
\text{II. } \angle 4 \text{ and } \angle 6 \text{ are Consecutive Interior Angles.} \\
\text{III. } \angle 1 \text{ and } \angle 7 \text{ are Alternate Exterior Angles.} \]

50. ALGEBRA If \(-2 + x = -6\), then \(-17 - x = \)

\[ \begin{align*}
A & \quad 13 \\
B & \quad -4 \\
C & \quad 13 \\
D & \quad 21
\end{align*} \]

51. AVIATION Airplanes are assigned an altitude level based on the direction they are flying. If one airplane is flying northwest at 34,000 feet and another airplane is flying east at 25,000 feet, describe the type of lines formed by the paths of the airplanes. Explain your reasoning. (Lesson 3-1)

52. If \( \angle 1 \) and \( \angle 2 \) form a linear pair and \( m\angle 2 = 67 \), find \( m\angle 1 \).

53. \( \angle 6 \) and \( \angle 8 \) are complementary; \( m\angle 8 = 47 \).

54. \( m\angle 4 = 32 \)

55. TRAINS A train company wants to provide routes to New York City, Dallas, Chicago, Los Angeles, San Francisco, and Washington, D.C. An engineer draws lines between each pair of cities on a map. No three of the cities are collinear. How many lines did the engineer draw? (Lesson 2-5)

56. \( \frac{6 - 5}{4 - 2} \)

57. \( \frac{-5 - 2}{4 - 7} \)

58. \( \frac{-11 - 4}{12 - (-9)} \)

59. \( \frac{16 - 12}{15 - 11} \)

60. \( \frac{10 - 22}{8 - 17} \)

61. \( \frac{8 - 17}{12 - (-3)} \)
The rate of change of the steepness of a line is called the **slope**. Slope can be used to investigate the relationship between real-world quantities.

**Set Up the Lab**
- Connect the data collection device to the graphing calculator. Place on a desk or table so that the data collection device can read the motion of a walker.
- Mark the floor at distances of 1 meter and 6 meters from the device.

**ACTIVITY**

**Step 1**
Have one group member stand at the 1-meter mark. When another group member presses the button to begin collecting data, the walker begins to walk away from the device. Walk at a slow, steady pace.

**Step 2**
Stop collecting data when the walker passes the 6-meter mark. Save the data as Trial 1.

**Step 3**
Repeat the experiment, walking more quickly. Save the data as Trial 2.

**Step 4**
For Trial 3, repeat the experiment by walking toward the data collection device slowly.

**Step 5**
Repeat the experiment, walking quickly toward the device. Save the data as Trial 4.

**Analyze the Results**

1. Compare and contrast the graphs for Trials 1 and 2. How do the graphs for Trials 1 and 3 compare?

2. Use the TRACE feature of the calculator to find the coordinates of two points on each graph. Record the coordinates in a table like the one shown. Then use the points to find the slope of the line.

3. Compare and contrast the slopes for Trials 1 and 2. How do the slopes for Trials 1 and 2 compare to the slopes for Trials 3 and 4?

4. The slope of a line describes the rate of change of the quantities represented by the $x$- and $y$-values. What is represented by the rate of change in this experiment?

5. **MAKE A CONJECTURE** What would the graph look like if you were to collect data while the walker was standing still? Use the data collection device to test your conjecture.
Slopes of Lines

**Why?**

Ski resorts assign ratings to their ski trails according to their difficulty. A primary factor in determining this rating is a trail’s steepness or slope gradient. A trail with a 6% or \( \frac{6}{100} \) grade falls 6 feet vertically for every 100 feet traveled horizontally.

The easiest trails, labeled ○, have slopes ranging from 6% to 25%, while more difficult trails, labeled ♦ or ♦♦, have slopes of 40% or greater.

**Slope of a Line** The steepness or slope of a hill is described by the ratio of the hill’s vertical rise to its horizontal run. In algebra, you learned that the slope of a line in the coordinate plane can be calculated using any two points on the line.

**Key Concept**

For Your

In a coordinate plane, the slope of a line is the ratio of the change along the y-axis to the change along the x-axis between any two points on the line.

The slope \( m \) of a line containing two points with coordinates \( (x_1, y_1) \) and \( (x_2, y_2) \) is given by the formula

\[
m = \frac{y_2 - y_1}{x_2 - x_1}, \text{ where } x_1 \neq x_2.
\]

**EXAMPLE 1  Find the Slope of a Line**

Find the slope of each line.

**a.** Substitute \((-1, -2)\) for \((x_1, y_1)\) and \((3, 3)\) for \((x_2, y_2)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope Formula}
\]

\[
= \frac{3 - (-2)}{3 - (-1)} \quad \text{Substitution}
\]

\[
= \frac{5}{4} \quad \text{Simplify.}
\]
Lesson 3-3
Slopes of Lines

b. Substitute \((-2, 3)\) for \((x_1, y_1)\) and \((1, -3)\) for \((x_2, y_2)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope Formula}
\]
\[
= \frac{-3 - 3}{1 - (-2)} \quad \text{Substitution}
\]
\[
= -2 \quad \text{Simplify.}
\]

c. Substitute \((-4, -3)\) for \((x_1, y_1)\) and \((3, -3)\) for \((x_2, y_2)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope Formula}
\]
\[
= \frac{-3 - (-3)}{3 - (-4)} \quad \text{Substitution}
\]
\[
= 0 \quad \text{or} \quad 0 \quad \text{Simplify.}
\]

This slope is undefined.

d. Substitute \((2, 1)\) for \((x_1, y_1)\) and \((2, -4)\) for \((x_2, y_2)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope Formula}
\]
\[
= \frac{-4 - 1}{2 - 2} \quad \text{Substitution}
\]
\[
= \frac{-5}{0} \quad \text{Simplify.}
\]

This slope is undefined.

Example 1 illustrates the four different types of slopes.

StudyTip
Dividing by 0
The slope \(-\frac{5}{0}\) is undefined because there is no number that you can multiply by 0 and get \(-5\). Since this is true for any number, all numbers divided by 0 will have an undefined slope. All vertical lines have undefined slopes.

Check Your Progress
1A. the line containing \((6, -2)\) and \((-3, -5)\) 1B. the line containing \((8, -3)\) and \((-6, -2)\)
1C. the line containing \((4, 2)\) and \((4, -3)\) 1D. the line containing \((-3, 3)\) and \((4, 3)\)

Concept Summary
Classifying Slopes

<table>
<thead>
<tr>
<th>Positive Slope</th>
<th>Negative Slope</th>
<th>Zero Slope</th>
<th>Undefined Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Positive Slope Diagram" /></td>
<td><img src="image2.png" alt="Negative Slope Diagram" /></td>
<td><img src="image3.png" alt="Zero Slope Diagram" /></td>
<td><img src="image4.png" alt="Undefined Slope Diagram" /></td>
</tr>
</tbody>
</table>

Slope can be interpreted as a rate of change, describing how a quantity \(y\) changes in relationship to quantity \(x\). The slope of a line can also be used to identify the coordinates of any point on the line.
EXAMPLE 2

Use Slope as Rate of Change

TRAVEL A pilot flies a plane from Columbus, Ohio, to Orlando, Florida. After 0.5 hour, the plane reaches its cruising altitude and is 620 miles from Orlando. Half an hour later, the plane is 450 miles from Orlando. How far was the plane from Orlando 1.25 hours after takeoff?

Understand Use the data given to graph the line that models the distance flown $y$ in miles as a function of time $x$ in hours.

Assume that speed is constant. Plot the points $(0.5, 620)$ and $(1.0, 450)$ and draw a line through them.

You want to find the distance from Orlando after 1.25 hours.

Plan Find the slope of the line graphed. Use this rate of change in the plane’s distance from Orlando per hour to find the distance from Orlando after 1.25 hours.

Solve Use the Slope Formula to find the slope of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{450 - 620}{1.0 - 0.5} = \frac{-170}{0.5} = -340$$

The plane traveled at an average speed of 340 miles per hour. The negative sign indicates a decrease in distance over time.

Use the slope of the line and one known point on the line to calculate the distance $y$ when the time $x$ is 1.25.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope Formula}$$

$$-340 = \frac{y_2 - 620}{1.25 - 0.5} \quad m = -340, x_1 = 0.5, y_1 = 620, \text{ and } x_2 = 1.25$$

$$-340 = \frac{y_2 - 620}{0.75} \quad \text{Simplify.}$$

$$-255 = y_2 - 620$$

$$365 = y_2 \quad \text{Multiply each side by 0.75.}$$

Thus, the distance traveled after 1.25 hours is 365 miles.

Check From the graph we can estimate that after 1.25 hours, the distance from Orlando was a little less than 400 miles. Since 365 is close to this estimate, our answer is reasonable.

Check Your Progress

DOWNLOADS In 2006, 500 million songs were legally downloaded from the Internet. In 2004, 200 million songs were legally downloaded.

2A. Use the data given to graph the line that models the number of songs legally downloaded $y$ as a function of time $x$ in years.

2B. Find the slope of the line and interpret its meaning.

2C. If this trend continues at the same rate, how many songs will be legally downloaded in 2010?
Parallel and Perpendicular Lines

**Postulates**

3.2 **Slopes of Parallel Lines** Two nonvertical lines have the same slope if and only if they are parallel. All vertical lines are parallel.

**Example** Parallel lines \( \ell \) and \( m \) have the same slope, 4.

3.3 **Slopes of Perpendicular Lines** Two nonvertical lines are perpendicular if and only if the product of their slopes is \(-1\). Vertical and horizontal lines are perpendicular.

**Example** Line \( m \perp \) line \( p \)

**Study Tip**

If a line \( \ell \) has a slope of \( \frac{a}{b} \), then the slope of a line perpendicular to line \( \ell \) is the opposite reciprocal, \( -\frac{b}{a} \), since \( \frac{a}{b} \left(-\frac{b}{a}\right) = -1 \).

**EXAMPLE 3**

Determine Line Relationships

Determine whether \( \overrightarrow{AB} \) and \( \overrightarrow{CD} \) are parallel, perpendicular, or neither for \( A(1, 1) \), \( B(-1, -5) \), \( C(3, 2) \), and \( D(6, 1) \). Graph each line to verify your answer.

**Step 1** Find the slope of each line.

\[
\text{slope of } \overrightarrow{AB} = \frac{-5 - 1}{-1 - 1} = \frac{-6}{-2} = 3 \\
\text{slope of } \overrightarrow{CD} = \frac{1 - 2}{6 - 3} = \frac{-1}{3}
\]

**Step 2** Determine the relationship, if any, between the lines.

The two lines do not have the same slope, so they are not parallel. To determine if the lines are perpendicular, find the product of their slopes.

\[
3 \left(-\frac{1}{3}\right) = -1
\]

**Product of slopes for \( \overrightarrow{AB} \) and \( \overrightarrow{CD} \)**

Since the product of their slopes is \(-1\), \( \overrightarrow{AB} \) is perpendicular to \( \overrightarrow{CD} \).

**CHECK** When graphed, the two lines appear to intersect in right angles.

**Check Your Progress**

Determine whether \( \overrightarrow{AB} \) and \( \overrightarrow{CD} \) are parallel, perpendicular, or neither.

3A. \( A(14, 13), B(-11, 0), C(-3, 7), D(-4, -5) \)

3B. \( A(3, 6), B(-9, 2), C(5, 4), D(2, 3) \)
EXAMPLE 4 Use Slope to Graph a Line

Graph the line that contains \( A(–3, 0) \) and is perpendicular to \( \overrightarrow{CD} \) with \( C(–2, –3) \) and \( D(2, 0) \).

The slope of \( \overrightarrow{CD} \) is \( \frac{0 – (–3)}{2 – (–2)} \) or \( \frac{3}{4} \).

Since \( \frac{3}{4} \) \( \frac{4}{–3} \) \( = –1 \), the slope of the line perpendicular to \( \overrightarrow{CD} \) through \( A \) is \( –\frac{4}{3} \) or \( -\frac{4}{3} \).

To graph the line, start at \( A \). Move down 4 units and then right 3 units. Label the point \( B \) and draw \( \overrightarrow{AB} \).

Check Your Progress

4. Graph the line that contains \( P(0, 1) \) and is perpendicular to \( \overrightarrow{QR} \) with \( Q(–6, –2) \) and \( R(0, –6) \).

Check Your Understanding

Example 1

Find the slope of each line.

1. \[ \text{Graph the line.} \]

2. \[ \text{Graph the line.} \]

3. \[ \text{Graph the line.} \]

Example 2

4. BOTANY Kudzu is a fast-growing vine found in the southeastern United States. An initial measurement of the length of a kudzu vine was 0.5 meter. Seven days later the plant was 4 meters long.

a. Graph the line that models the length of the plant over time.

b. What is the slope of your graph? What does it represent?

c. Assuming that the growth rate of the plant continues, how long will the plant be after 15 days?

Example 3

Determine whether \( \overrightarrow{WX} \) and \( \overrightarrow{YZ} \) are parallel, perpendicular, or neither. Graph each line to verify your answer.

5. \( W(2, 4), X(4, 5), Y(4, 1), Z(8, –7) \)

6. \( W(1, 3), X(–2, –5), Y(–6, –2), Z(8, 3) \)

7. \( W(–7, 6), X(–6, 9), Y(6, 3), Z(3, –6) \)

8. \( W(1, –3), X(0, 2), Y(–2, 0), Z(8, 2) \)

Example 4

Graph the line that satisfies each condition.

9. passes through \( A(3, –4) \), parallel to \( \overrightarrow{BC} \) with \( B(2, 4) \) and \( C(5, 6) \)

10. slope = 3, passes through \( A(–1, 4) \)

11. passes through \( P(7, 3) \), perpendicular to \( \overrightarrow{LM} \) with \( L(–2, –3) \) and \( M(–1, 5) \)
Find the slope of each line.

12. $\frac{y}{x}$

13. $\frac{y}{x}$

14. $\frac{y}{x}$

15. $\frac{y}{x}$

16. $\frac{y}{x}$

17. $\frac{y}{x}$

Determine the slope of the line that contains the given points.

18. C(3, 1), D(−2, 1)

19. E(5, −1), F(2, −4)

20. G(−4, 3), H(−4, 7)

21. J(−3, −3), K(−8, −3)

22. L(8, −3), M(−4, −12)

23. P(−3, −5), Q(−3, −1)

24. R(2, −6), S(−6, 5)

25. T(−6, −11), V(−12, −10)

26. MOUNTAIN BIKING In 2004, 8 million Americans over the age of 7 participated in mountain biking, and in 2006, 8.5 million participated.

a. Create a graph to show the number of participants in mountain biking based on the change in participation from 2004 to 2006.

b. Based on the data, what is the growth per year of the sport?

c. If participation continues at the same rate, what will be the participation in 2013 to the nearest 10,000?


a. Graph a trend line to predict the price of the MP3 player for 2003 through 2007.

b. Based on the data, how much does the price drop per year?

c. If the trend continues, what will be the cost of an MP3 player in 2010?

28. A(1, 5), B(4, 4), C(9, −10), D(−6, −5)

29. A(−6, −9), B(8, 19), C(0, −4), D(2, 0)

30. A(4, 2), B(−3, 1), C(6, 0), D(−10, 8)

31. A(8, −2), B(4, −1), C(3, 11), D(−2, −9)

32. A(8, 4), B(4, 3), C(4, −9), D(2, −1)

33. A(4, −2), B(−2, −8), C(4, 6), D(8, 5)
Example 4  

Graph the line that satisfies each condition.

34. passes through $A(2, -5)$, parallel to $\overrightarrow{BC}$ with $B(1, 3)$ and $C(4, 5)$

35. slope $= -2$, passes through $H(-2, -4)$

36. passes through $K(3, 7)$, perpendicular to $\overrightarrow{LM}$ with $L(-1, -2)$ and $M(-4, 8)$

37. passes through $X(1, -4)$, parallel to $\overrightarrow{YZ}$ with $Y(5, 2)$ and $Z(-3, -5)$

38. slope $= \frac{2}{3}$, passes through $J(-5, 4)$

39. passes through $D(-5, -6)$, perpendicular to $\overrightarrow{FG}$ with $F(-2, -9)$ and $G(1, -5)$

40. **STADIUMS** The RCA Dome is home to the Indianapolis Colts. The attendance in 2001 was 450,746, and the attendance in 2005 was 457,373.

   a. What is the approximate rate of change in attendance from 2001 to 2005?
   
   b. If this rate of change continues, predict the attendance for 2012.
   
   c. Will the attendance continue to increase indefinitely? Explain.
   
   d. The Colts are in the process of building a new, larger stadium. Do you think their decision is reasonable? Why or why not?

Determine which line passing through the given points has a steeper slope.

41. Line 1: $(0, 5)$ and $(6, 1)$
   Line 2: $(-4, 10)$ and $(8, -5)$

42. Line 1: $(0, -4)$ and $(2, 2)$
   Line 2: $(0, -4)$ and $(4, 5)$

43. Line 1: $(-6, 7)$ and $(9, -3)$
   Line 2: $(-9, 9)$ and $(3, 5)$

44. Line 1: $(-9, -4)$ and $(7, 0)$
   Line 2: $(0, 1)$ and $(7, 4)$

45. **ENDANGERED SPECIES** Michigan provides habitat for two endangered species, the bald eagle and the gray wolf. The graph shows the Michigan population of each species in 1992 and 2006.

   a. Which species experienced a greater rate of change in population?
   
   b. Make a line graph showing the growth of both populations.
   
   c. If both species continue to grow at their respective rates, what will the population of each species be in 2012?

Find the value of $x$ or $y$ that satisfies the given conditions. Then graph the line.

46. The line containing $(4, -1)$ and $(x, -6)$ has a slope of $-\frac{5}{2}$.

47. The line containing $(-4, 9)$ and $(4, 3)$ is parallel to the line containing $(-8, 1)$ and $(4, y)$.

48. The line containing $(8, 7)$ and $(7, -6)$ is perpendicular to the line containing $(2, 4)$ and $(x, 3)$.

49. The line containing $(1, -3)$ and $(3, y)$ is parallel to the line containing $(5, -6)$ and $(9, y)$.

50. **SCHOOLS** In 2000, Jefferson High School had 1125 students. By 2006, the student body had increased to 1425 students. When Fairview High School was built in 2001, it had 1275 students. How many students did Fairview High School have in 2006 if the student body grew at the same rate as Jefferson High School?
Maggie and Mikayla want to go to the music store near Maggie’s house after school. They can walk 3.5 miles per hour and ride their bikes 10 miles per hour.

a. Create a table to show how far Maggie and Mikayla can travel walking and riding their bikes. Include distances for 0, 1, 2, 3, and 4 hours.

b. Create a graph to show how far Maggie and Mikayla can travel based on time for both walking and riding their bikes. Be sure to label the axes of your graph.

c. What does the slope represent in your graph?

d. Maggie’s mom says they can only go if they can make it to the music store and back in less than two hours. If they want to spend at least 30 minutes in the music store and it is four miles away, can they make it? Should they walk or ride their bikes? Explain your reasoning.

H.O.T. Problems

52. WRITE A QUESTION A classmate says that all lines have positive or negative slope. Write a question that would challenge his conjecture.

53. FIND THE ERROR Terrell and Hale calculated the slope of the line passing through the points Q(3, 5) and R(−2, 2). Is either of them correct? Explain your reasoning.

\[
\begin{align*}
\text{Terrell} & : m = \frac{5 - 2}{3 - (-2)} = \frac{3}{5} \\
\text{Hale} & : m = \frac{5 - 2}{-2 - 2} = \frac{-3}{5}
\end{align*}
\]

54. REASONING Draw a square \(ABCD\) with vertices at \(A(2, -4)\) and \(C(10, 4)\).

a. Find the other two vertices of the square and label them \(B\) and \(D\).

b. Show that \(\overline{AD} \parallel \overline{BC}\) and \(\overline{AB} \parallel \overline{DC}\).

c. Show that the measure of each angle inside the square is equal to 90°.

55. WRITING IN MATH Describe the slopes of the Sears Tower and the Leaning Tower of Pisa.

56. CHALLENGE In this lesson you learned that \(m = \frac{y_2 - y_1}{x_2 - x_1}\). Use an algebraic proof to show that the slope can also be calculated using the equation \(m = \frac{y_1 - y_2}{x_1 - x_2}\).

57. WRITING IN MATH Find two additional points that lie along the same line as \(X(3, -1)\) and \(Y(-1, 7)\). Generalize a method you can use to find additional points on the line from any given point.
58. The graph of which equation passes through \((-3, -2)\) and is perpendicular to the graph of \(y = \frac{3}{4}x + 8\)?
   A. \(y = -\frac{4}{3}x - 6\)
   B. \(y = -\frac{4}{3}x + 5\)
   C. \(y = \frac{3}{4}x + \frac{1}{4}\)
   D. \(y = -\frac{3}{4}x - 5\)

59. SHORT RESPONSE A set of 25 cards is randomly placed face down on a table. 15 cards have only the letter A written on the face, and 10 cards have only the letter B. Patrick turned over 1 card. What are the odds of this card having the letter B written on its face?

60. ALGEBRA Jamie is collecting money to buy an $81 gift for her teacher. She has already contributed $24. She will collect $3 from each contributing student. From how many students must Jamie collect money?
   F. 3 students
   G. 9 students
   H. 12 students
   J. 19 students

61. SAT/ACT The area of a circle is \(20\pi\) square centimeters. What is its circumference?
   A. \(\sqrt{5}\pi\) cm
   B. \(2\sqrt{5}\pi\) cm
   C. \(4\sqrt{5}\pi\) cm
   D. \(20\pi\) cm

Spiral Review

In the figure, \(a \parallel b\), \(c \parallel d\), and \(m\angle 4 = 57\).

Find the measure of each angle. (Lesson 3-2)

62. \(\angle 5\)

63. \(\angle 1\)

64. \(\angle 8\)

65. \(\angle 10\)

Refer to the diagram at the right. (Lesson 3-1)

66. Name all segments parallel to \(TU\).

67. Name all planes intersecting plane \(BCR\).

68. Name all segments skew to \(DE\).

Determine whether the stated conclusion is valid based on the given information. If not, write invalid. Explain your reasoning. (Lesson 2-4)

69. Given: \(\angle B\) and \(\angle C\) are vertical angles.
   Conclusion: \(\angle B \cong \angle C\)

70. Given: \(\angle W \cong \angle Y\)
   Conclusion: \(\angle W\) and \(\angle Y\) are vertical angles.

71. CONSTRUCTION There are four buildings on the Mansfield High School Campus, no three of which stand in a straight line. How many sidewalks need to be built so that each building is directly connected to every other building? (Lesson 2-6)

Skills Review

Solve for \(y\).

72. \(3x + y = 5\)

73. \(4x + 2y = 6\)

74. \(4y - 3x = 5\)
 Identify the transversal connecting each pair of angles. Then classify the relationship between each pair of angles as alternate interior, alternate exterior, corresponding, or consecutive interior angles. (Lesson 3-1)  

1. $\angle 6$ and $\angle 3$  
2. $\angle 1$ and $\angle 14$  
3. $\angle 10$ and $\angle 11$  
4. $\angle 5$ and $\angle 7$  

Refer to the figure to identify each of the following. (Lesson 3-1)  

5. A plane parallel to plane $ABCD$  
6. A segment skew to $\overline{GH}$ that contains point $D$  
7. All segments parallel to $\overline{HE}$  

8. **MULTIPLE CHOICE** Which term best describes $\angle 4$ and $\angle 8$? (Lesson 3-1)  

A corresponding  
B alternate exterior  
C alternate interior  
D consecutive interior  

In the figure, $m\angle 4 = 104$, $m\angle 14 = 118$. Find the measure of each angle. Tell which postulate(s) or theorem(s) you used. (Lesson 3-2)  

9. $\angle 2$  
10. $\angle 9$  
11. $\angle 10$  
12. $\angle 7$  

13. Find $x$. (Lesson 3-2)  

14. **MODEL TRAINS** Amy is going to set up two parallel train tracks with a third track running diagonally across the first two. To properly place a switch, she needs the angle between the diagonal and the top of the second track to be twice as large as the angle between the diagonal and top of the first track. What is the value of $x$? (Lesson 3-2)  

Determine whether $\overrightarrow{AB}$ and $\overrightarrow{XY}$ are parallel, perpendicular, or neither. Graph each line to verify your answer. (Lesson 3-3)  

15. $A(2, 0), B(4, -5), X(-3, 3), Y(-5, 8)$  
16. $A(1, 1), B(6, -9), X(4, -10), Y(7, -4)$  

Find the slope of each line. (Lesson 3-3)  

17. line $\ell$  
18. A line parallel to $m$  
19. A line perpendicular to $n$  

20. **SALES** The 2001 and 2004 sales figures for Vaughn Electronics are in the table below. (Lesson 3-3)  

<table>
<thead>
<tr>
<th>Year</th>
<th>Approximate Sales ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003</td>
<td>240,000</td>
</tr>
<tr>
<td>2006</td>
<td>330,000</td>
</tr>
</tbody>
</table>

a. What is the rate of change in approximate sales from 2003 to 2006?  
b. If this rate of change continues, predict the approximate sales for the year 2010.
Equations of Lines

Why?

On an interstate near Lauren’s hometown, the minimum fine for speeding ten or fewer miles per hour over the speed limit of 65 miles per hour is $42.50. There is an additional charge of $2 for each mile per hour over this initial ten miles per hour. The total charge, not including court costs, can be represented by the equation $C = 42.5 + 2m$.

Write Equations of Lines

You may remember from algebra that an equation of a nonvertical line can be written in different but equivalent forms.

**Key Concept**

**Nonvertical Line Equations**

The **slope-intercept form** of a linear equation is $y = mx + b$, where $m$ is the slope of the line and $b$ is the $y$-intercept.

The **point-slope form** of a linear equation is $y - y_1 = m(x - x_1)$, where $(x_1, y_1)$ is any point on the line and $m$ is the slope of the line.

When given the slope and either the $y$-intercept or a point on a line, you can use these forms to write the equation of the line.

**EXAMPLE 1**

**Slope and $y$-intercept**

Write an equation in slope-intercept form of the line with slope 3 and $y$-intercept of $-2$. Then graph the line.

$y = mx + b$  
$y = 3x + (-2)$  
$m = 3$, $b = -2$  
$y = 3x - 2$  
Simplify.

Plot a point at the $y$-intercept, $-2$. Use the slope of 3 or $\frac{3}{1}$ to find another point 3 units up and 1 unit to the right of the $y$-intercept. Then draw the line through these two points.

**Check Your Progress**

1. Write an equation in slope-intercept form of the line with slope $\frac{1}{2}$ and $y$-intercept of 8. Then graph the line.
Lesson 3-4 Equations of Lines

**Watch Out!**

**Substituting Negative Coordinates**  When substituting negative coordinates, use parentheses to avoid making errors with the signs.

**EXAMPLE 2**  **Slope and a Point on the Line**

Write an equation in point-slope form of the line with slope $-\frac{3}{4}$ that contains $(-2, 5)$. Then graph the line.

\[
y - y_1 = m(x - x_1) \quad \text{Point-Slope form}
\]

\[
y - 5 = -\frac{3}{4}(x - (-2)) \quad m = -\frac{3}{4}, (x_1, y_1) = (-2, 5)
\]

Simplify.

Graph the given point $(-2, 5)$. Use the slope $-\frac{3}{4}$ or $\frac{-3}{4}$ to find another point $3$ units down and $4$ units to the right. Then draw the line through these two points.

**Check Your Progress**

2. Write an equation in point-slope form of the line with slope of $4$ that contains $(-3, -6)$. Then graph the line.

When the slope of a line is not given, use two points on the line to calculate the slope. Then use the point-slope or slope-intercept form to write an equation of the line.

**EXAMPLE 3**  **Two Points**

Write an equation of the line through each pair of points in slope-intercept form.

a. $(0, 3)$ and $(-2, -1)$

Step 1  Find the slope of the line through the points.

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 3}{-2 - 0} = \frac{-4}{-2} = 2 \quad \text{or} \quad 2
\]

Use the Slope Formula.

Step 2  Write an equation of the line.

\[
y = mx + b
\]

\[
y = 2x + 3
\]

Slope-Intercept form

$m = 2; (0, 3)$ is the $y$-intercept.

b. $(-7, 4)$ and $(9, -4)$

Step 1  \[m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 4}{9 - (-7)} = \frac{-8}{16} = -\frac{1}{2}\]

Use the Slope Formula.

Step 2  \[y - y_1 = m(x - x_1)\]

\[
y - 4 = -\frac{1}{2}[x - (-7)]
\]

\[
y - 4 = -\frac{1}{2}(x + 7)
\]

Simplify.

\[
y - 4 = -\frac{1}{2}x - \frac{7}{2}
\]

Distribute.

\[
y = -\frac{1}{2}x + \frac{1}{2}
\]

**Check Your Progress**

3A. $(-2, 4)$ and $(8, 10)$  3B. $(-1, 3)$ and $(7, 3)$

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EXAMPLE 4  **Horizontal Line**

Write an equation of the line through \((-2, 6)\) and \((5, 6)\) in slope-intercept form.

**Step 1**

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 6}{5 - (-2)} = \frac{0}{7} = 0
\]

This is a horizontal line.

**Step 2**

\[
y - y_1 = m(x - x_1)
\]

Point-Slope form

\[
y - 6 = 0[x - (-2)]
\]

Simplify.

\[
y - 6 = 0
\]

Add 6 to each side.

\[
y = 6
\]

The equations of horizontal and vertical lines involve only one variable.

---

**Check Your Progress**

4. Write an equation of the line through \((5, 0)\) and \((0, -1)\) in slope-intercept form.

---

**Key Concept**  **Horizontal and Vertical Line Equations**

The equation of a horizontal line is \(y = b\), where \(b\) is the \(y\)-intercept of the line.

**Example** \(y = -3\)

The equation of a vertical line is \(x = a\), where \(a\) is the \(x\)-intercept of the line.

**Example** \(x = -2\)

Parallel lines that are not vertical have equal slopes. Two nonvertical lines are perpendicular if the product of their slope is \(-1\). Vertical and horizontal lines are always perpendicular to one another.

---

**EXAMPLE 5  Write Parallel or Perpendicular Equations of Lines**

Write an equation in slope-intercept form for a line perpendicular to the line \(y = -3x + 2\) through \((4, 0)\).

The slope of \(y = -3x + 2\) is \(-3\), so the slope of a line perpendicular to it is \(\frac{1}{3}\).

\[
y = mx + b \quad \text{Slope-Intercept form}
\]

\[
0 = \frac{1}{3}(4) + b \quad m = \frac{1}{3} \text{ and } (x, y) = (4, 0)
\]

\[
0 = \frac{4}{3} + b \quad \text{Simplify.}
\]

\[
-\frac{4}{3} = b \quad \text{Subtract } \frac{4}{3} \text{ from each side.}
\]

So the equation is \(y = \frac{1}{3}x + \left( -\frac{4}{3} \right)\) or \(y = \frac{1}{3}x - 1\frac{1}{3}\).

---

**Check Your Progress**

5. Write an equation in slope-intercept form for a line parallel to \(y = -\frac{3}{4}x + 3\) and containing \((-3, 6)\).
Write Equations to Solve Problems  Many real-world situations can be modeled using a linear equation.

**Real-World Example 6  Write Linear Equations**

**Text Messaging**  Benito’s current wireless phone plan, Plan X, costs $39.95 per month for unlimited calls and $0.05 per text message. He is considering switching to a Plan Y that costs $35 per month for unlimited calls plus $0.10 for each text message. Which plan offers him the better rate?

**Understand**  Plan X costs $39.95 per month plus $0.05 per text message. Plan Y costs $35 per month plus $0.10 per text message. You want to compare the two plans to determine when the cost of one plan is less than the other.

**Plan**  Write an equation to model the total monthly cost $C$ of each plan for $t$ text messages sent or received. Then graph the equations in order to compare the two plans.

**Solve**  The rates of increase, or slopes $m$, in the total costs are 0.05 for Plan X and 0.10 for Plan Y. When the number of text messages is 0, the total charge is just the monthly fee. So the $y$-intercept $b$ is 39.95 for Plan X and 35 for Plan Y.

**Plan X**  
\[ C = mt + b \]  
Slope-intercept form  
\[ C = 0.05t + 39.95 \]  
Substitute for $m$ and $b$.  

**Plan Y**  
\[ C = mt + b \]  
\[ C = 0.10t + 35 \]

Graph the two equations on the same coordinate plane.

From the graph, it appears that if Benito sends or receives less than about 100 text messages, Plan Y offers the lower rate. For more than 100 messages, Plan X is lower.

**Check**  Check your estimate. For 100 text messages, Plan X costs $0.05(100) + 39.95$ or $44.95$, and Plan Y costs $0.10(100) + 35$ or $45$. Adjusting our estimate, we find that when the number of messages is 99, both plans cost $44.90.

**Check Your Progress**

6. Suppose the rate for Plan Y was $44 a month and $0.02 per text message. Which plan offers him the better rate? Justify your answer.
Check Your Understanding

Example 1  
Write an equation in slope-intercept form of the line having the given slope and y-intercept. Then graph the line.

1. \( m: 4, \ y\)-intercept: \(-3\)  
2. \( m: \frac{1}{2}, \ y\)-intercept: \(-1\)  
3. \( m: -\frac{2}{3}, \ y\)-intercept: \(5\)

Example 2  
Write an equation in point-slope form of the line having the given slope that contains the given point. Then graph the line.

4. \( m = 5, \ (3, -2)\)  
5. \( m = \frac{1}{4}, \ (-2, -3)\)  
6. \( m = -4.25, \ (-4, 6)\)

Examples 3 and 4  
Write an equation of the line through each pair of points in slope-intercept form.

7. \((0, -1)\) and \((4, 4)\)  
8. \((4, 3)\) and \((1, -6)\)  
9. \((6, 5)\) and \((-1, -4)\)

Example 5  
Write an equation in slope-intercept form for a line containing \((3, 2)\) that is perpendicular to the line with equation \(y = -2x + 6\).

10. Write an equation in slope-intercept form for a line containing \((-1, 5)\) that is parallel to the line with equation \(y = 4x - 5\).

Example 6  
MUSIC  
Kameko currently subscribes to Ace Music, an online music service, but she is considering switching to another online service, Orange Tunes. The plan for each online music service is described below.

\[ \begin{align*} 
\text{ACE Music} & : \text{Subscription fee} \$5/\text{mo} \\
& : \text{\$0.79 per song downloaded} \\
\text{ORANGE TUNES} & : 40 \text{ downloads per month} \\
& : \text{\$10/\text{mo} Subscription fee} 
\end{align*} \]

a. Write an equation to represent the total monthly cost for each plan.

b. Graph the equations.

c. If Kameko downloads 15 songs per month, should she keep her current plan, or change to the other plan? Explain.

Practice and Problem Solving

Example 1  
Write an equation in slope-intercept form of the line having the given slope and y-intercept or given points. Then graph the line.

13. \( m: -5, \ y\)-intercept: \(-2\)  
14. \( m: -7, \ y\)-intercept: \(-4\)  
15. \( m: 9, \ y\)-intercept: \(-2\)  
16. \( m: 12, \ y\)-intercept: \(\frac{4}{5}\)  
17. \( m: -\frac{3}{4}, \ (0, 4)\)  
18. \( m: \frac{5}{11}, \ (0, -3)\)

Example 2  
Write an equation in point-slope form of the line having the given slope that contains the given point. Then graph the line.

19. \( m = 2, \ (3, 11)\)  
20. \( m = 4, \ (-4, 8)\)  
21. \( m = -7, \ (1, 9)\)  
22. \( m = \frac{5}{7}, \ (-2, -5)\)  
23. \( m = -\frac{4}{5}, \ (-3, -6)\)  
24. \( m = -2.4, \ (14, -12)\)

Examples 3 and 4  
Write an equation of the line through each pair of points in slope-intercept form.

25. \((-1, -4)\) and \((3, -4)\)  
26. \((2, -1)\) and \((2, 6)\)  
27. \((-3, -2)\) and \((-3, 4)\)  
28. \((0, 5)\) and \((3, 3)\)  
29. \((-12, -6)\) and \((8, 9)\)  
30. \((2, 4)\) and \((-4, -11)\)
Write an equation in slope-intercept form for each line shown or described.

31. $EF$

32. $MN$

33. contains $(-1, -2)$ and $(3, 4)$

34. contains $(-4, -5)$ and $(-8, -13)$

35. $x$-intercept $= 3$, $y$-intercept $= -2$

36. $x$-intercept $= -\frac{1}{2}$, $y$-intercept $= 4$

Write an equation in slope-intercept form for each line described.

37. passes through $(-7, -4)$, perpendicular to $y = \frac{1}{2}x + 9$

38. passes through $(-1, -10)$, parallel to $y = 7$

39. passes through $(6, 2)$, parallel to $y = -\frac{2}{3}x + 1$

40. passes through $(-2, 2)$, perpendicular to $y = -5x - 8$

Example 5
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Example 6
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41. PLANNING Karen is planning a graduation party for the senior class. She plans to rent a meeting room at the convention center that costs $400. There is an additional fee of $5.50 for each person who attends the party.

a. Write an equation to represent the cost $y$ of the party if $x$ people attend.

b. Graph the equation.

c. There are 285 people in Karen’s class. If $\frac{2}{3}$ of these people attend, how much will the party cost?

d. If the senior class has raised $2000 for the party, how many people can attend?

42. SAVINGS Victor is saving his money to buy a new satellite radio for his car. He wants to save enough money for the radio and one year of satellite radio service before he buys it. He started saving for the radio with $\$50$ that he got for his birthday. Since then, he has been adding $\$15$ every week after he cashes his paycheck.

a. Write an equation to represent Victor’s savings $y$ after $x$ weeks.

b. Graph the equation.

c. How long will it take Victor to save $\$150$?

d. A satellite radio costs $\$180$. Satellite radio service costs $\$10$ per month. If Victor started saving two weeks ago, how much longer will it take him to save enough money? Explain.

Whereas FM radio signals can only broadcast about 30 to 40 miles, satellite radio can broadcast its signal more than 22,000 miles.

Source: How Stuff Works

Name the line(s) on the graph shown that match each description.

43. parallel to $y = 2x - 3$

44. perpendicular to $y = \frac{1}{2}x + 7$

45. intersecting but not perpendicular to $y = \frac{1}{2}x - 5$

Determine whether the lines are parallel, perpendicular, or neither.

46. $y = 2x + 4$, $y = 2x - 10$

47. $y = -\frac{1}{2}x - 12$, $y = 2x + 7$

48. $y - 4 = 3(x + 5)$, $y + 3 = -\frac{1}{3}(x + 1)$

49. $y - 3 = 6(x + 2)$, $y + 3 = -\frac{1}{3}(x - 4)$

Lesson 3-4 Equations of Lines 201
50. Write an equation in slope-intercept form for a line containing (4, 2) that is parallel to the line $y - 2 = 3(x + 7)$.

51. Write an equation for a line containing $(-8, 12)$ that is perpendicular to the line containing the points $(3, 2)$ and $(-7, 2)$.

52. Write an equation in slope-intercept form for a line containing $(5, 3)$ that is parallel to the line $y + 11 = \frac{1}{2}(4x + 6)$.

53. **POTTERY** A local community arts center offers pottery classes. A $40 enrollment fee covers supplies and materials, including one ten-pound bag of clay. Any extra bags of clay cost $15 each. Write an equation to represent the cost of the class and $x$ bags of clay.

54. **MULTIPLE REPRESENTATIONS** Ms. Smith has asked Dan to work on Friday night valet parking cars for a birthday party. Ms. Smith has given Dan two payment options: she can choose to be paid $3 per car or a $75 fee plus $1.50 per car.
   a. **TABULAR** Create tables to show how much Dan would be paid for 20, 50, and 100 cars for both options.
   b. **NUMERICAL** Write an equation to represent Dan’s earnings for each payment option.
   c. **GRAPHICAL** Graph the equations for both options.
   d. **ANALYTICAL** If 35 people attend the party, which option is more profitable? If 75 people attend? Explain.
   e. **VERBAL** Write a statement to describe which payment option is more profitable based on the number of cars that Dan parks.
   f. **LOGICAL** If Ms. Smith sent out 50 invitations, which option should Dan choose? Explain your reasoning.

55. **CHALLENGE** Find the value of $n$ so that the line perpendicular to the line with the equation $-2y + 4 = 6x + 8$ passes through the points at $(n, -4)$ and $(2, -8)$.

56. **REASONING** Determine whether the points at $(-2, 2), (2, 5),$ and $(6, 8)$ are collinear. Justify your answer.

57. **OPEN ENDED** Write equations for two different pairs of perpendicular lines that intersect at the point at $(-3, -7)$.

58. **FIND THE ERROR** Mark and Josefina wrote an equation of a line with slope $-5$ that passes through the point at $(-2, 4)$. Is either of them correct? Explain your reasoning.

Mark
\[
\begin{align*}
y - 4 &= -5(x - (-2)) \\
y - 4 &= -5(x + 2) \\
y - 4 &= -5x - 10 \\
y &= -5x - 6
\end{align*}
\]

Josefina
\[
\begin{align*}
y - 4 &= -5(x - (-2)) \\
y - 4 &= -5(x + 2)
\end{align*}
\]

59. **WRITING IN MATH** When is it easier to use the point-slope form to write an equation of a line and when is it easier to use the slope-intercept form?
60. Which graph best represents a line passing through the point \((-2, -3)\)?

A

B

C

D

61. Which equation describes the line that passes through the point at \((-2, 1)\) and is perpendicular to the line \(y = \frac{1}{3}x + 5\)?

F  \(y = 3x + 7\)

G  \(y = \frac{1}{3}x + 7\)

H  \(y = -3x - 5\)

J  \(y = -\frac{1}{3}x - 5\)

62. GRIDDED RESPONSE At Jefferson College, 80% of students have cell phones. Of the students who have cell phones, 70% have computers. What percent of the students at Jefferson College have both a cell phone and a computer?

63. SAT/ACT Which expression is equivalent to \(4(x - 6) - \frac{1}{2}(x^2 + 8)\)?

A  \(4x^2 + 4x - 28\)

B  \(-\frac{1}{2}x^2 + 4x - 28\)

C  \(-\frac{1}{2}x^2 + 6x - 24\)

D  \(3x - 20\)

---

Spiral Review

Determine the slope of the line that contains the given points. (Lesson 3-3)

64. \(J(4, 3), K(5, -2)\)

65. \(X(0, 2), Y(-3, -4)\)

66. \(A(2, 5), B(5, 1)\)

Find \(x\) and \(y\) in each figure. (Lesson 3-2)

67. \((6y + 10)^\circ, (8x - 12)^\circ, 4x^\circ\)

68. \((4x - 8)^\circ, (y^2)^\circ, 2x^\circ, 48^\circ\)

69. DRIVING Lacy’s home is located at the midpoint between Newman’s Gas Station and Gas-O-Rama. Newman’s Gas Station is a quarter mile away from Lacy’s home. How far away is Gas-O-Rama from Lacy’s home? How far apart are the two gas stations? (Lesson 1-3)

---

Skills Review

Determine the relationship between each pair of angles. (Lesson 3-1)

70. \(\angle 1 \text{ and } \angle 12\)

71. \(\angle 7 \text{ and } \angle 10\)

72. \(\angle 4 \text{ and } \angle 8\)

73. \(\angle 2 \text{ and } \angle 11\)
You can apply what you have learned about slope and equations of lines to geometric figures on a plane.

**ACTIVITY**

Find the equation of a line that is a perpendicular bisector of a segment $AB$ with endpoints $A(-3, 3)$ and $B(4, 0)$.

**Step 1** A segment bisector contains the midpoint of the segment. Use the Midpoint Formula to find the midpoint $M$ of $AB$.

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = M\left(\frac{-3 + 4}{2}, \frac{3 + 0}{2}\right) = M\left(\frac{1}{2}, \frac{3}{2}\right)$$

**Step 2** A perpendicular bisector is perpendicular to the segment through the midpoint. In order to find the slope of the bisector, first find the slope of $AB$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0 - 3}{4 - (-3)}$$

$$= \frac{-3}{7}$$

$x_1 = -5, x_2 = 1, y_1 = 4, y_2 = 12$

Simplify.

**Step 3** Now use the point-slope form to write the equation of the line. The slope of the bisector is $\frac{7}{3}$ since $-\frac{3(\frac{2}{3})}{7} = -1$.

$$y - y_1 = m(x - x_1)$$

$$y - \frac{3}{2} = \frac{7}{3}(x - \frac{1}{2})$$

$$y - \frac{3}{2} = \frac{7}{3}x - \frac{7}{6}$$

Distributive Property

$$y = \frac{7}{3}x + \frac{1}{2}$$

Add $\frac{3}{2}$ to each side.

---

**Exercises**

Find the equation of the perpendicular bisector $\overline{PQ}$ for the given endpoints.

1. $P(5, 2), Q(7, 4)$
2. $P(-3, 9), Q(-1, 5)$
3. $P(-6, -1), Q(8, 7)$
4. $P(-2, 1), Q(0, -3)$
5. $P(0, 1.6), Q(0.5, 2.1)$
6. $P(-7, 3), Q(5, 3)$

7. Extend what you have learned to find the equations of the lines that contain the sides of $\triangle XYZ$ with vertices $X(-2, 0), Y(1, 3)$, and $Z(3, -1)$.
Proving Lines Parallel

Why?

When you see a roller coaster track, the two sides of the track are always the same distance apart, even though the track curves and turns. The tracks are carefully constructed to be parallel at all points so that the car is secure on the track.

Identify Parallel Lines

The two sides of the track of a roller coaster are parallel, and all of the supports along the track are also parallel. Each of the angles formed between the track and the supports are corresponding angles. We have learned that corresponding angles are congruent when lines are parallel. The converse of this relationship is also true.

Postulate 3.4

Converse of Corresponding Angles Postulate

If two lines are cut by a transversal so that corresponding angles are congruent, then the lines are parallel.

Examples

If \( \angle 1 \cong \angle 3, \angle 2 \cong \angle 4, \angle 5 \cong \angle 7, \angle 6 \cong \angle 8 \), then \( a \parallel b \).

The Converse of the Corresponding Angles Postulate can be used to construct parallel lines.

Construction

Parallel Line Through a Point Not on the Line

Step 1
Use a straightedge to draw \( AB \). Draw a point \( C \) that is not on \( AB \). Draw \( CA \).

Step 2
Copy \( \angle CAB \) so that \( C \) is the vertex of the new angle. Label the intersection points \( D \) and \( E \).

Step 3
Draw \( CD \). Because \( \angle ECD \cong \angle CAB \) by construction and they are corresponding angles, \( AB \parallel CD \).
Parallel lines that are cut by a transversal create several pairs of congruent angles. These special angle pairs can also be used to prove that a pair of lines are parallel.

**Theorems**

### Proving Lines Parallel

#### 3.5 Alternate Exterior Angles Converse

If two lines in a plane are cut by a transversal so that a pair of alternate exterior angles is congruent, then the two lines are parallel.

#### 3.6 Consecutive Interior Angles Converse

If two lines in a plane are cut by a transversal so that a pair of consecutive interior angles is supplementary, then the lines are parallel.

#### 3.7 Alternate Interior Angles Converse

If two lines in a plane are cut by a transversal so that a pair of alternate interior angles is congruent, then the lines are parallel.

#### 3.8 Perpendicular Transversal Converse

In a plane, if two lines are perpendicular to the same line, then they are parallel.

You will prove Theorems 3.5, 3.6, 3.7, and 3.8 in Exercises 6, 23, 31, and 30, respectively.

### EXAMPLE 1

**Identify Parallel Lines**

Given the following information, is it possible to prove that any of the lines shown are parallel? If so, state the postulate or theorem that justifies your answer.

**a.** \( \angle 1 \cong \angle 6 \)

\( \angle 1 \) and \( \angle 6 \) are alternate exterior angles of lines \( \ell \) and \( n \).

Since \( \angle 1 \cong \angle 6 \), \( \ell \parallel n \) by the Converse of the Alternate Exterior Angles Theorem.

**b.** \( \angle 2 \cong \angle 3 \)

\( \angle 2 \) and \( \angle 3 \) are alternate interior angles of lines \( \ell \) and \( m \).

Since \( \angle 2 \cong \angle 3 \), \( \ell \parallel m \) by the Converse of the Alternate Interior Angles Theorem.
Lesson 3-5  Proving Lines Parallel

**Test-Taking Tip**

**Finding What Is Asked For**

Be sure to reread test questions carefully to be sure you are answering the question that was asked. In Example 2, a common error would be to stop after you have found the value of $x$ and say that the solution of the problem is 14.

---

**KCCT EXAMPLE 2**  MA-HS-3.1.1

**OPEN ENDED** Find $m\angle MRQ$ so that \(a \parallel b\). Show your work.

**Read the Test Item**

From the figure, you know that $m\angle MRQ = 5x + 7$ and $m\angle RPN = 7x - 21$. You are asked to find the measure of $\angle MRQ$.

**Solve the Test Item**

$\angle MRQ$ and $\angle RPN$ are alternate interior angles. For lines $a$ and $b$ to be parallel, alternate interior angles must be congruent, so $\angle MRQ \cong \angle RPN$. By the definition of congruence, $m\angle MRQ = m\angle RPN$. Substitute the given angle measures into this equation and solve for $x$.

\[
m\angle MRQ = m\angle RPN
\]

\[
5x + 7 = 7x - 21
\]

\[
7 = 2x - 21
\]

\[
28 = 2x
\]

\[
x = 14
\]

Now, use the value of $x$ to find $\angle MRQ$.

\[
m\angle MRQ = 5x + 7
\]

\[
= 5(14) + 7
\]

\[
= 77
\]

**CHECK** Check your answer by using the value of $x$ to find $m\angle RPN$.

\[
m\angle RP = 7x - 21
\]

\[
= 7(14) - 21
\]

\[
= 77
\]

Since $m\angle MRQ = m\angle RPN$, $\angle MRQ \cong \angle RPN$ and $a \parallel b$. ✓

---

**Check Your Progress**

2. Find $y$ so that $e \parallel f$. Show your work.
Prove Lines Parallel The angle pair relationships formed by a transversal can be used to prove that two lines are parallel.

EXAMPLE 3 Prove Lines Parallel

HOME FURNISHINGS In the ladder shown, each rung is perpendicular to the two rails. Is it possible to prove that the two rails are parallel and that all of the rungs are parallel? If so, explain how. If not, explain why not.

Since both rails are perpendicular to each rung, the rails are parallel by the Perpendicular Transversal Converse. Since any pair of rungs is perpendicular to the rails, they are also parallel.

Check Your Progress

3. ROWING In order to move in a straight line with maximum efficiency, rower’s oars should be parallel. Refer to the photo at the right. Is it possible to prove that any of the oars are parallel? If so, explain how. If not, explain why not.

Check Your Understanding

Example 1 p. 206

Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.

1. \( \angle 1 \cong \angle 3 \)
2. \( \angle 2 \cong \angle 5 \)
3. \( \angle 3 \cong \angle 10 \)
4. \( m\angle 6 + m\angle 8 = 180 \)

Example 2 p. 207

5. SHORT RESPONSE Find \( x \) so that \( m || n \). Show your work.

Example 3 p. 208

6. PROOF Copy and complete the proof of Theorem 3.5.

Given: \( \angle 1 \cong \angle 2 \)

Prove: \( \ell \parallel m \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( \angle 1 \cong \angle 2 )</td>
<td>a. Given</td>
</tr>
<tr>
<td>b. ( \angle 2 \cong \angle 3 )</td>
<td>b. <em><strong>?</strong></em></td>
</tr>
<tr>
<td>c. ( \angle 1 \cong \angle 3 )</td>
<td>c. Transitive Property</td>
</tr>
<tr>
<td>d. <em><strong>?</strong></em></td>
<td>d. <em><strong>?</strong></em></td>
</tr>
</tbody>
</table>
7. RECREATION Is it possible to prove that the backrest and footrest of the lounging beach chair are parallel? If so, explain how. If not, explain why not.

Example 1 p. 206
Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.

8. \( \angle 1 \equiv \angle 2 \)
9. \( \angle 2 \equiv \angle 9 \)
10. \( \angle 5 \equiv \angle 7 \)
11. \( m\angle 7 + m\angle 8 = 180 \)
12. \( m\angle 3 + m\angle 6 = 180 \)
13. \( \angle 3 \equiv \angle 5 \)
14. \( \angle 3 \equiv \angle 7 \)
15. \( \angle 4 \equiv \angle 5 \)

Example 2 p. 207
Find \( x \) so that \( m \parallel n \). Identify the postulate or theorem you used.

16. \( m \) \( (3x - 14)^\circ \) \( n \) \( (2x + 25)^\circ \)
17. \( m \) \( (5x - 20)^\circ \) \( n \)
18. \( m \) \( (21 + 2x)^\circ \) \( n \) \( (x + 84)^\circ \)
19. \( m \) \( (7x - 2)^\circ \) \( n \) \( (10 - 3x)^\circ \)
20. \( m \) \( (2x + 45)^\circ \) \( n \) \( (3x)^\circ \)
21. \( m \) \( (6x - 144)^\circ \) \( n \) \( (2x)^\circ \)

22. FRAMING Wooden picture frames are often constructed using a miter box or miter saw. These tools allow you to cut at an angle of a given size. If each of the four pieces of framing material is cut at a 45° angle, will the sides of the frame be parallel? Explain your reasoning.

Example 3 p. 208
23. PROOF Copy and complete the proof of Theorem 3.6.

Given: \( \angle 1 \) and \( \angle 2 \) are supplementary.

Prove: \( \ell \parallel m \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. <em><strong><strong>?</strong></strong></em>__</td>
<td>a. Given</td>
</tr>
<tr>
<td>b. ( \angle 2 ) and ( \angle 3 ) form a linear pair.</td>
<td>b. <em><strong><strong>?</strong></strong></em>__</td>
</tr>
<tr>
<td>c. <em><strong><strong>?</strong></strong></em>__</td>
<td>c. <em><strong><strong>?</strong></strong></em>__</td>
</tr>
<tr>
<td>d. ( \angle 1 \equiv \angle 3 )</td>
<td>d. <em><strong><strong>?</strong></strong></em>__</td>
</tr>
<tr>
<td>e. ( \ell \parallel m )</td>
<td>e. <em><strong><strong>?</strong></strong></em>__</td>
</tr>
</tbody>
</table>
24. **CRAFTS** Jacqui is making a stained glass piece. She cuts the top and bottom pieces at a 30° angle. If the corners are right angles, explain how Jacqui knows that each pair of opposite sides are parallel.

**PROOF** Write a two-column proof for each of the following.

25. Given: \( \angle 1 \cong \angle 3 \)
\[ AC \parallel BD \]
Prove: \( \overline{AB} \parallel \overline{CD} \)

![Diagram](image1)

26. Given: \( \overline{WX} \parallel \overline{YZ} \)
\[ \angle 2 \cong \angle 3 \]
Prove: \( \overline{WY} \parallel \overline{XZ} \)

![Diagram](image2)

27. Given: \( \triangle ABC \cong \triangle ADC \)
\[ m\angle A + m\angle ABC = 180 \]
Prove: \( \overline{AB} \parallel \overline{CD} \)

![Diagram](image3)

28. Given: \( \angle 1 \cong \angle 2 \)
\[ \overline{LJ} \perp \overline{ML} \]
Prove: \( \overline{KM} \perp \overline{ML} \)

![Diagram](image4)

29. **MAILBOXES** Mail slots are used to make the organization and distribution of mail easier. In the mail slots shown, each slot is perpendicular to each of the sides. Explain why you can conclude that the slots are parallel.

30. **PROOF** Write a paragraph proof of Theorem 3.8.

31. **PROOF** Write a two-column proof of Theorem 3.7.

32. **STAIRS** Based upon the information given in the photo of the staircase at the right, what is the relationship between each step? Explain your answer.

Determine whether lines \( r \) and \( s \) are parallel. Justify your answer.

33. \[ r \]
\[ 32^\circ \]
\[ 32^\circ \]
\[ 64^\circ \]
34. \[ r \]
\[ s \]
\[ 57^\circ \]
\[ 41^\circ \]
\[ 23^\circ \]
\[ 75^\circ \]
35. \[ r \]
\[ s \]
\[ 65^\circ \]
\[ 27^\circ \]
\[ 92^\circ \]
36. **MUTIPLE REPRESENTATIONS** In this problem, you will explore the shortest distance between two parallel lines.

   a. **GEOMETRIC** Draw three sets of parallel lines \( k \) and \( \ell \), \( s \) and \( t \), and \( \chi \) and \( y \). Draw the shortest segment \( BC \) and label points \( A \) and \( D \) as shown below.

   \[
   \begin{array}{c}
   B \quad A \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \Quad...
44. Which of the following facts would be sufficient to prove that line \( d \) is parallel to \( XZ \)?

\[ \angle 1 \cong \angle 3 \quad \text{C} \quad \angle 1 \cong \angle Z \]
\[ \angle 3 \cong \angle Z \quad \text{D} \quad \angle 2 \cong \angle X \]

45. **ALGEBRA** The expression \( \sqrt{52} + \sqrt{117} \) is equivalent to

\[ F \quad 13 \quad \text{H} \quad 6\sqrt{13} \]
\[ G \quad 5\sqrt{13} \quad \text{J} \quad 13\sqrt{13} \]

46. What is the approximate surface area of the figure?

\[ \text{A} \quad 101.3 \text{ in}^2 \quad \text{C} \quad 202.5 \text{ in}^2 \]
\[ \text{B} \quad 108 \text{ in}^2 \quad \text{D} \quad 216 \text{ in}^2 \]

47. **SAT/ACT** If \( x^2 = 25 \) and \( y^2 = 9 \), what is the greatest possible value of \( (x - y)^2 \)?

\[ \text{F} \quad 4 \quad \text{H} \quad 64 \]
\[ \text{G} \quad 58 \quad \text{J} \quad 70 \]

---

**Spiral Review**

Write an equation in slope-intercept form of the line having the given slope and \( y \)-intercept. (Lesson 3-4)

48. \( m: 2.5 \), \((0, 0.5)\)

49. \( m: \frac{4}{5} \), \((0, -9)\)

50. \( m: -\frac{7}{8} \), \((0, -\frac{5}{6})\)

51. **ROAD TRIP** Anne is driving 400 miles to visit Niagara Falls. She manages to travel the first 100 miles of her trip in two hours. If she continues at this rate, how long will it take her to drive the remaining distance? (Lesson 3-3)

52. Given: \( \angle 1 \) and \( \angle 2 \) are complementary angles.
   **Conjecture:** \( \angle 1 \) and \( \angle 2 \) form a right angle.

53. Given: points \( W, X, Y, \) and \( Z \)
   **Conjecture:** \( W, X, Y, \) and \( Z \) are noncollinear.

Find the perimeter or circumference and area of each figure. Round to the nearest tenth. (Lesson 1-6)

54.

55.

56.

---

**Skills Review**

57. Find \( x \) and \( y \) so that \( \overline{BE} \) and \( \overline{AD} \) are perpendicular. (Lesson 1-5)
Perpendiculars and Distance

Why?

A **plumb bob** is made of string with a specially designed weight. When the weight is suspended and allowed to swing freely, the point of the bob is precisely below the point to which the string is fixed.

The plumb bob is useful in establishing what is the true vertical or **plumb** when constructing a wall or when hanging wallpaper.

**Distance From a Point to a Line**  The plumb bob also indicates the shortest distance between the point at which it is attached on the ceiling and a level floor below. This perpendicular distance between a point and a line is the shortest in all cases.

**Key Concept**

The distance between a line and a point not on the line is the length of the segment perpendicular to the line from the point.

**Perpendicular Postulate**

If given a line and a point not on the line, then there exists exactly one line through the point that is perpendicular to the given line.

**Distance Between a Point and a Line**

The construction of a line perpendicular to an existing line through a point not on the existing line in Extend Lesson 1-5 establishes that there is at least one line through a point \(P\) that is perpendicular to a line \(AB\). The following postulate states that this line is the only line through \(P\) perpendicular to \(AB\).
**Real-World Career**

**Landscape Architect**
Landscape architects enjoy working with their hands and possess strong analytical skills. Creative vision and artistic talent also are desirable qualities. Typically, a bachelor’s degree is required of landscape architects, but a master’s degree may be required for specializations such as golf course design.

---

**StudyTip**

**Drawing the Shortest Distance** You can use tools like the corner of a piece of paper to help you draw a perpendicular segment from a point to a line, but only a compass and a straightedge can be used to construct this segment.

---

**Example 1: Construct Distance From Point to a Line**

**Landscaping** A landscape architect notices that one part of a yard does not drain well. She wants to tap into an existing underground drain represented by line \( m \). Construct and name the segment with the length that represents the shortest amount of pipe she will need to lay to connect this drain to point \( A \).

1. The distance from a line to a point not on the line is the length of the segment perpendicular to the line from the point. Locate points \( B \) and \( C \) on line \( m \) equidistant from point \( A \).
2. Locate a second point not on line \( m \) equidistant from \( A \) and \( B \). Construct \( \overline{AD} \) so that \( \overline{AD} \perp \overline{BC} \).
3. The measure of \( \overline{AD} \) represents the shortest amount of pipe the architect will need to lay to connect the drain to point \( A \).

---

**Check Your Progress**

1. Copy the figure. Then construct and name the segment that represents the distance from \( Q \) to \( \overline{PR} \).
EXAMPLE 2  Distance from a Point to a Line on Coordinate Plane

COORDINATE GEOMETRY Line $\ell$ contains points at $(-5, 3)$ and $(4, -6)$. Find the distance between line $\ell$ and point $P(2, 4)$.

Step 1  Find the equation of the line $\ell$.

Begin by finding the slope of the line through points $(-5, 3)$ and $(4, -6)$.

\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - 3}{4 - (-5)} = \frac{-9}{9} = -1 \]

Then write the equation of this line using the point $(4, -6)$ on the line.

\[ y = mx + b \quad \text{Slope-intercept form} \]
\[ -6 = -1(4) + b \quad m = -1, (x, y) = (4, -6) \]
\[ -6 = -4 + b \quad \text{Simplify.} \]
\[ -2 = b \quad \text{Add 4 to each side.} \]

The equation of line $\ell$ is $y = -x - 2$ or $y = -x - 2$.

Step 2  Write an equation of the line $w$ perpendicular to line $\ell$ through $P(2, 4)$.

Since the slope of line $\ell$ is $-1$, the slope of a line $p$ is 1. Write the equation of line $w$ through $P(2, 4)$ with slope 1.

\[ y = mx + b \quad \text{Slope-intercept form} \]
\[ 4 = 1(2) + b \quad m = 1, (x, y) = (2, 4) \]
\[ 4 = 2 + b \quad \text{Simplify.} \]
\[ 2 = b \quad \text{Subtract 2 from each side.} \]

The equation of line $w$ is $y = x + 2$.

Step 3  Solve the system of equations to determine the point of intersection.

line $\ell$: \hspace{1cm} $y = -x - 2$

line $w$: \hspace{1cm} $(+) y = x + 2$

\[ 2y = 0 \quad \text{Add the two equations.} \]
\[ y = 0 \quad \text{Divide each side by 2.} \]

Solve for $x$.

\[ 0 = x + 2 \quad \text{Substitute 0 for y in the second equation.} \]
\[ -2 = x \quad \text{Subtract 2 from each side.} \]

The point of intersection is $(-2, 0)$. Let this be point $Q$.

Step 4  Use the Distance Formula to determine the distance between $P(2, 4)$ and $Q(-2, 0)$.

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance formula} \]
\[ = \sqrt{(-2 - 2)^2 + (0 - 4)^2} \]
\[ = \sqrt{32} \quad \text{Simplify.} \]

The distance between the point and the line is $\sqrt{32}$ or about 5.66 units.
Distance Between Parallel Lines  By definition, parallel lines do not intersect. An alternate definition states that two lines in a plane are parallel if they are everywhere equidistant. Equidistant means that the distance between two lines measured along a perpendicular line to the lines is always the same.

This leads us to the definition of the distance between two parallel lines.

Key Concept  Distance Between Parallel Lines

The distance between two parallel lines is the perpendicular distance between one of the lines and any point on the other line.

Recall from Lesson 1-1 that a locus is the set of all points that satisfy a given condition. Parallel lines can be described as the locus of points in a plane equidistant from a given line.

Theorem 3.9  Two Lines Equidistant from a Third

In a plane, if two lines are each equidistant from a third line, then the two lines are parallel to each other.

You will prove Theorem 3.9 in Exercise 30.

EXAMPLE 3  Distance Between Parallel Lines

Find the distance between the parallel lines $\ell$ and $m$ with equations $y = 2x + 1$ and $y = 2x - 3$, respectively.

You will need to solve a system of equations to find the endpoints of a segment that is perpendicular to both $\ell$ and $m$. From their equations, we know that the slope of line $\ell$ and line $m$ is 2.

Sketch line $p$ through the $y$-intercept of line $m$, $(0, -3)$, perpendicular to lines $m$ and $\ell$. 
Step 1  Write an equation of line \( p \). The slope of \( p \) is the opposite reciprocal of 2, or \( -\frac{1}{2} \). Use the \( y \)-intercept of line \( m \), \((0, -3)\), as one of the endpoints of the perpendicular segment.

\[
(y - y_1) = m(x - x_1) \quad \text{Point-slope form}
\]

\[
[y - (-3)] = -\frac{1}{2}(x - 0)
\]

\[
x_1 = 0, \ y_1 = 3, \ \text{and} \ m = -\frac{1}{2}
\]

\[
y + 3 = -\frac{1}{2}x
\]

Simplify.

\[
y = -\frac{1}{2}x - 3
\]

Subtract 3 from each side.

Step 2  Use a system of equations to determine the point of intersection of lines \( \ell \) and \( p \).

\( \ell \): \( y = 2x + 1 \)

\( p \): \( y = -\frac{1}{2}x - 3 \)

Substitute \( 2x + 1 \) for \( y \) in the second equation.

\[
2x + 1 = -\frac{1}{2}x - 3
\]

Group like terms on each side.

\[
\frac{5}{2}x = -4
\]

Simplify on each side.

\[
x = -\frac{8}{5}
\]

Multiply each side by \( \frac{2}{5} \).

\[
y = -\frac{1}{2}\left(-\frac{8}{5}\right) - 3
\]

Substitute \( -\frac{8}{5} \) for \( x \) in the equation for \( p \).

Simplify.

\[
y = \frac{11}{5}
\]

The point of intersection is \( \left(-\frac{8}{5}, \frac{11}{5}\right) \) or \((-1.6, -2.2)\).

Step 3  Use the Distance Formula to determine the distance between \((0, -3)\) and \((-1.6, -2.2)\).

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}
\]

\[
x_2 = -1.6, \ x_1 = 0, \ y_2 = -2.2, \ \text{and} \ y_1 = -3
\]

\[
d \approx 1.8
\]

Simplify using a calculator.

The distance between the lines is about 1.8 units.
Check Your Understanding

Copy each figure. Construct the segment that represents the distance indicated.

1. \( Y \) to \( \overline{TS} \)

2. \( C \) to \( \overline{AB} \)

3. MARCHING BAND  After forming a line, every even member of a marching band turns to face the home team’s end zone and marches 5 paces straight forward. At the same time, every odd member turns in the opposite direction and marches 5 paces straight forward. Assuming that each band member covers the same distance, what formation should result? Justify your answer.

COORDINATE GEOMETRY  Find the distance from \( P \) to \( \ell \).

4. Line \( \ell \) contains points \((4, 3)\) and \((-2, 0)\). Point \( P \) has coordinates \((3, 10)\).

5. Line \( \ell \) contains points \((-6, 1)\) and \((9, -4)\). Point \( P \) has coordinates \((4, 1)\).

6. Line \( \ell \) contains points \((4, 18)\) and \((-2, 9)\). Point \( P \) has coordinates \((-9, 5)\).

Find the distance between each pair of parallel lines with the given equations.

7. \( y = -2x + 4 \)

   \( y = -2x + 14 \)

8. \( y = 7 \)

   \( y = -3 \)

Practice and Problem Solving

Copy each figure. Construct the segment that represents the distance indicated.

9. \( Q \) to \( \overline{RS} \)

10. \( A \) to \( \overline{BC} \)

11. \( H \) to \( \overline{FG} \)

12. \( K \) to \( \overline{LM} \)
13. **DRIVEWAYS** In the diagram at the right, is the driveway shown the shortest possible one from the house to the road? Explain why or why not.

14. **SCHOOL** Rondell is crossing the courtyard in front of his school. Three possible paths are shown in the diagram at the right. Which of the three paths shown is the shortest? Explain your reasoning.

**Example 2**

**COORDINATE GEOMETRY** Find the distance from \( P \) to \( \ell \).

15. Line \( \ell \) contains points \((0, -3)\) and \((7, 4)\). Point \( P \) has coordinates \((4, 3)\).

16. Line \( \ell \) contains points \((11, -1)\) and \((-3, -11)\). Point \( P \) has coordinates \((-1, 1)\).

17. Line \( \ell \) contains points \((-2, 1)\) and \((4, 1)\). Point \( P \) has coordinates \((5, 7)\).

18. Line \( \ell \) contains points \((4, -1)\) and \((4, 9)\). Point \( P \) has coordinates \((1, 6)\).

19. Line \( \ell \) contains points \((1, 5)\) and \((4, -4)\). Point \( P \) has coordinates \((-1, 1)\).

20. Line \( \ell \) contains points \((-8, 1)\) and \((3, 1)\). Point \( P \) has coordinates \((-2, 4)\).

**Example 3**

Find the distance between each pair of parallel lines with the given equations.

21. \( y = -2 \)

22. \( x = 3 \)

23. \( y = 5x - 22 \)

24. \( y = \frac{1}{3}x - 3 \)

25. \( x = 7 \)

26. \( y = 5x + 4 \)

27. \( y = \frac{1}{3}x + 2 \)

28. \( 3x + y = 3 \)

29. \( y = \frac{1}{4}x + 3.5 \)

30. \( y = y \)

31. \( y = -3, (5, 2) \)

32. \( y = \frac{1}{6}x + 6, (-6, 5) \)

33. \( x = 4, (-2, 5) \)

34. **POSTERS** Alma is hanging two posters on the wall in her room as shown. How can Alma use perpendicular distances to confirm that the posters are parallel?
**SCHOOL SPIRIT** Brock is decorating a hallway bulletin board to display pictures of students demonstrating school spirit. He cuts off one length of border to match the board’s width, and then uses that strip as a template to cut a second strip that is exactly the same length for the bottom.

When stapling the bottom border in place, he notices that the strip he cut is about a quarter of an inch too short. Describe what he can conclude about the bulletin board. Explain your reasoning.

**CONSTRUCTION** Line \( \ell \) contains points at \((-4, 3)\) and \((2, -3)\). Point \( P \) at \((-2, 1)\) is on line \( \ell \). Complete the following construction.

**Step 1**
Graph line \( \ell \) and point \( P \), and put the compass at point \( P \). Using the same compass setting, draw arcs to the left and right of \( P \). Label these points \( A \) and \( B \).

**Step 2**
Open the compass to a setting greater than \( AP \). Put the compass at point \( A \) and draw an arc above line \( \ell \).

**Step 3**
Using the same compass setting, put the compass at point \( B \) and draw an arc above line \( \ell \). Label the point of intersection \( Q \). Then draw \( PQ \).

36. What is the relationship between line \( \ell \) and \( PQ \)? Verify your conjecture using the slopes of the two lines.

37. Repeat the construction above using a different line and point on that line.

38. **COORDINATE GEOMETRY** \( \overline{AB} \) has a slope of 2 and midpoint \( M(3, 2) \). A segment perpendicular to \( AB \) has midpoint \( P(4, -1) \) and shares endpoint \( B \) with \( AB \).
   a. Graph the segments.
   b. Find the coordinates of \( A \) and \( B \).

39. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the areas of triangles formed by points on parallel lines.
   a. **GEOMETRIC** Draw two parallel lines and label them as shown.

   ![Parallel Lines](image)

   b. **VERBAL** Where would you place point \( C \) on line \( m \) to ensure that triangle \( ABC \) would have the largest area? Explain your reasoning.

   c. **ANALYTICAL** If \( AB = 11 \) inches, what is the maximum area of \( \triangle ABC \)?
40. **PERPENDICULARITY AND PLANES** Make a copy of the diagram below to answer each question, marking the diagram with the given information.

![Diagram of perpendicular lines and planes]

a. If two lines are perpendicular to the same plane, then they are coplanar. If both line \(a\) and line \(b\) are perpendicular to plane \(P\), what must also be true?

b. If a plane intersects two parallel planes, then the intersections form two parallel lines. If planes \(R\) and \(Q\) are parallel and they intersect plane \(P\), what must also be true?

c. If two planes are perpendicular to the same line, then they are parallel. If both plane \(Q\) and plane \(R\) are perpendicular to line \(\ell\), what must also be true?

### H.O.T. Problems

**Use Higher-Order Thinking Skills**

41. **FIND THE ERROR** Han draws the segments \(\overline{AB}\) and \(\overline{CD}\) shown below using a straightedge. He claims that these two lines, if extended, will never intersect. Shenequa claims that they will. Is either of them correct? Justify your answer.

![Diagram of lines AB and CD]

42. **WRITING IN MATH** Describe a method that could be used to find the distance between a point and a plane.

43. **CHALLENGE** Suppose a line perpendicular to a pair of parallel lines intersects the lines at the points \((a, 4)\) and \((0, 6)\). If the distance between the parallel lines is \(\sqrt{5}\), find the value of \(a\) and the equations of the parallel lines.

44. **REASONING** Determine whether the following statement is *sometimes*, *always*, or *never* true. Explain.

   \[\text{The distance between a line and a plane can be found.}\]

45. **OPEN ENDED** Draw an irregular convex polygon using a straightedge.

   a. Construct the distance between one vertex and a nonadjacent side.

   b. Use measurement to justify that the angle you constructed is perpendicular to the side chosen. That is, justify that the line you constructed through the vertex is perpendicular to the side chosen.

   c. Use mathematics to justify this construction.

46. **CHALLENGE** Rewrite Theorem 3.9 in terms of two planes that are equidistant from a third plane. Sketch an example.

47. **WRITING IN MATH** Summarize the steps necessary to find the distance between a pair of parallel lines given the equations of the two lines.
48. **Extended Response** Segment $AB$ is perpendicular to segment $CD$. Segment $AB$ and segment $CD$ bisect each other at point $X$.
   a. Draw a figure to represent the problem.
   b. Find $BD$ if $AB = 12$ and $CD = 16$.
   c. Find $BD$ if $AB = 24$ and $CD = 18$.

49. A city park is square and has an area of 81,000 square feet. Which of the following is the closest to the length of one side of the park?
   - F 100 ft
   - H 300 ft
   - G 200 ft
   - J 400 ft

50. **Algebra** Pablo bought a sweater on sale for 25% off the original price and another 40% off the discounted price. If the sweater originally cost $48, what was the final price of the sweater?
   - A $14.40
   - B $21.60
   - C $31.20
   - D $36.00

51. **SAT/ACT** After $N$ cookies are divided equally among 8 children, 3 remain. How many would remain if $(N + 6)$ cookies were divided equally among the 8 children?
   - F 0
   - H 2
   - G 1
   - J 4

52. Refer to the figure at the right. Determine whether $a \parallel b$. Justify your answer. (Lesson 3-5)

Write an equation in point-slope form of the line having the given slope that contains the given point. (Lesson 3-4)

- **53.** $m = \frac{1}{4}$, $(3, -1)$
- **54.** $m = 0$, $(-2, 6)$
- **55.** $m = -1$, $(-2, 3)$
- **56.** $m = -2$, $(-6, -7)$

Prove the following. (Lesson 2-7)

**57.** If $AB = BC$, then $AC = 2BC$.

**58.** Given: $\overline{JK} \cong \overline{KL}$, $\overline{HJ} \cong \overline{GH}$, $\overline{KL} \cong \overline{HJ}$
Prove: $\overline{GH} \cong \overline{JK}$

59. **Maps** Darnell sketched a map for his friend of the cross streets nearest to his home. Describe two different angle relationships between the streets. (Lesson 1-5)

**Skills Review**

Use the Distance Formula to find the distance between each pair of points. (Lesson 3-3)

- **60.** $A(0, 0)$, $B(15, 20)$
- **61.** $O(-12, 0)$, $P(-8, 3)$
- **62.** $C(11, -12)$, $D(6, 2)$
- **63.** $R(-2, 3)$, $S(3, 15)$
- **64.** $M(1, -2)$, $N(9, 13)$
- **65.** $Q(-12, 2)$, $T(-9, 6)$
Chapter Summary

Key Concepts

Transversals (Lessons 3-1 and 3-2)
- When a transversal intersects two lines, the following types of angles are formed: exterior, interior, consecutive interior, alternate interior, alternate exterior, and corresponding.
- If two parallel lines are cut by a transversal, then:
  - each pair of corresponding angles is congruent,
  - each pair of alternate interior angles is congruent,
  - each pair of consecutive interior angles is supplementary, and
  - each pair of alternate exterior angles is congruent.

Slope (Lessons 3-3 and 3-4)
- The slope $m$ of a line containing two points with coordinates $(x_1, y_1)$ and $(x_2, y_2)$ is $m = \frac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$.

Proving Lines Parallel (Lesson 3-5)
- If two lines in a plane are cut by a transversal so that any one of the following is true, then the two lines are parallel: a pair of corresponding angles is congruent, a pair of alternate interior angles is congruent, or a pair of consecutive interior angles is supplementary.
- In a plane, if two lines are perpendicular to the same line, then they are parallel.

Distance (Lesson 3-6)
- The distance from a line to a point not on the line is the length of the segment perpendicular to the line from the point.
- The distance between two parallel lines is the distance between one of the lines and any point on the other line.

Key Vocabulary

- alternate exterior angles (p. 172)
- alternate interior angles (p. 172)
- consecutive interior angles (p. 172)
- corresponding angles (p. 172)
- equidistant (p. 216)
- parallel lines (p. 171)
- parallel planes (p. 171)
- point-slope form (p. 196)
- rate of change (p. 187)
- skew lines (p. 171)
- slope (p. 186)
- slope-intercept form (p. 196)
- transversal (p. 172)

Vocabulary Check

State whether each sentence is true or false. If false, replace the underlined word or number to make a true sentence.

1. If $\angle 1 \cong \angle 5$, then lines $p$ and $q$ are skew lines.
2. Angles 4 and 6 are alternate interior angles.
3. Angles 1 and 7 are alternate exterior angles.
4. If lines $p$ and $q$ are parallel, then angles 3 and 6 are congruent.
5. The distance from point $X$ to line $q$ is the length of the segment perpendicular to line $q$ from $X$.
6. Line $t$ is called the transversal for lines $p$ and $q$.
7. If $p \parallel q$, then $\angle 2$ and $\angle 8$ are supplementary.
8. Angles 4 and 8 are corresponding angles.
Lesson-by-Lesson Review

3-1 Parallel Lines and Transversals (pp. 171–176)

Classify the relationship between each pair of angles as alternate interior, alternate exterior, corresponding, or consecutive interior angles.

9. \(\angle 1\) and \(\angle 5\)
10. \(\angle 4\) and \(\angle 6\)
11. \(\angle 2\) and \(\angle 8\)
12. \(\angle 4\) and \(\angle 5\)

EXAMPLE 1

Refer to the figure below. Classify the relationship between each pair of angles as alternate interior, alternate exterior, corresponding, or consecutive interior angles.

a. \(\angle 3\) and \(\angle 6\) consecutive interior
b. \(\angle 2\) and \(\angle 6\) corresponding
c. \(\angle 1\) and \(\angle 7\) alternate exterior
d. \(\angle 3\) and \(\angle 5\) alternate interior

3-2 Angles and Parallel Lines (pp. 178–184)

In the figure, \(m \angle 1 = 123\). Find the measure of each angle. Tell which postulate(s) or theorem(s) you used.

14. \(\angle 5\)
15. \(\angle 14\)
16. \(\angle 16\)
17. \(\angle 11\)
18. \(\angle 4\)
19. \(\angle 6\)

20. MAPS The diagram shows the layout of Elm, Plum, and Oak streets. Find the value of \(x\).

EXAMPLE 2

ALGEBRA Use the figure below to find the indicated variable. Explain your reasoning.

If \(m \angle 5 = 7x - 5\) and \(m \angle 4 = 2x + 23\), find \(x\).

\[
\begin{align*}
m \angle 4 + m \angle 5 &= 180 \\
(7x - 5) + (2x + 23) &= 180 \\
9x + 18 &= 180 \\
9x &= 162 \\
x &= 18
\end{align*}
\]

Since lines \(\ell\) and \(m\) are parallel, \(\angle 4\) and \(\angle 5\) are supplementary by the Consecutive Interior Angles Theorem.
**3-3 Slopes of Lines (pp. 186–194)**

Determine whether \( \overrightarrow{AB} \) and \( \overrightarrow{XY} \) are parallel, perpendicular, or neither. Graph each line to verify your answer.

21. \( A(5, 3), B(8, 0), \ X(-7, 2), \ Y(1, 10) \)
22. \( A(-3, 9), B(0, 7), \ X(4, 13), \ Y(-5, 7) \)
23. \( A(8, 1), B(-2, 7), \ X(-6, 2), \ Y(-1, -1) \)

Graph the line that satisfies each condition.

24. contains \((-3, 4)\) and is parallel to \( \overrightarrow{AB} \) with \(A(2, 5)\) and \(B(9, 2)\)
25. contains \((1, 3)\) and is perpendicular to \( \overrightarrow{PQ} \) with \(P(4, -6)\) and \(Q(6, -1)\)

**AIRPLANES** Two Oceanic Airlines planes are flying at the same altitude. Using satellite imagery, each plane’s position can be mapped onto a coordinate plane. Flight 815 was mapped at \((23, 17)\) and \((5, 11)\) while Flight 44 was mapped at \((3, 15)\) and \((9, 17)\). Determine whether their paths are parallel, perpendicular, or neither.

**3-4 Equations of Lines (pp. 196–203)**

Write an equation in point-slope form of the line having the given slope that contains the given point.

27. \( m = 2, \ (4, -9) \)
28. \( m = -\frac{3}{4}, \ (8, -1) \)

Write an equation in slope-intercept form of the line having the given slope and \(y\)-intercept.

29. \( m: 5, \ y\)-intercept: \(-3\)
30. \( m: \frac{1}{2}, \ y\)-intercept: \(4\)

Write an equation in slope-intercept form for each line.

31. \((-3, 12)\) and \((15, 0)\)
32. \((-7, 2)\) and \((5, 8)\)

**WINDOW CLEANING** Ace Window Cleaning Service charges \$50 for the service call and \$20 for each hour spent on the job. Write an equation in slope-intercept form that represents the total cost \(C\) in terms of the number of hours \(h\).

**EXAMPLE 3**

Graph the line that contains \(C(0, -4)\) and is perpendicular to \( \overrightarrow{AB} \) with \(A(5, -4)\) and \(B(0, -2)\).

**EXAMPLE 4**

Write an equation of the line through \((2, 5)\) and \((6, 3)\) in slope-intercept form.

**Step 1** Find the slope of the line through the points.

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 5}{6 - 2} = \frac{-2}{4} = -\frac{1}{2}
\]

**Step 2** Write an equation of the line.

\[
y - y_1 = m(x - x_1)
\]

\[
y - 5 = -\frac{1}{2}(x - 2)
\]

\[
y = -\frac{1}{2}x + 1
\]

Add 5 to each side.
EXAMPLE 5

Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.

a. \( \angle 1 \cong \angle 7 \)

\( \angle 1 \) and \( \angle 7 \) are alternate exterior angles of lines \( b \) and \( d \).

Since \( \angle 1 \cong \angle 7 \), \( b \parallel d \) by the Converse of the Alternate Exterior Angles Theorem.

b. \( \angle 4 \cong \angle 5 \)

\( \angle 4 \) and \( \angle 5 \) are alternate interior angles of lines \( c \) and \( d \).

Since \( \angle 4 \cong \angle 5 \), \( c \parallel d \) by the Converse of the Alternate Interior Angles Theorem.

EXAMPLE 6

Copy the figure. Draw the segment that represents the distance from point \( A \) to \( CD \).

The distance from a line to a point not on the line is the length of the segment perpendicular to the line that passes through the point.

The distance from \( A \) to \( CD \) is the length of the segment perpendicular to \( CD \) from \( A \).
Classify the relationship between each pair of angles as alternate interior, alternate exterior, corresponding, or consecutive interior angles.

1. \( \angle 6 \) and \( \angle 3 \)
2. \( \angle 4 \) and \( \angle 7 \)
3. \( \angle 5 \) and \( \angle 4 \)

Find the distance between each pair of parallel lines with the given equations.

16. \( y = x - 11 \)  
   \( y = x - 7 \)
17. \( y = -2x + 1 \)  
   \( y = -2x + 16 \)

18. MULTIPLE CHOICE  Which segment is skew to \( \overline{CD} \)?

   - A \( \overline{ZY} \)
   - B \( \overline{AB} \)
   - C \( \overline{DE} \)
   - D \( \overline{VZ} \)

19. Find \( x \) so that \( \overrightarrow{a} \parallel \overrightarrow{b} \). Identify the postulate or theorem you used.

   \( (4x + 11)^\circ \)  
   \( (8x + 1)^\circ \)

COORDINATE GEOMETRY  Find the distance from \( P \) to \( \ell \).

20. Line \( \ell \) contains points \((-4, 2)\) and \((3, -5)\). Point \( P \) has coordinates \((1, 2)\).

21. Line \( \ell \) contains points \((6, 5)\) and \((2, 3)\). Point \( P \) has coordinates \((2, 6)\).

Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.

22. \( \angle 4 \) and \( \angle 10 \)
23. \( \angle 9 \) and \( \angle 6 \)
24. \( \angle 7 \) and \( \angle 11 \)

25. JOBS  Hailey works at a gift shop after school. She is paid $10 per hour plus a 15% commission on merchandise she sells. Write an equation that represents her earnings in a week if she sold $550 worth of merchandise.
Gridded Response Questions

In addition to multiple-choice, short-answer, and extended-response questions, you will likely encounter gridded-response questions on standardized tests. After solving a gridded-response question, you must print your answer on an answer sheet and mark in the correct circles on the grid to match your answer. Answers to gridded-response questions may be whole numbers, decimals, or fractions.

Strategies for Solving Gridded-Response Questions

**Step 1**
Read the problem carefully and solve.
- Be sure your answer makes sense.
- If time permits, check your answer.

**Step 2**
Print your answer in the answer boxes.
- Print only one digit or symbol in each answer box.
- Do not write any digits or symbols outside the answer boxes.
- Answers as a whole numbers, decimal, or fraction.

**Step 3**
Fill in the grid.
- Fill in only one bubble for every answer box that you have written in. Be sure not to fill in a bubble under a blank answer box.
- Fill in each bubble completely and clearly.
EXAMPLE

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

**GRIDDED RESPONSE** In the figure below, \( \angle ABC \) is intersected by parallel lines \( \ell \) and \( m \). What is the measure of \( \angle ABC \)? Express your answer in degrees.

\[
\ell \quad 38^\circ \quad A
\]

\[
B \quad 38^\circ \quad \text{---}
\]

\[
m \quad 33^\circ \quad C
\]

Redraw the figure and add a third line parallel to lines \( \ell \) and \( m \) through point \( B \). Find the angle measures using alternate interior angles.

Solve the Problem

\[
\ell \quad 38^\circ \quad A
\]

\[
B \quad 38^\circ \quad \text{---}
\]

\[
m \quad 33^\circ \quad C
\]

\[
m\angle ABC = 38 + 33 = 71
\]

Print your answer in the answer box and fill in the grid.

Exercises

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. **GRIDDED RESPONSE** What is the slope of the line that contains the points \( R(-2, 1) \) and \( S(10, 6) \)? Express your answer as a fraction.

2. **GRIDDED RESPONSE** Solve for \( x \) in the figure below.
Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. If $a \parallel b$ in the diagram below, which of the following is not true?

   $\angle 1 \cong \angle 3$
   $\angle 2 \cong \angle 5$
   $\angle 4 \cong \angle 7$
   $\angle 8 \cong \angle 2$

   [Diagram]

   A $\angle 1 \cong \angle 3$
   B $\angle 4 \cong \angle 7$
   C $\angle 2 \cong \angle 5$
   D $\angle 8 \cong \angle 2$

2. The sum of two odd numbers is odd.

Which of the following best describes a counterexample to the assertion above?

   F $3 + 3 = 6$
   H $6 + 2 = 8$
   G $5 + 4 = 9$
   J $4 + 9 = 13$

3. What is the slope of the line?

   [Graph]

   A $-\frac{2}{3}$
   B $\frac{1}{2}$
   C $-\frac{2}{5}$
   D $-\frac{1}{6}$

4. Line $k$ contains points at $(4, 1)$ and $(-5, -5)$. Find the distance between line $k$ and point $F(-4, 0)$.

   F 3.3 units
   H 4.0 units
   G 3.6 units
   J 4.2 units

5. The globe has a diameter of 22 inches. What is the volume of the globe?

   A $1520.5 \text{ in}^3$
   B $1741.4 \text{ in}^3$
   C $5575.3 \text{ in}^3$
   D $6014.8 \text{ in}^3$

6. What is $m \angle 1$ in the figure below?

   [Diagram]

   F 85
   H 95
   G 90
   J 100

7. Jason is saving money to buy a car stereo. He has $45 saved, and he can save $15 per week. If the stereo that he wants is $210, how many weeks will it take Jason to buy the stereo?

   A 10
   B 11
   C 12
   D 13

Test-TakingTip

Question 6 Drawing a diagram can help you solve problems. Draw a third parallel line through the vertex of angle 1. Then use the properties of parallel lines and transversals to solve the problem.
Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

8. **GRIDDED RESPONSE** For a given line and a point not on the line, how many lines exist that pass through the point and are parallel to the given line?

9. Find the slope of the line that contains the points (4, 3) and (−2, −5).

10. Use the proof to answer the question.

   **Given:** \( \angle 1 \cong \angle 2 \)
   
   **Prove:** \( a \parallel b \)

   **Proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
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<tbody>
<tr>
<td>1. ( \angle 1 \cong \angle 2 )</td>
<td>1. Given</td>
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<tr>
<td>2. ( \angle 2 \cong \angle 3 )</td>
<td>2. ?</td>
</tr>
<tr>
<td>3. ( \angle 1 \cong \angle 3 )</td>
<td>3. Transitive Prop.</td>
</tr>
<tr>
<td>4. ( a \parallel b )</td>
<td>4. If corresponding angles are congruent, then the lines are parallel.</td>
</tr>
</tbody>
</table>

11. **GRIDDED RESPONSE** Write the contrapositive of the statement.

   *If a figure is a square, then the figure is a parallelogram.*

12. Refer to the figure to identify each of the following.

   ![Figure](image)

   a. all segments parallel to \( \overline{MQ} \)
   
   b. all planes intersecting plane \( \overline{SRN} \)
   
   c. a segment skew to \( \overline{ON} \)

13. Use this graph to answer each question.

   ![Graph](image)

   a. What is the equation of line \( m \)?
   
   b. What is the slope of a line that is parallel to line \( m \)?
   
   c. What is the slope of a line that is perpendicular to line \( m \)?

Need Extra Help?

If you missed Question...

<table>
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<tr>
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**MA-HS-3.1.1**

**MA-HS-3.3.1**

**MA-HS-3.3.1**

**MA-HS-3.1.3**

**MA-HS-3.3.1**

**MA-HS-3.1.3**

**MA-HS-3.3.1**

**MA-HS-3.1.5**

**MA-HS-3.3.1**

**MA-HS-3.3.1**

**MA-HS-3.3.1**

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**MA-HS-3.3.1**

**MA-HS-3.3.1**