Why?

OCEANS Tsunamis, or large waves, are generated by undersea earthquakes. A radical equation can be used to find the speed of a tsunami in meters per second or the depth of the ocean in meters.
Get Ready for Chapter 10

Diagnose Readiness  You have two options for checking Prerequisite Skills.

Text Option

Take the Quick Check below. Refer to the Quick Review for help.

QuickCheck

Find each square root. If necessary, round to the nearest hundredth. (Lesson 0-2)

1. \(\sqrt{82}\)  
2. \(\sqrt{26}\)  
3. \(\sqrt{15}\)  
4. \(\sqrt{99}\)  
5. SANDBOX  Isaac is making a square sandbox with an area of 100 square feet. How long is a side of the sandbox?

EXAMPLE 1

Find the square root of \(\sqrt{50}\). If necessary, round to the nearest hundredth.

\[\sqrt{50} = 7.071067812\ldots\]

Use a calculator.

To the nearest hundredth, \(\sqrt{50} = 7.07\).

EXAMPLE 2

Simplify \(3x + 7y - 4x - 8y\).

\[3x + 7y - 4x - 8y = (3x - 4x) + (7y - 8y)\]

Combine like terms.

\[= -x - y\]

Simplify.

EXAMPLE 3

Solve \(x^2 - 5x + 6 = 0\).

\[x^2 - 5x + 6 = 0\]

Original equation

\[(x - 3)(x - 2) = 0\]

Factor.

\[x - 3 = 0\] or \[x - 2 = 0\]

Zero Product Property

\[x = 3\] or \[x = 2\]

Solve each equation.

EXAMPLE 4

Use cross products to determine whether each pair of ratios forms a proportion. Write yes or no. (Lesson 2-6)

16. \(\frac{2}{3}\) and \(\frac{4}{9}\)  
17. \(\frac{3}{4}\) and \(\frac{15}{20}\)  
18. MAPS  On a map, 1 inch = 10 miles. If the distance between cities is 50 miles, how many inches will it be on the map? (Lesson 2-6)

Online Option

Take a self-check Chapter Readiness Quiz at glencoe.com.
Get Started on Chapter 10

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 10. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

Foldables Study Organizer

Radical Functions and Geometry Make this Foldable to help you organize your Chapter 10 notes about radical functions and geometry. Begin with four sheets of grid paper.

1. Fold in half along the width.
2. Staple along the fold.
3. Turn the fold to the left and write the title of the chapter on the front. On each left-hand page of the booklet, write the title of a lesson from the chapter.

New Vocabulary

<table>
<thead>
<tr>
<th>English</th>
<th>Español</th>
</tr>
</thead>
<tbody>
<tr>
<td>radicand</td>
<td>radicando</td>
</tr>
<tr>
<td>radical function</td>
<td>función radicales</td>
</tr>
<tr>
<td>conjugate</td>
<td>conjugado</td>
</tr>
<tr>
<td>radical equations</td>
<td>ecuaciones radicales</td>
</tr>
<tr>
<td>hypotenuse</td>
<td>hipotenusa</td>
</tr>
<tr>
<td>legs</td>
<td>catetos</td>
</tr>
<tr>
<td>converse</td>
<td>recíproco</td>
</tr>
<tr>
<td>midpoint</td>
<td>punto medio</td>
</tr>
<tr>
<td>similar triangles</td>
<td>semejantes</td>
</tr>
<tr>
<td>cosine</td>
<td>coseno</td>
</tr>
<tr>
<td>tangent</td>
<td>tangente</td>
</tr>
<tr>
<td>trigonometry</td>
<td>trigonometría</td>
</tr>
<tr>
<td>inverse cosine</td>
<td>coseno inverso</td>
</tr>
<tr>
<td>inverse sine</td>
<td>seno inverso</td>
</tr>
<tr>
<td>inverse tangent</td>
<td>tangente inverso</td>
</tr>
</tbody>
</table>

Review Vocabulary

- FOIL method • p. 448 • metodo FOIL to multiply two binomials, find the sum of the products of the First terms, Outer terms, Inner terms, and Last terms
- perfect square • p. P7 • cuadrado perfecto a number with a square root that is a rational number
- proportion • p. 111 • proporción an equation of the form \[
\frac{a}{b} = \frac{c}{d}
\]
  stating that two ratios are equivalent
  \[
  \frac{a}{b} = \frac{c}{d} \Rightarrow ad = bc
  \]
Square Root Functions

Why?

Scientists use sounds of whales to track their movements. The distance to a whale can be found by relating time to the speed of sound in water.

The speed of sound in water is defined by the square root function \( c = \sqrt{\frac{K}{d}} \), where \( K \) represents the compressibility of the water and \( d \) represents the density of the water.

Dilations of Radical Functions

A square root function contains the square root of a variable. Square root functions are a type of radical function. The expression under the radical sign is called the radicand. For a square root to be a real number, the radicand cannot be negative. Values that make the radicand negative are not included in the domain.

Key Concept

Square Root Function

Parent function: \( f(x) = \sqrt{x} \)

Type of graph: curve

Domain: \( \{x | x \geq 0\} \)

Range: \( \{y | y \geq 0\} \)

EXAMPLE 1

Dilation of the Square Root Function

Graph \( f(x) = 2\sqrt{x} \). State the domain and range.

Step 1 Make a table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>0</td>
<td>( \approx 1.4 )</td>
<td>2</td>
<td>( \approx 2.8 )</td>
<td>( \approx 3.5 )</td>
<td>4</td>
</tr>
</tbody>
</table>

Step 2 Plot points. Draw a smooth curve.

The domain is \( \{x | x \geq 0\} \), and the range is \( \{y | y \geq 0\} \).
Reflections and Translations of Radical Functions  Recall that when the value of $a$ is negative in the quadratic function $f(x) = ax^2$, the graph is a reflection of the parent graph. The behavior of the radical function is similar when $a$ is negative.

**Key Concept**

**Step 1** Draw the graph of $y = a\sqrt{x}$. The graph starts at the origin and passes through $(1, a)$. If $a > 0$, the graph is in quadrant I. If $a < 0$, the graph is reflected across the $x$-axis and is in quadrant IV.

**Step 2** Translate the graph $|c|$ units up if $c > 0$ and down if $c < 0$.

**Step 3** Translate the graph $|h|$ units left if $h > 0$ and right if $h < 0$.

**Graphing $y = a\sqrt{x} + h + c$**

**EXAMPLE 2** Reflection of the Square Root Function

Graph $y = -3\sqrt{x}$. Compare to the parent graph. State the domain and range.

Make a table of values. Then plot the points on a coordinate system and draw a smooth curve that connects them.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>0</td>
<td>≈−2.1</td>
<td>−3</td>
<td>−6</td>
</tr>
</tbody>
</table>

Notice that the graph is in the 4th quadrant. It is a dilation of the graph of $y = \sqrt{x}$ that has been reflected across the $x$-axis. The domain is $\{x \mid x \geq 0\}$, and the range is $\{y \mid y \leq 0\}$.

**Check Your Progress**

2A. $y = -2\sqrt{x}$
2B. $y = -4\sqrt{x}$

**EXAMPLE 3** Translation of the Square Root Function

Graph each function. Compare to the parent graph. State the domain and range.

**a.** $g(x) = \sqrt{x} + 1$

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>4</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>0</td>
<td>≈1.7</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Notice that the values of $g(x)$ are 1 greater than those of $f(x) = \sqrt{x}$. This is a vertical translation 1 unit up from the parent function. The domain is $\{x \mid x \geq 0\}$, and the range is $\{y \mid y \geq 1\}$.

**b.** $h(x) = \sqrt{x} - 2$

<table>
<thead>
<tr>
<th>$x$</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>0</td>
<td>≈1.4</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

This is a horizontal translation 2 units to the right of the parent function. The domain is $\{x \mid x \geq 2\}$, and the range is $\{y \mid y \geq 0\}$.
Lesson 10-1 Square Root Functions

Check Your Progress

3A. \( g(x) = \sqrt{x} - 4 \)

3B. \( h(x) = \sqrt{x + 3} \)

Physical concepts such as motion can be modeled by radical functions. Often these functions are transformations of the parent square root function.

Real-World EXAMPLE 4 Analyze a Radical Function

BRIDGES The Golden Gate Bridge is about 67 meters above the water. The velocity, \( v \) of an object dropped from \( h \) meters is given by \( v = \sqrt{2gh} \), where \( g \) is the constant, 9.8 meters per second squared. Graph the function. If an object is dropped from the bridge, what is its velocity when it hits the water?

Use a graphing calculator to graph the function.

To find the velocity of the object, substitute 67 meters for \( h \).

\[
v = \sqrt{2gh}
\]

\[
= \sqrt{(2)(9.8)(67)}
\]

\[
≈ 36.2 \text{ m/s}
\]

The velocity of the object is 36.2 meters per second when dropped from a height of 67 meters.

Check Your Progress

4. Use the graph above to estimate the height from which an object was dropped if it is moving at 20 meters per second.

Transformations such as reflections, translations, and dilations can be combined in one equation.

EXAMPLE 5 Transformations of the Square Root Function

Graph \( y = -2\sqrt{x} + 1 \), and compare to the parent graph. State the domain and range.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>4</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>1</td>
<td>-1</td>
<td>-3</td>
<td>-5</td>
</tr>
</tbody>
</table>

This graph is a dilation of the graph of \( y = \sqrt{x} \) that has been reflected across the \( x \)-axis. It has also been translated 1 unit up. The domain is \( \{x \mid x \geq 0\} \), and the range is \( \{y \mid y \leq 1\} \).

Check Your Progress

5A. \( y = \frac{1}{2}\sqrt{x} - 1 \)

5B. \( y = -2\sqrt{x} - 1 \)
Check Your Understanding

Examples 1 and 3
pp. 605–606
Graph each function. Compare to the parent graph. State the domain and range.

1. \( y = 3\sqrt{x} \)
2. \( y = -5\sqrt{x} \)
3. \( y = \frac{1}{3}\sqrt{x} \)
4. \( y = -\frac{1}{2}\sqrt{x} \)
5. \( y = \sqrt{x} + 3 \)
6. \( y = \sqrt{x} - 2 \)
7. \( y = \sqrt{x} + 2 \)
8. \( y = \sqrt{x} - 3 \)

Example 4
p. 607
9. FREE FALL The time in seconds that it takes an object to fall a distance \( d \) is given by the function \( t = \frac{1}{4}\sqrt{d} \) (assuming zero wind resistance). Graph the function, and state the domain and range.

Example 5
p. 607
Graph each function, and compare to the parent graph. State the domain and range.

10. \( y = \frac{1}{2}\sqrt{x} + 2 \)
11. \( y = -\frac{1}{4}\sqrt{x} - 1 \)
12. \( y = -2\sqrt{x} + 1 \)
13. \( y = 3\sqrt{x} - 2 \)

Practice and Problem Solving

Examples 1 and 3
pp. 605–606
Graph each function. Compare to the parent graph. State the domain and range.

14. \( y = 5\sqrt{x} \)
15. \( y = \frac{1}{2}\sqrt{x} \)
16. \( y = -\frac{1}{3}\sqrt{x} \)
17. \( y = 7\sqrt{x} \)
18. \( y = -\frac{1}{4}\sqrt{x} \)
19. \( y = -\sqrt{x} \)
20. \( y = -\frac{1}{5}\sqrt{x} \)
21. \( y = -7\sqrt{x} \)
22. \( y = \sqrt{x} + 2 \)
23. \( y = \sqrt{x} + 4 \)
24. \( y = \sqrt{x} - 1 \)
25. \( y = \sqrt{x} - 3 \)
26. \( y = \sqrt{x} + 1.5 \)
27. \( y = \sqrt{x} - 2.5 \)
28. \( y = \sqrt{x} + 4 \)
29. \( y = \sqrt{x} - 4 \)
30. \( y = \sqrt{x} + 1 \)
31. \( y = \sqrt{x} - 0.5 \)
32. \( y = \sqrt{x} + 5 \)
33. \( y = \sqrt{x} - 1.5 \)
34. GEOMETRY The perimeter of a square is given by the function \( P = 4\sqrt{A} \), where \( A \) is the area of the square.
   a. Graph the function.
   b. Determine the perimeter of a square with an area of 225 m².
   c. When will the perimeter and the area be the same value?

Example 4
p. 607
35. \( y = -2\sqrt{x} + 2 \)
36. \( y = -3\sqrt{x} - 3 \)
37. \( y = \frac{1}{2}\sqrt{x} + 2 \)
38. \( y = -\sqrt{x} - 1 \)
39. \( y = \frac{1}{4}\sqrt{x} - 1 + 2 \)
40. \( y = \frac{1}{2}\sqrt{x} - 2 + 1 \)
41. ENERGY An object has kinetic energy when it is in motion. The velocity in meters per second of an object of mass \( m \) kilograms with an energy of \( E \) joules is given by the function \( v = \sqrt{\frac{2E}{m}} \). Use a graphing calculator to graph the function that represents the velocity of a basketball with a mass of 0.6 kilogram.
Lesson 10-1  Square Root Functions

42. **GEOMETRY** The radius of a circle is given by \( R = \sqrt{\frac{A}{\pi}} \), where \( A \) is the area of the circle.
   a. Graph the function.
   b. Use a graphing calculator to determine the radius of a circle that has an area of 27 in\(^2\).

43. **SPEED OF SOUND** The speed of sound in air is determined by the temperature of the air. The speed \( c \) in meters per second is given by \( c = 331.5 \sqrt{1 + \frac{t}{273.15}} \), where \( t \) is the temperature of the air in degrees Celsius.
   a. Use a graphing calculator to graph the function.
   b. How fast does sound travel when the temperature is 55°C?
   c. How is the speed of sound affected when the temperature increases by 10 degrees? Explain.

44. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the relationship between the graphs of square root functions and parabolas.
   a. **GRAPHICAL** Graph \( y = x^2 \) on a coordinate system.
   b. **ALGEBRAIC** Write a piecewise-defined function to describe the graph of \( y = x^2 \) in each quadrant.
   c. **GRAPHICAL** On the same coordinate system, graph \( y = \sqrt{x} \) and \( y = -\sqrt{x} \).
   d. **GRAPHICAL** Graph the line \( y = x \). Plot the points (2, 4), (4, 2), and (1, 1).
   e. **ANALYTICAL** Compare the graph of the parabola to the graphs of the square root functions.

**H.O.T. Problems**

**CHALLENGE** Determine whether each statement is true or false. Provide an example or counterexample to support your answer.

45. The domain of a radical function will always be nonnegative.
46. The range of a radical function will always be nonnegative.
47. **REASONING** Write a radical function that translates \( y = \sqrt{x} \) four units to the right. Graph the function.
48. **CHALLENGE** Write a radical function with a domain of all real numbers greater than or equal to 2 and a range of all real numbers less than or equal to 5.
49. **WHICH DOES NOT BELONG?** Identify the equation that does not belong with the others. Explain your reasoning.

- \( y = 3\sqrt{x} \)
- \( y = 0.7\sqrt{x} \)
- \( y = \sqrt{x} + 3 \)
- \( y = \frac{\sqrt{x}}{6} \)

50. **OPEN ENDED** Write a function that is a reflection, translation, and a dilation of the parent graph \( y = \sqrt{x} \).
51. **REASONING** If the range of the function \( y = a\sqrt{x} \) is \( \{y \mid y \leq 0\} \), what can you conclude about the value of \( a \)? Explain your reasoning.
52. **WRITING IN MATH** Compare and contrast the graphs of \( f(x) = \sqrt{x} + 2 \) and \( g(x) = \sqrt{x} + 2 \).
53. Which function best represents the graph?
   A \( y = x^2 \)  
   B \( y = 2^x \)  
   C \( y = \sqrt{x} \)  
   D \( y = x \)

54. The statement “\( x < 10 \) and \( 3x - 2 \geq 7 \)” is true when \( x \) is equal to
   F 0  
   G 2  
   H 8  
   J 12

55. Which of the following is the equation of a line parallel to \( y = -\frac{1}{2}x + 3 \) and passing through \((-2, -1)\)?
   A \( y = \frac{1}{2}x \)  
   B \( y = 2x + 3 \)  
   C \( y = -\frac{1}{2}x + 2 \)  
   D \( y = -\frac{1}{2}x - 2 \)

56. SHORT RESPONSE A landscaper needs to mulch 6 rectangular flower beds that are 8 feet by 4 feet and 4 circular flower beds each with a radius of 3 feet. One bag of mulch covers 25 square feet. How many bags of mulch are needed to cover the flower beds?
For a square root to be a real number, the radicand cannot be negative. When graphing a radical equation, determine when the radicand would be negative and exclude those values from the domain.

**ACTIVITY 1**  
**Parent Function**

Graph \( y = \sqrt{x} \).

Enter the equation in the \( Y = \) list.

**KEYSTROKES:**  
\[ \text{Y= 2nd} \left[ \sqrt{\text{X,T,0,n}} \right] \text{ GRAPH} \]

1A. Examine the graph. What is the domain of the function?

1B. What is the range of the function?

**ACTIVITY 2**  
**Translation of Parent Function**

Graph \( y = \sqrt{x - 2} \).

Enter the equation in the \( Y = \) list.

**KEYSTROKES:**  
\[ \text{Y= 2nd} \left[ \sqrt{\text{X,T,0,n} - 2} \right] \text{ GRAPH} \]

2A. What are the domain and range of the function?

2B. How does the graph of \( y = \sqrt{x - 2} \) compare to the graph of the parent function \( y = \sqrt{x} \)?

**Exercises**

Graph each equation and sketch the graph on your paper. State the domain and range of the graph. Then describe how the graph differs from the parent function \( y = \sqrt{x} \).

1. \( y = \sqrt{x} - 1 \)
2. \( y = \sqrt{x} + 3 \)
3. \( y = \sqrt{x} - 2 \)
4. \( y = -\sqrt{x} \)
5. \( y = \sqrt{2x} \)
6. \( y = \sqrt{2 - x} \)
7. \( y = \sqrt{x - 3} + 2 \)

9. Does \( x = y^2 \) represent function? Explain your reasoning.
10. Does the equation \( x^2 + y^2 = 4 \) determine \( y \) as a function of \( x \)? Explain.
11. Does the equation \( x^2 + y^2 = 2 \) determine \( y \) as a function of \( x \)? Explain.

Write a function with a graph that fits each description.

12. the graph of \( y = \sqrt{x} \) shifted 4 units to the left
13. the graph of \( y = \sqrt{x} \) shifted up 7 units
14. the graph of \( y = \sqrt{x} \) shifted down 6 units
15. the graph of \( y = \sqrt{x} \) shifted 5 units to the right and up 3 units
Simplifying Radical Expressions

Why?
The Sunshine Skyway Bridge across Tampa Bay in Florida, is supported by 21 steel cables, each 9 inches in diameter.

To find the diameter a steel cable should have to support a given weight, you can use the equation

\[ d = \sqrt{\frac{w}{8}} \]

where \( d \) is the diameter of the cable in inches and \( w \) is the weight in tons.

Product Property of Square Roots

A radical expression contains a radical, such as a square root. Recall the expression under the radical sign is called the radicand. A radicand is in simplest form if the following three conditions are true.

- No radicands have perfect square factors other than 1.
- No radicands contain fractions.
- No radicals appear in the denominator of a fraction.

The following property can be used to simplify square roots.

**Key Concept**

**Product Property of Square Roots**

For any nonnegative real numbers \( a \) and \( b \), the square root of \( ab \) is equal to the square root of \( a \) times the square root of \( b \).

\[ \sqrt{ab} = \sqrt{a} \cdot \sqrt{b} \]

**Examples**

\[ \sqrt{4 \cdot 9} = \sqrt{36} \text{ or } 6 \]

\[ \sqrt{4 \cdot 9} = \sqrt{4} \cdot \sqrt{9} = 2 \cdot 3 \text{ or } 6 \]

**EXAMPLE 1**

Simplify Square Roots

Simplify \( \sqrt{80} \).

\[
\sqrt{80} = \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 5} \\
= \sqrt{2^4} \cdot \sqrt{5} \\
= 2^2 \cdot \sqrt{5} \text{ or } 4\sqrt{5}
\]

**Check Your Progress**

1A. \( \sqrt{54} \) 
1B. \( \sqrt{180} \)

New Vocabulary

radical expression
rationaling the
denominator
conjugate

KY Program of Studies

HS-AT-S-VEO5 Students will understand the properties of integer exponents and roots and apply these properties to simplify algebraic expressions.

HS-AT-S-VEO10 Students will determine when an expression is undefined. Also addresses HS-NPO-S-NO4.
Lesson 10-2  Simplifying Radical Expressions

EXAMPLE 2  Multiply Square Roots

Simplify \( \sqrt{2} \cdot \sqrt{14} \).

\[
\sqrt{2} \cdot \sqrt{14} = \sqrt{2} \cdot \sqrt{2} \cdot \sqrt{7} = 2\sqrt{7}
\]

Product Property of Square Roots

Check Your Progress

2A. \( \sqrt{5} \cdot \sqrt{10} \)

2B. \( \sqrt{6} \cdot \sqrt{8} \)

Consider the expression \( \sqrt{x^2} \). It may seem that \( x = \sqrt{x^2} \), but when finding the principal square root of an expression containing variables, you have to be sure that the result is not negative. Consider \( x = -3 \).

\[
\sqrt{x^2} = x \quad \text{Replace } x \text{ with } -3.
\]

\[
\sqrt{(-3)^2} = -3 \
\]

\[
\sqrt{9} = 3 
\]

Notice in this case, if the right hand side of the equation began as \( |x| \), the equation would be true. For expressions where the exponent of the variable inside a radical is even and the simplified exponent is odd, you must use absolute value.

\[
\sqrt{x^2} = |x| \quad \sqrt{x^4} = x^2 \quad \sqrt{x^6} = |x^3| 
\]

EXAMPLE 3  Simplify a Square Root with Variables

Simplify \( \sqrt{90x^3y^4z^5} \).

\[
\sqrt{90x^3y^4z^5} = \sqrt{2 \cdot 3^2 \cdot 5 \cdot x^3 \cdot y^4 \cdot z^5} = \sqrt{2} \cdot \sqrt{3^2 \cdot 5} \cdot \sqrt{x^3} \cdot \sqrt{y^4} \cdot \sqrt{z^5} = \sqrt{2} \cdot 3 \cdot \sqrt{5} \cdot |x| \cdot \sqrt{x} \cdot y^2 \cdot z^2 \cdot \sqrt{z} = 3y^2z^2 |x| \sqrt{10xz}
\]

Check Your Progress

3A. \( \sqrt{32r^2k^4t^5} \)

3B. \( \sqrt{56xy^{10}z^5} \)

Quotient Property of Square Roots  To divide square roots and simplify radical expressions, you can use the Quotient Property of Square Roots.

Key Concept  Quotient Property of Square Roots

For any real numbers \( a \) and \( b \), where \( a \geq 0 \) and \( b > 0 \), the square root of \( \frac{a}{b} \) is equal to the square root of \( a \) divided by the square root of \( b \).

Symbols

\[
\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}
\]
You can use the properties of square roots to **rationalize the denominator** of a fraction with a radical. This involves multiplying the numerator and denominator by a factor that eliminates radicals in the denominator.

**STANDARDIZED TEST EXAMPLE 4**

Which expression is equivalent to \( \frac{\sqrt{35}}{15} \)?

- **A** \( \frac{5\sqrt{21}}{15} \)
- **B** \( \frac{\sqrt{21}}{3} \)
- **C** \( \frac{\sqrt{525}}{15} \)
- **D** \( \frac{\sqrt{35}}{15} \)

**Read the Test Item**

The radical expression needs to be simplified.

**Solve the Test Item**

\[
\sqrt{\frac{35}{15}} = \frac{\sqrt{35}}{\sqrt{15}}
\]

Quotient Property of Square Roots

\[
= \frac{\sqrt{35}}{\sqrt{15}} \cdot \frac{\sqrt{15}}{\sqrt{15}}
\]

Multiply by \( \frac{\sqrt{15}}{\sqrt{15}} \).

\[
= \sqrt{\frac{35 \cdot 15}{15^2}}
\]

Product Property of Square Roots

\[
= \sqrt{\frac{525}{15^2}}
\]

Prime factorization

\[
= \frac{5\sqrt{21}}{15} \text{ or } \frac{\sqrt{21}}{3}
\]

\( \sqrt{5^2} = 5 \)

The correct choice is **B**.

**Check Your Progress**

4. Simplify \( \frac{\sqrt{6y}}{\sqrt{12}} \).

Binomials of the form \( a\sqrt{b} + c\sqrt{d} \) and \( a\sqrt{b} - c\sqrt{d} \) are called **conjugates**. For example, \( 2 + \sqrt{7} \) and \( 2 - \sqrt{7} \) are conjugates. The product of two conjugates is a rational number and can be found using the pattern for the difference of squares.

**EXAMPLE 5**

**Use Conjugates to Rationalize a Denominator**

Simplify \( \frac{3}{5 + \sqrt{2}} \).

\[
\frac{3}{5 + \sqrt{2}} = \frac{3}{5 + \sqrt{2}} \cdot \frac{5 - \sqrt{2}}{5 - \sqrt{2}}
\]

The conjugate of \( 5 + \sqrt{2} \) is \( 5 - \sqrt{2} \).

\[
= \frac{15 - 3\sqrt{2}}{25 - 2}
\]

\( (\sqrt{2})^2 = 2 \)

**Check Your Progress**

Simplify each expression.

5A. \( \frac{3}{2 + \sqrt{2}} \)

5B. \( \frac{7}{3 - \sqrt{7}} \)
Check Your Understanding

Examples 1 and 3  
pp. 612–613

Simplify each expression.

1. \( \sqrt{24} \)  
2. \( 3\sqrt{16} \)  
3. \( 2\sqrt{25} \)

4. \( \sqrt{10} \cdot \sqrt{14} \)  
5. \( \sqrt{3} \cdot \sqrt{18} \)  
6. \( 3\sqrt{10} \cdot 4\sqrt{10} \)

7. \( \sqrt{60x^4y^7} \)  
8. \( \sqrt{88m^3p^2r^5} \)  
9. \( \sqrt{99ab^5c^2} \)

Example 4  
p. 614

10. MULTIPLE CHOICE  Which expression is equivalent to \( \sqrt{45} - 10 \)?

   A. \( \frac{5\sqrt{2}}{10} \)  
   B. \( \sqrt{450} - 10 \)  
   C. \( \frac{\sqrt{50}}{10} \)  
   D. \( 3\sqrt{2} - 2 \)

Example 5  
p. 614

Simplify each expression.

11. \( \frac{3}{3 + \sqrt{5}} \)  
12. \( \frac{5}{2 - \sqrt{6}} \)  
13. \( \frac{2}{1 - \sqrt{10}} \)

14. \( \frac{1}{4 + \sqrt{12}} \)  
15. \( \frac{4}{6 - \sqrt{7}} \)  
16. \( \frac{6}{5 + \sqrt{11}} \)

Practice and Problem Solving

Examples 1 and 3  
pp. 612–613

Simplify each expression.

17. \( \sqrt{52} \)  
18. \( \sqrt{56} \)  
19. \( \sqrt{72} \)

20. \( 3\sqrt{18} \)  
21. \( \sqrt{243} \)  
22. \( \sqrt{245} \)

23. \( \sqrt{5} \cdot \sqrt{10} \)  
24. \( \sqrt{10} \cdot \sqrt{20} \)  
25. \( 3\sqrt{8} \cdot 2\sqrt{7} \)

26. \( 4\sqrt{2} \cdot 5\sqrt{8} \)  
27. \( 3\sqrt{25t^2} \)  
28. \( 5\sqrt{81q^5} \)

29. \( \sqrt{28a^2b^3} \)  
30. \( \sqrt{75qr^3} \)  
31. \( 7\sqrt{63m^3p} \)

32. \( 4\sqrt{66g^2l^4} \)  
33. \( \sqrt{2ab^2} \cdot \sqrt{10a^5b} \)  
34. \( \sqrt{4c^3d^3} \cdot \sqrt{8c^3d} \)

35. ROLLER COASTER  The velocity \( v \) of a roller coaster in feet per second at the bottom of a hill is \( v = \sqrt{64h} \), where \( h \) is the height of the hill in feet.

   a. Simplify the equation.
   b. Determine the velocity of a roller coaster at the bottom of a 134-foot hill.

36. FIREFIGHTING  When fighting a fire, the velocity \( v \) of water being pumped into the air is modeled by the function \( v = \sqrt{2hg} \), where \( h \) represents the maximum height of the water and \( g \) represents the acceleration due to gravity (32 ft/s²).

   a. Solve the function for \( h \).
   b. The Hollowville Fire Department must purchase a pump that will propel water 80 feet into the air. Will a pump that is advertised to project water with a velocity of 70 feet per second meet the fire department’s need? Explain.
   c. The Jackson Fire Department must purchase a pump that will propel water 90 feet into the air. Will a pump that is advertised to project water with a velocity of 77 feet per second meet the fire department’s need? Explain.

Real-World Link

In 1736, Benjamin Franklin founded the first volunteer fire organization, the Union Fire Company, in Philadelphia.  
Source: Firehouse Magazine
Simplify each expression.

37. \( \sqrt{\frac{32}{t^4}} \)

38. \( \sqrt{\frac{27}{m^5}} \)

39. \( \frac{\sqrt{68ac^3}}{\sqrt{27a^2}} \)

40. \( \frac{\sqrt{8^5}}{\sqrt{8}} \)

41. \( \sqrt{\frac{3}{16} \cdot \sqrt{\frac{9}{5}}} \)

42. \( \frac{\sqrt{\frac{7}{2}} \cdot \sqrt{\frac{5}{3}}}{-2 + \sqrt{6}} \)

43. \( \frac{7}{5 + \sqrt{3}} \)

44. \( \frac{9}{6 - \sqrt{8}} \)

45. \( \frac{3\sqrt{3}}{-2 + \sqrt{6}} \)

46. \( \frac{\sqrt{3}}{\sqrt{7} - \sqrt{2}} \)

47. \( \frac{5}{\sqrt{6} + \sqrt{3}} \)

48. \( \frac{2\sqrt{5}}{2\sqrt{7} + 3\sqrt{3}} \)

49. **ELECTRICITY** The amount of current in amperes \( I \) that an appliance uses can be calculated using the formula \( I = \sqrt{\frac{P}{R}} \), where \( P \) is the power in watts and \( R \) is the resistance in ohms.
   
   a. Simplify the formula.

   b. How much current does an appliance use if the power used is 75 watts and the resistance is 5 ohms?

50. **KINETIC ENERGY** The speed \( v \) of a ball can be determined by the equation \( v = \sqrt{\frac{2k}{m}} \), where \( k \) is the kinetic energy and \( m \) is the mass of the ball.
   
   a. Simplify the formula if the mass of the ball is 3 kilograms.

   b. If the ball is traveling 7 meters per second, what is the kinetic energy of the ball in Joules?

51. **SUBMARINES** A lookout on a submarine is \( h \) feet above the surface of the water. The greatest distance \( d \) in miles that the lookout can see on a clear day is modeled by the formula \( d = \sqrt{\frac{3h}{2}} \). Determine how high the submarine would have to raise its periscope to see a ship, if the submarine is the given distances away from a ship.

<table>
<thead>
<tr>
<th>Distance</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

52. **REASONING** Explain how to solve \((3x - 2)^2 = (2x + 6)^2\).

53. **CHALLENGE** Solve \( |y^3| = \frac{1}{3\sqrt{3}} \) for \( y \).

54. **REASONING** Marge takes any number, subtracts 4, multiplies by 4, takes the square root, and takes the reciprocal to get \( \frac{1}{2} \). What number did she start with? Write a formula to describe the process.

55. **OPEN ENDED** Write two binomials of the form \( a\sqrt{b} + c\sqrt{f} \) and \( a\sqrt{b} - c\sqrt{f} \). Then find their product.

56. **CHALLENGE** Use the Quotient Property of Square Roots to derive the Quadratic Formula by solving the quadratic equation \( ax^2 + bx + c = 0 \). *(Hint: Begin by completing the square.)*

57. **WRITING IN MATH** Summarize how to write a radical expression in simplest form.
58. Jerry’s electric bill is $23 less than his natural gas bill. The two bills are a total of $109. Which of the following equations can be used to find the amount of his natural gas bill?

A \( g + g = 109 \)  
B \( 23 + 2g = 109 \)  
C \( g - 23 = 109 \)  
D \( 2g - 23 = 109 \)

59. Solve \( a^2 - 2a + 1 = 25 \).

F \(-4, -6\)  
G \(4, -6\)  
H \(-4, 6\)  
J \(4, 6\)

60. The expression \( \sqrt{160x^2y^5} \) is equivalent to which of the following?

A \( 16 \left| x \right| y^2 \sqrt{10y} \)  
B \( \left| x \right| y^2 \sqrt{160y} \)  
C \( 4 \left| x \right| y^2 \sqrt{10y} \)  
D \( 10 \left| x \right| y^2 \sqrt{4y} \)

61. GRIDDED RESPONSE Miki earns $10 an hour and 10% commission on sales. If Miki worked 38 hours and had a total sales of $1275 last week, how much did she make?

\[ \text{Miki's earnings} = 38 \times 10 + 0.10 \times 1275 \]

Spiral Review

Graph each function. Compare to the parent graph. State the domain and range. (Lesson 10-1)

62. \( y = 2\sqrt{x} - 1 \)
63. \( y = \frac{1}{2}\sqrt{x} \)
64. \( y = 2\sqrt{x} + 2 \)
65. \( y = -\sqrt{x} + 1 \)
66. \( y = -3\sqrt{x} - 3 \)
67. \( y = -2\sqrt{x} + 1 \)

Look for a pattern in each table of values to determine which model best describes the data. (Lesson 9-9)

68. \[
\begin{array}{cccc}
  x & 0 & 1 & 2 \\
  y & 1 & 3 & 9 \\
\end{array}
\]
69. \[
\begin{array}{cccc}
  x & -3 & 0 & 1 \\
  y & 18 & 2 & 2 \\
\end{array}
\]
70. \[
\begin{array}{cccc}
  x & 1 & 2 & 3 \\
  y & 1 & 3 & 5 \\
\end{array}
\]

71. POPULATION The country of Latvia has been experiencing a 1.1% annual decrease in population. In 2005, its population was 2,290,237. If the trend continues, predict Latvia’s population in 2015. (Lesson 9-7)

72. \( x^2 - 25 = 0 \)
73. \( r^2 + 25 = 0 \)
74. \( 4w^2 + 100 = 40w \)
75. \( 2r^2 + r - 14 = 0 \)
76. \( 5v^2 - 7v = 1 \)
77. \( 11z^2 - z = 3 \)

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary. (Lesson 9-5)

78. \( n^2 - 81 \)
79. \( 4 - 9a^2 \)
80. \( 2x^5 - 98x^3 \)
81. \( 32x^4 - 2y^4 \)
82. \( 4t^2 - 27 \)
83. \( x^3 - 3x^2 - 9x + 27 \)

Factor each polynomial, if possible. If the polynomial cannot be factored, write prime. (Lesson 8-5)

84. Gardening Cleveland is planting 120 jalapeno pepper plants in a rectangular arrangement in his garden. In what ways can he arrange them so that he has at least 4 rows of plants, the same number of plants in each row, and at least 6 plants in each row? (Lesson 8-1)

Skills Review

Write the prime factorization of each number. (Concepts and Skills Bank Lesson 6)

85. \( 24 \)
86. \( 88 \)
87. \( 180 \)
88. \( 31 \)
89. \( 60 \)
90. \( 90 \)
You have studied the properties of exponents that are whole numbers. Some exponents are rational numbers or fractions. You can use a calculator to explore the meaning of rational exponents.

**ACTIVITY**

**Rational Exponents**

**Step 1** Evaluate \(16^{\frac{1}{2}}\) and \(\sqrt[2]{16}\).

**KEYSTROKES:**
- \(16\) \(\boxed{\text{and}}\) \((\frac{1}{2})\) \(\boxed{\text{ent}}\)
- \(2\text{nd} [\sqrt{\text{]]}}\) \(16\) \(\boxed{\text{ent}}\)

Record the results in a table like the one at the right.

**Step 2** Use a calculator to evaluate each expression.

Record each result in your table. To find a root other than a square root, choose the \(\sqrt[n]{\text{function}}\) from the \([\text{MATH}]\) Menu.

1A. Study the table. What do you observe about the value of an expression of the form \(a^{\frac{1}{n}}\)?

1B. What do you observe about the value of an expression of the form \(a^{\frac{m}{n}}\)?

**Exercises**

1. Recall the Power of a Power Property. For any number \(a\) and all integers \(m\) and \(n\), 
\((a^m)^n = (a^n)^m\). Assume that fractional exponents behave as whole number exponents and find the value of \((\sqrt{b})^2\).

\[
\left(\sqrt{b}\right)^2 = \frac{1}{2} \cdot 2 = b^{\frac{1}{2}} \quad \text{Power of a Power Property}
\]

Thus, \(b^\frac{1}{2}\) is a number whose square equals \(b\). So it makes sense to define \(b^\frac{1}{2} = \sqrt{b}\).

Use a similar process to define \(b^\frac{1}{n}\).

2. Define \(b^\frac{m}{n}\). Justify your answer.

Write each root as an expression using a fractional exponent. Then evaluate the expression.

3. \(\sqrt{36}\)

4. \(\sqrt{121}\)

5. \(\sqrt[4]{256}\)

6. \(\sqrt[3]{32}\)

7. \(\sqrt[3]{8^2}\)

8. \(\sqrt[4]{1296}\)

9. \(\sqrt[4]{16^3}\)

10. \(\sqrt[3]{8^3}\)
Then
You simplified radical expressions.
(Lesson 10-2)

Now
• Add and subtract radical expressions.
• Multiply radical expressions.

KY Program of Studies
HS-NPO-S-NO4 Students will apply absolute value, integer exponents, roots and factorials to solve problems.
HS-AT-S-VEO5 Students will understand the properties of integer exponents and roots and apply these properties to simplify algebraic expressions. Also addresses HS-NPO-S-NO10.

KY Math Online
Glencoe.com
• Extra Examples
• Personal Tutor
• Self-Check Quiz
• Homework Help

Operations with Radical Expressions

Conchita is going to run in her neighborhood to get ready for the soccer season. She plans to run the course that she has laid out three times each day.

How far does Conchita have to run to complete the course that she laid out?

How far does she run every day?

Add or Subtract Radical Expressions
To add or subtract radical expressions, the radicands must be alike in the same way that monomial terms must be alike to add or subtract.

\[
\begin{align*}
4a + 2a &= (4 + 2)a \\
&= 6a \\
9b - 2b &= (9 - 2)b \\
&= 7b
\end{align*}
\]

Notice that when adding and subtracting radical expressions, the radicand does not change. This is the same as when adding or subtracting monomials.

EXAMPLE 1 Add and Subtract Expressions with Like Radicands

Simplify each expression.

a. \[5\sqrt{2} + 7\sqrt{2} - 6\sqrt{2}\]
\[5\sqrt{2} + 7\sqrt{2} - 6\sqrt{2} = (5 + 7 - 6)\sqrt{2} \\
= 6\sqrt{2}\]

b. \[10\sqrt{7} + 5\sqrt{11} + 4\sqrt{7} - 6\sqrt{11}\]
\[10\sqrt{7} + 5\sqrt{11} + 4\sqrt{7} - 6\sqrt{11} = (10 + 4)\sqrt{7} + (5 - 6)\sqrt{11} \\
= 14\sqrt{7} - \sqrt{11}\]

Check Your Progress

1A. \[3\sqrt{2} - 5\sqrt{2} + 4\sqrt{2}\]  
1B. \[6\sqrt{11} + 2\sqrt{11} - 9\sqrt{11}\]  
1C. \[15\sqrt{3} - 14\sqrt{5} + 6\sqrt{5} - 11\sqrt{3}\]  
1D. \[4\sqrt{3} + 3\sqrt{7} - 6\sqrt{3} + 3\sqrt{7}\]

Not all radical expressions have like radicands. Simplifying the expressions may make it possible to have like radicands so that they can be added or subtracted.
EXAMPLE 2  Add and Subtract Expressions with Unlike Radicands

Simplify $2\sqrt{18} + 2\sqrt{32} + \sqrt{72}$.

$2\sqrt{18} + 2\sqrt{32} + \sqrt{72} = 2(\sqrt{3^2} \cdot \sqrt{2}) + 2(\sqrt{4^2} \cdot \sqrt{2}) + (\sqrt{6^2} \cdot \sqrt{2})$

$= 2\times(3\sqrt{2}) + 2\times(4\sqrt{2}) + (6\sqrt{2})$

$= 6\sqrt{2} + 8\sqrt{2} + 6\sqrt{2}$

$= 20\sqrt{2}$

Check Your Progress

2A. $4\sqrt{54} + 2\sqrt{24}$

2B. $4\sqrt{12} - 6\sqrt{48}$

2C. $3\sqrt{45} + \sqrt{20} - \sqrt{245}$

2D. $\sqrt{24} - \sqrt{54} + \sqrt{96}$

Multiply Radical Expressions

Multiplying radical expressions is similar to multiplying monomial algebraic expressions.

**Monomials**

$$(2x)(3x) = 2 \cdot 3 \cdot x \cdot x$$

$$= 6x^2$$

**Radical Expressions**

$$(2\sqrt{x})(3\sqrt{x}) = 2 \cdot 3 \cdot \sqrt{x} \cdot \sqrt{x}$$

$$= 6\sqrt{x}$$

You can also apply the Distributive Property to radical expressions.

EXAMPLE 3  Multiply Radical Expressions

Simplify each expression.

a. $3\sqrt{2} \cdot 2\sqrt{6}$

$$3\sqrt{2} \cdot 2\sqrt{6} = (3 \cdot 2)(\sqrt{2} \cdot \sqrt{6})$$

$$= (6)(\sqrt{12})$$

$$= (6)(2\sqrt{3})$$

$$= 12\sqrt{3}$$

b. $3\sqrt{5}(2\sqrt{5} + 5\sqrt{3})$

$$3\sqrt{5}(2\sqrt{5} + 5\sqrt{3}) = (3\sqrt{5} \cdot 2\sqrt{5}) + (3\sqrt{5} \cdot 5\sqrt{3})$$

$$= [(3 \cdot 2)(\sqrt{5} \cdot \sqrt{5})] + [(3 \cdot 5)(\sqrt{5} \cdot \sqrt{3})]$$

$$= [(6)(\sqrt{25})] + [(15)(\sqrt{15})]$$

$$= [(6)(5)] + [(15)(\sqrt{15})]$$

$$= 30 + 15\sqrt{15}$$

Check Your Progress

3A. $2\sqrt{6} \cdot 7\sqrt{3}$

3B. $9\sqrt{5} \cdot 11\sqrt{15}$

3C. $3\sqrt{2}(4\sqrt{3} + 6\sqrt{2})$

3D. $5\sqrt{3}(3\sqrt{2} - \sqrt{3})$
**Multiply Radical Expressions**

**GEOMETRY** Find the area of the rectangle in simplest form.

\[
A = (5\sqrt{2} - \sqrt{3})(\sqrt{5} + 4\sqrt{3})
\]

First Terms Outer Terms Inner Terms Last Terms
\[= (5\sqrt{2})(\sqrt{5}) + (5\sqrt{2})(4\sqrt{3}) - (\sqrt{3})(\sqrt{5}) - (\sqrt{3})(4\sqrt{3})\]

\[= 5\sqrt{10} + 20\sqrt{6} - \sqrt{15} - 4\sqrt{9} \quad \text{Multiply.}\]

\[= 5\sqrt{10} + 20\sqrt{6} - \sqrt{15} - 12 \quad \text{Simplify.}\]

**Check Your Progress**

4. **GEOMETRY** The area \(A\) of a rhombus can be found using the equation \(A = \frac{1}{2}d_1d_2\), where \(d_1\) and \(d_2\) are the lengths of the diagonals of the rhombus. What is the area of the rhombus at the right?

**Concept Summary**

**Operations with Radical Expressions**

<table>
<thead>
<tr>
<th>Operation</th>
<th>Symbols</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>addition</td>
<td>(a\sqrt{b} + c\sqrt{b} = (a + c)\sqrt{b}) like radicands</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(4\sqrt{3} + 6\sqrt{3} = (4 + 6)\sqrt{3})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(= 10\sqrt{3})</td>
</tr>
<tr>
<td>subtraction</td>
<td>(a\sqrt{b} - c\sqrt{b} = (a - c)\sqrt{b}) like radicands</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(12\sqrt{5} - 8\sqrt{5} = (12 - 8)\sqrt{5})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(= 4\sqrt{5})</td>
</tr>
<tr>
<td>multiplication</td>
<td>(a\sqrt{b}f\sqrt{g} = af\sqrt{bg}) Radicands do not have to be like radicands.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3\sqrt{2}(5\sqrt{7}) = (3 \cdot 5)(\sqrt{2}\sqrt{7}))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(= 15\sqrt{14})</td>
</tr>
</tbody>
</table>

**Check Your Understanding**

**Examples 1 and 3**

pp. 619–620

1. \(3\sqrt{5} + 6\sqrt{5}\)
2. \(8\sqrt{3} + 5\sqrt{3}\)
3. \(\sqrt{7} - 6\sqrt{7}\)
4. \(10\sqrt{2} - 6\sqrt{2}\)
5. \(4\sqrt{5} + 2\sqrt{20}\)
6. \(\sqrt{12} - \sqrt{3}\)
7. \(\sqrt{8} + \sqrt{12} + \sqrt{18}\)
8. \(\sqrt{27} + 2\sqrt{3} - \sqrt{12}\)
9. \(9\sqrt{2}(4\sqrt{6})\)
10. \(4\sqrt{3}(8\sqrt{3})\)
11. \(\sqrt{3}(\sqrt{7} + 3\sqrt{2})\)
12. \(\sqrt{5}(\sqrt{2} + 4\sqrt{2})\)
13. **GEOMETRY** The area \(A\) of a triangle can be found by using the formula \(A = \frac{1}{2}bh\), where \(b\) represents the base and \(h\) is the height. What is the area of the triangle at the right?

**Example 4**

p. 621

\[\frac{4\sqrt{3} + \sqrt{5}}{2\sqrt{3} + \sqrt{5}}\]
Simplify each expression.

14. \(7\sqrt{5} + 4\sqrt{5}\)

15. \(2\sqrt{6} + 9\sqrt{6}\)

16. \(3\sqrt{5} - 2\sqrt{20}\)

17. \(3\sqrt{50} - 3\sqrt{32}\)

18. \(7\sqrt{3} - 2\sqrt{2} + 3\sqrt{2} + 5\sqrt{3}\)

19. \(\sqrt{5}(\sqrt{2} + 4\sqrt{2})\)

20. \(\sqrt{6}(2\sqrt{10} + 3\sqrt{2})\)

21. \(4\sqrt{5}(3\sqrt{5} + 8\sqrt{2})\)

22. \(5\sqrt{3}(6\sqrt{10} - 6\sqrt{3})\)

23. \((\sqrt{3} - \sqrt{2})(\sqrt{15} + \sqrt{12})\)

24. \((3\sqrt{11} + 3\sqrt{15})(3\sqrt{3} - 2\sqrt{2})\)

25. \((5\sqrt{2} + 3\sqrt{5})(2\sqrt{10} - 5)\)

26. **GEOMETRY** Find the perimeter and area of a rectangle with a width of \(2\sqrt{7} - 2\sqrt{5}\) and a length of \(3\sqrt{7} + 3\sqrt{5}\).

27. \(\sqrt{\frac{1}{5}} - \sqrt{5}\)

28. \(\sqrt{\frac{2}{3}} + \sqrt{6}\)

29. \(2\sqrt{\frac{1}{2}} + 2\sqrt{2} - \sqrt{8}\)

30. \(8\sqrt{\frac{5}{4}} + 3\sqrt{20} - 10\sqrt{\frac{1}{5}}\)

31. \((3 - \sqrt{5})^2\)

32. \((\sqrt{2} + \sqrt{3})^2\)

**ROLLER COASTERS** The velocity \(v\) in feet per second of a roller coaster at the bottom of a hill is related to the vertical drop \(h\) in feet and the velocity \(v_0\) of the coaster at the top of the hill by the formula \(v_0 = \sqrt{v^2 - 64h}\).

a. What velocity must a coaster have at the top of a 225-foot hill to achieve a velocity of 120 feet per second at the bottom?

b. Explain why \(v_0 = v - 8\sqrt{r}\) is not equivalent to the formula given.

34. **SAVING** Tadi invests $225 in a savings account. In two years, Tadi has $270 in his account. You can use the formula \(r = \sqrt{\frac{v_2}{v_0}} - 1\) to find the average annual interest rate \(r\) that the account has earned. The initial investment is \(v_0\), and \(v_2\) is the amount in two years. What was the average annual interest rate that Tadi’s account earned?

35. **ELECTRICITY** Electricians can calculate the electrical current in amps \(A\) by using the formula \(A = \frac{\sqrt{w}}{\sqrt{r}}\), where \(w\) is the power in watts and \(r\) the resistance in ohms. How much electrical current is running through a microwave oven that has 850 watts of power and 5 ohms of resistance? Write in simplest radical form, and then estimate the amount of current to the nearest tenth.

**H.O.T. Problems** Use **Higher-Order Thinking Skills**

36. **CHALLENGE** Determine whether the following statement is true or false. Provide an example or counterexample to support your answer.

\[x + y > \sqrt{x^2 + y^2}\] when \(x > 0\) and \(y > 0\)

37. **REASONING** Show that if you multiply \(a\sqrt{b} + c\sqrt{f}\) and \(a\sqrt{b} - c\sqrt{f}\), the product has no radicals. Explain why this occurs.

38. **OPEN ENDED** Write an equation that shows a sum of two radicals with different radicands. Explain how you could combine these terms.

39. **WRITING IN MATH** Describe step by step how to multiply two radical expressions, each with two terms. Write an example to demonstrate your description.
40. **SHORT RESPONSE** The population of a town is 13,000 and is increasing by about 250 people per year. This can be represented by the equation \( p = 13,000 + 250y \), where \( y \) is the number of years from now and \( p \) represents the population. In how many years will the population of the town be 14,500?

41. **GEOMETRY** Which expression represents the sum of the lengths of the 12 edges on this rectangular solid?

- A \( 2(a + b + c) \)
- B \( 3(a + b + c) \)
- C \( 4(a + b + c) \)
- D \( 12(a + b + c) \)

42. Which of the following is equivalent to \( 8(3 - y) + 5(3 - y) \)?

- F \( 39 - y \)
- H \( 40(30 - y) \)
- G \( 13(3 - y) \)
- J \( 13(6 - 2y) \)

43. The current \( I \) in a simple electrical circuit is given by the formula \( I = \frac{V}{R} \), where \( V \) is the voltage and \( R \) is the resistance of the circuit. If the voltage remains unchanged, what effect will doubling the resistance of the circuit have on the current?

- A The current will remain the same.
- B The current will double its previous value.
- C The current will be half its previous value.
- D The current will be two units more than its previous value.

**Spiral Review**

Simplify. (Lesson 10-2)

44. \( \sqrt{18} \)
45. \( \sqrt{24} \)
46. \( \sqrt{60} \)
47. \( \sqrt{50a^2b^5} \)
48. \( \sqrt{169x^4y^7} \)
49. \( \sqrt{63e^3d^4f^5} \)

Graph each function. Compare to the parent graph. State the domain and range. (Lesson 10-1)

50. \( y = 2\sqrt{x} \)
51. \( y = -3\sqrt{x} \)
52. \( y = \sqrt{x + 1} \)
53. \( y = \sqrt{x - 4} \)
54. \( y = \sqrt{x + 3} \)
55. \( y = \sqrt{x} - 2 \)

56. **INVESTMENT** Determine the value of an investment if $400 is invested at an interest rate of 7.25% compounded quarterly for 7 years. (Lesson 9-7)

Factor each trinomial. (Lesson 8-3)

57. \( x^2 + 12x + 27 \)
58. \( y^2 + 13y + 30 \)
59. \( p^2 - 17p + 72 \)
60. \( x^2 + 6x - 7 \)
61. \( y^2 - y - 42 \)
62. \( -72 + 6w + w^2 \)

**Skills Review**

Solve each equation. Round each solution to the nearest tenth, if necessary. (Lesson 2-3)

63. \( -4c - 1.2 = 0.8 \)
64. \( -2.6q - 33.7 = 84.1 \)
65. \( 0.3m + 4 = 9.6 \)
66. \( -10 - \frac{n}{5} = 6 \)
67. \( -\frac{4h - (-5)}{-7} = 13 \)
68. \( 3.6t + 6 - 2.5t = 8 \)
Radical Equations

**Why?**

The waterline length of a sailboat is the length of the line made by the water’s edge when the boat is full. A sailboat’s hull speed is the fastest speed that it can travel. You can estimate hull speed $h$ by using the formula $h = 1.34\sqrt{\ell}$, where $\ell$ is the length of the sailboat’s waterline.

**Radical Equations**

Equations that contain variables in the radicand, like $h = 1.34\sqrt{\ell}$, are called radical equations. To solve, isolate the desired variable on one side of the equation first. Then square each side of the equation to eliminate the radical.

**Real-World Example 1**

SAILING Idris and Sebastian are sailing in a friend’s sailboat. They measure the hull speed at 9 nautical miles per hour. Find the length of the sailboat’s waterline. Round to the nearest foot.

**Understand**

You know how fast the boat will travel and that it relates to the length.

**Plan**

The boat travels at 9 nautical miles per hour. The formula for hull speed is $h = 1.34\sqrt{\ell}$.

**Solve**

$$h = 1.34\sqrt{\ell}$$  \hspace{1cm} \text{Formula for hull speed}

$$9 = 1.34\sqrt{\ell}$$ \hspace{1cm} \text{Substitute 9 for } h.

$$\frac{9}{1.34} = \frac{1.34\sqrt{\ell}}{1.34}$$ \hspace{1cm} \text{Divide each side by 1.34.}

$$6.71641791 = \sqrt{\ell}$$ \hspace{1cm} \text{Simplify.}

$$\left(6.71641791\right)^2 = \left(\sqrt{\ell}\right)^2$$ \hspace{1cm} \text{Square each side of the equation.}

$$45.11026954 \approx \ell$$ \hspace{1cm} \text{Simplify.}

The sailboat’s waterline length is about 45 feet.

**Check**

Check the results by substituting your estimate back into the original formula. Remember that your result should be about 9.

$$h = 1.34\sqrt{\ell}$$ \hspace{1cm} \text{Formula for hull speed}

$$9 \pm 1.34\sqrt{45}$$ \hspace{1cm} \text{Multiply.}

$$9 \pm 1.34(6.708203932)$$ \hspace{1cm} \text{Multiply.}

$$9 \approx 8.98899327 \checkmark$$ \hspace{1cm} \text{Multiply.}
Check Your Progress

1. **DRIVING** The equation \( v = \sqrt{2.5r} \) represents the maximum velocity that a car can travel safely on an unbanked curve when \( v \) is the maximum velocity in miles per hour and \( r \) is the radius of the turn in feet. If a road is designed for a maximum speed of 65 miles per hour, what is the radius of the turn?

When a radicand is an expression, isolate the radical first. Then square both sides of the equation.

**EXAMPLE 2** Expression as a Radicand

Solve \( \sqrt{a + 5} + 7 = 12 \).

- \( \sqrt{a + 5} + 7 = 12 \) Original equation
- \( \sqrt{a + 5} = 5 \) Subtract 7 from each side.
- \( (\sqrt{a + 5})^2 = 5^2 \) Square each side.
- \( a + 5 = 25 \) Simplify.
- \( a = 20 \) Subtract 5 from each side.

**EXAMPLE 3** Variable on Each Side

Solve \( \sqrt{k + 1} = k - 1 \). Check your solution.

- \( \sqrt{k + 1} = k - 1 \) Original equation
- \( (\sqrt{k + 1})^2 = (k - 1)^2 \) Square each side.
- \( k + 1 = k^2 - 2k + 1 \) Simplify.
- \( 0 = k^2 - 3k \) Subtract \( k \) and 1 from each side.
- \( 0 = k(k - 3) \) Factor.
- \( k = 0 \) or \( k - 3 = 0 \) Zero Product Property
- \( k = 3 \) Solve.

**CHECK** \( \sqrt{k + 1} = k - 1 \)

- \( \sqrt{0 + 1} \neq 0 - 1 \) Simplify.
- \( 1 \neq -1 \) False

Since 0 does not satisfy the original equation, 3 is the only solution.

**Check Your Progress**

Solve each equation. Check your solution.

- **3A.** \( \sqrt{t + 5} = t + 3 \)
- **3B.** \( x - 3 = \sqrt{x - 1} \)
Check Your Understanding

Example 1  p. 624

1. **GEOMETRY** The surface area of a basketball is $x$ square inches. What is the radius of the basketball if the formula for the surface area of a sphere is $SA = 4\pi r^2$?

Examples 2 and 3  p. 625

Solve each equation. Check your solution.

2. $\sqrt{10h} + 1 = 21$  
3. $\sqrt{7r} + 2 + 3 = 7$  
4. $5 + \sqrt{g - 3} = 6$  
5. $\sqrt{3x} - 5 = x - 5$  
6. $\sqrt{2n} + 3 = n$  
7. $\sqrt{a} - 2 + 4 = a$

Practice and Problem Solving

Example 1  p. 624

8. **EXERCISE** Suppose the function $S = \pi \sqrt{\frac{9.8\ell}{7}}$, where $S$ represents speed in meters per second and $\ell$ is the leg length of a person in meters, can approximate the maximum speed that a person can run.
   
   a. What is the maximum running speed of a person with a leg length of 1.1 meters to the nearest tenth of a meter?
   
   b. What is the leg length of a person with a running speed of 2.7 meters per second to the nearest tenth of a meter?
   
   c. As a person’s leg length increases, does their speed increase or decrease? Explain.

Examples 2 and 3  p. 625

Solve each equation. Check your solution.

9. $\sqrt{a} + 11 = 21$  
10. $\sqrt{f} - 4 = 7$  
11. $\sqrt{n} - 3 = 6$

12. $\sqrt{c} + 10 = 4$  
13. $\sqrt{h} - 5 = 2\sqrt{3}$  
14. $\sqrt{k} + 7 = 3\sqrt{2}$

15. $y = \sqrt{12} - y$  
16. $\sqrt{u} + 6 = u$  
17. $\sqrt{r} + 3 = r - 3$

18. $\sqrt{1 - 2t} = 1 + t$  
19. $5\sqrt{a} - 3 + 4 = 14$  
20. $2\sqrt{x} - 11 - 8 = 4$

21. **RIDES** The amount of time $t$, in seconds, that it takes a simple pendulum to complete a full swing is called the period of the pendulum. It is given by $t = 2\pi \sqrt{\frac{\ell}{32}}$, where $\ell$ is the length of the pendulum, in feet.
   
   a. The Giant Swing completes a period in about 8 seconds. About how long is the pendulum’s arm? Round to the nearest foot?
   
   b. Does increasing the length of the pendulum increase or decrease the period? Explain.

Solve each equation. Check your solution.

22. $\sqrt{6a} - 6 = a + 1$  
23. $\sqrt{x^2 + 9x + 15} = x + 5$  
24. $6\sqrt{\frac{5k}{4}} - 3 = 0$

25. $\sqrt{\frac{5y}{6}} - 10 = 4$  
26. $\sqrt{2a^2 - 121} = a$  
27. $\sqrt{5x^2 - 9} = 2x$

28. **GEOMETRY** The formula for the slant height $c$ of a cone is $c = \sqrt{h^2 + r^2}$, where $h$ is the height of the cone and $r$ is the radius of its base. Find the height of the cone if the slant height is 4 and the radius is 2. Round to the nearest tenth.

---

Real-World Link

The Giant Swing at Silver Dollar City in Branson, Missouri, swings riders at 45 miles per hour and reaches a height of 7 stories.

Source: Silver Dollar City Amusement Park
MULTIPLE REPRESENTATIONS In this problem, you will solve a radical equation by graphing. Consider the equation \( \sqrt{2x - 7} = x - 7 \).

a. **GRAPHICAL** Clear the Y= list. Enter the left side of the equation as \( Y_1 = \sqrt{2x - 7} \). Enter the right side of the equation as \( Y_2 = x - 7 \). Press GRAPH.

b. **GRAPHICAL** Sketch what is shown on the screen.

c. **ANALYTICAL** Use the intersect feature on the CALC menu to find the point of intersection.

d. **ANALYTICAL** Solve the radical equation algebraically. How does your solution compare to the solution from the graph?

30. **PACKAGING** A cylindrical container of chocolate drink mix has a volume of 162 cubic inches. The radius \( r \) of the container can be found by using the formula \( r = \sqrt{\frac{V}{\pi h}} \), where \( V \) is the volume of the container and \( h \) is the height.

a. If the radius is 2.5 inches, find the height of the container. Round your answer to the nearest hundredth.

b. If the height of the container is 10 inches, find the radius of the container. Round to the nearest hundredth

H.O.T. Problems

31. **FIND THE ERROR** Jada and Fina solved \( \sqrt{6 - b} = \sqrt{b + 10} \). Is either of them correct? Explain.

![Jada and Fina solutions](image)

32. **REASONING** Which equation has the same solution set as \( \sqrt{4} = \sqrt{x + 2} \)? Explain
   A. \( \sqrt{4} = \sqrt{x} + \sqrt{2} \)  
   B. \( 4 = x + 2 \)  
   C. \( 2 - \sqrt{2} = \sqrt{x} \)

33. **REASONING** Explain how solving the equation \( 5 = \sqrt{x} + 1 \) is different from solving the equation \( 5 = \sqrt{x} + 1 \).

34. **OPEN ENDED** Write a radical equation with a variable on each side. Then solve the equation.

35. **REASONING** Is the following equation sometimes, always or never true? Explain.
   \( \sqrt{(x - 2)^2} = x - 2 \)

36. **CHALLENGE** Solve \( \sqrt{x + 9} = \sqrt{3} + \sqrt{x} \).

37. **WRITING IN MATH** Write some general rules about how to solve radical equations. Demonstrate your rules by solving a radical equation.
38. SHORT RESPONSE  Zach needs to drill a hole at each of the points A, B, C, D, and E on circle P.

If Zack drills holes so that \( \angle APE = 110^\circ \) and the other four angles are equal in measure, what is \( \angle CPD \)?

39. Which expression is undefined when \( w = 3 \)?

- A \( \frac{w - 3}{w + 1} \)
- B \( \frac{w^2 - 3w}{3w} \)
- C \( \frac{w + 1}{w^2 - 3w} \)
- D \( \frac{3w}{3w^2} \)

40. What is the slope of a line that is parallel to the line?

41. What are the solutions of \( \sqrt{x + 3} - 1 = x - 4 \)?

- A 1, 6
- B -1, -6
- C 1
- D 6

Simplify each expression. (Lesson 10-2)

42. ELECTRICITY  The voltage \( V \) required for a circuit is given by \( V = \sqrt{PR} \), where \( P \) is the power in watts and \( R \) is the resistance in ohms. How many more volts are needed to light a 100-watt light bulb than a 75-watt light bulb if the resistance of both is 110 ohms? (Lesson 10-3)

Simplify each expression. (Lesson 10-2)

43. \( \sqrt{6} \cdot \sqrt{8} \)
44. \( \sqrt{3} \cdot \sqrt{6} \)
45. \( 7\sqrt{3} \cdot 2\sqrt{6} \)
46. \( \sqrt{\frac{27}{a^2}} \)
47. \( \sqrt{\frac{5c^5}{4d^3}} \)
48. \( \sqrt[4]{9x^3y} \cdot \sqrt[4]{16x^2y^2} \)

49. PHYSICAL SCIENCE  A projectile is shot straight up from ground level. Its height \( h \), in feet, after \( t \) seconds is given by \( h = 96t - 16t^2 \). Find the value(s) of \( t \) when \( h \) is 96 feet. (Lesson 9-5)

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write prime. (Lesson 8-4)

50. \( 2x^2 + 7x + 5 \)
51. \( 6p^2 + 5p - 6 \)
52. \( 5d^2 + 6d - 8 \)
53. \( 8k^2 - 19k + 9 \)
54. \( 9g^2 - 12g + 4 \)
55. \( 2a^2 - 9a - 18 \)

Determine whether each expression is a monomial. Write yes or no. Explain. (Lesson 7-1)

56. 12    57. \( 4x^3 \)    58. \( a - 2b \)    59. \( 4n + 5p \)    60. \( \frac{x}{y^2} \)    61. \( \frac{1}{5}abc^{14} \)

Simplify. (Lesson 1-1)

62. \( 9^2 \)    63. \( 10^6 \)    64. \( 4^5 \)    65. \( (8v)^2 \)    66. \( \left( \frac{w^3}{9} \right)^2 \)    67. \( (10y^2)^3 \)
Graph each function. Compare to the parent graph. State the domain and range. (Lesson 10-1)

1. \( y = 2\sqrt{x} \)
2. \( y = -4\sqrt{x} \)
3. \( y = \frac{1}{2}\sqrt{x} \)
4. \( y = \sqrt{x} - 3 \)
5. \( y = \sqrt{x} - 1 \)
6. \( y = 2\sqrt{x} - 2 \)

7. **GEOMETRY** The length of the side of a square is given by the function \( s = \sqrt{A} \), where \( A \) is the area of the square. What is the length of the side of a square that has an area of 121 square inches? (Lesson 10-1)
   
   A. 121 inches  
   B. 11 inches  
   C. 44 inches  
   D. 10 inches

Simplify each expression. (Lesson 10-2)

8. \( 2\sqrt{25} \)
9. \( \sqrt{12} \cdot \sqrt{8} \)
10. \( \sqrt[3]{72xy^5z^6} \)
11. \( \frac{3}{1 + \sqrt{5}} \)
12. \( \frac{1}{5 - \sqrt{7}} \)

13. **SATELLITES** A satellite is launched into orbit 200 kilometers above Earth. The orbital velocity of a satellite is given by the formula \( v = \sqrt{\frac{GM_E}{r}} \). \( v \) is velocity in meters per second, \( G \) is a given constant, \( m_E \) is the mass of Earth, and \( r \) is the radius of the satellite’s orbit. (Lesson 10-2)
   
   a. The radius of Earth is 6,380,000 meters. What is the radius of the satellite’s orbit in meters?
   
   b. The mass of Earth is \( 5.97 \times 10^{24} \) kilogram, and the constant \( G \) is \( 6.67 \times 10^{-11} \) N \( \cdot \) m\(^2\) / kg\(^2\), where N is in Newtons. Use the formula to find the orbital velocity of the satellite in meters per second.

14. Which expression is equivalent to \( \sqrt{\frac{16}{32}} \)? (Lesson 10-2)
   
   F. \( \frac{1}{2} \)  
   G. 2  
   H. \( \frac{\sqrt{2}}{2} \)  
   J. 4

Simplify each expression. (Lesson 10-3)

15. \( 3\sqrt{2} + 5\sqrt{2} \)
16. \( \sqrt{11} - 3\sqrt{11} \)
17. \( 6\sqrt{2} + 4\sqrt{50} \)
18. \( \sqrt{27} - \sqrt{48} \)
19. \( 4\sqrt{3}(2\sqrt{6}) \)
20. \( 3\sqrt{20}(2\sqrt{5}) \)
21. \( (\sqrt{5} + \sqrt{7})(\sqrt{20} + \sqrt{3}) \)

22. **GEOMETRY** Find the area of the rectangle. (Lesson 10-3)

23. \( \sqrt{5}x - 1 = 4 \)
24. \( \sqrt{a} - 2 = 6 \)
25. \( \sqrt{15} - x = 4 \)
26. \( \sqrt{3x^2 - 32} = x \)
27. \( \sqrt{2x - 1} = 2x - 7 \)
28. \( \sqrt{x + 1} + 2 = 4 \)

29. **GEOMETRY** The surface area \( S \) of a cone can be found by using the formula \( S = \pi r\sqrt{r^2 + h^2} \), where \( r \) is the radius of the base and \( h \) is the height of the cone. Find the height of the cone. (Lesson 10-4)

\[ S = 121 \text{ in}^2 \]
Then
You solved quadratic equations by using the Square Root Property. (Lesson 8-6)

Now
- Solve problems by using the Pythagorean Theorem.
- Determine whether a triangle is a right triangle.

KY Program of Studies
HS-M-S-MPA6 Students will apply definitions and properties of right triangle relationships to determine length and angle measures to solve realistic problems.
HS-M-S-MPA7 Students will apply special right triangles and the converse of the Pythagorean theorem to solve realistic problems. Also addresses HS-NPO-S-E6 and HS-G-S-CG3.

New Vocabulary
- hypotenuse
- legs
- converse
- Pythagorean triple

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- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

The Pythagorean Theorem

**Why?**

The designer television shown is made of black and white leather just like a real soccer ball. Televisions are measured along the diagonal of the screen. If the height and width of the screen is known, the Pythagorean Theorem can be used to find the measure of the diagonal.

The Pythagorean Theorem In a right triangle, the side opposite the right angle is the **hypotenuse**. This side is always the longest. The other two sides are the **legs**. The Pythagorean Theorem describes the relationship of the lengths of the sides of a right triangle.

**Key Concept**

**The Pythagorean Theorem**

**Words** If a triangle is a right triangle, then the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

**Symbols** \[ c^2 = a^2 + b^2 \]

**EXAMPLE 1** Find the Length of a Side

Find each missing length. If necessary, round to the nearest hundredth.

a. \[
\begin{align*}
10 & \quad \quad \quad c \\
24 & \quad \quad \quad b \\
\end{align*}
\]

**Pythagorean Theorem**

\[ c^2 = a^2 + b^2 \]

\[ c^2 = 10^2 + 24^2 \]

\[ c^2 = 100 + 576 \]

\[ c^2 = 676 \]

\[ c = \pm \sqrt{676} \]

\[ c = \pm 26 \]

Evaluate squares. Simplify. Take the square root of each side. 

A length cannot be negative. The missing length is 26 units.

b. \[
\begin{align*}
15 & \quad \quad \quad c \\
7 & \quad \quad \quad b \\
\end{align*}
\]

**Pythagorean Theorem**

\[ c^2 = a^2 + b^2 \]

\[ 15^2 = 7^2 + b^2 \]

\[ 225 = 49 + b^2 \]

\[ 176 = b^2 \]

\[ \pm \sqrt{176} = b \]

\[ \pm 13.27 \approx b \]

Evaluate squares. Subtract 49 from each side. Take the square root of each side. Use a calculator to evaluate \( \sqrt{176} \).

The missing length is 13.27 units.

**Check Your Progress**

1A. \[
\begin{align*}
8 & \quad \quad \quad c \\
6 & \quad \quad \quad b \\
\end{align*}
\]

1B. \[
\begin{align*}
12 & \quad \quad \quad a \\
16 & \quad \quad \quad b \\
\end{align*}
\]
EXAMPLE 2 Find the Length of a Side

SAILING The sail of a keelboat forms a right angle as shown at the right. Find the height of the sail.

\[20^2 = h^2 + 10^2\]  \hspace{1cm} \text{Pythagorean Theorem}
\[400 = h^2 + 100\]  \hspace{1cm} \text{Evaluate squares.}
\[300 = h^2\]  \hspace{1cm} \text{Subtract 100 from each side.}
\[\pm \sqrt{300} = h\]  \hspace{1cm} \text{Take the square root of each side.}
\[\pm 17.32 \approx h\]  \hspace{1cm} \text{Use the calculator to evaluate } \sqrt{300}.
\[17.32 \approx h\]  \hspace{1cm} \text{Use the positive value.}

The sail is approximately 17.32 feet high.

Check Your Progress

2. Suppose the longest side of the sail is 30 feet long and the shortest side is 14 feet long. Find the height of the sail.

Right Triangles If you exchange the hypothesis and conclusion of an if-then statement, the result is the converse of the statement. The converse of the Pythagorean Theorem can be used to determine whether a triangle is a right triangle.

Key Concept Converse of the Pythagorean Theorem

If a triangle has side lengths \(a, b, \) and \(c\) such that \(c^2 = a^2 + b^2\), then the triangle is a right triangle. If \(c^2 \neq a^2 + b^2\), then the triangle is not a right triangle.

A Pythagorean triple is a group of three whole numbers that satisfy the equation \(c^2 = a^2 + b^2\), where \(c\) is the greatest number. Examples include \((3, 4, 5)\) and \((5, 12, 13)\). Multiples of Pythagorean triples also satisfy the converse of the Pythagorean Theorem, so \((6, 8, 10)\) is also a Pythagorean triple.

EXAMPLE 3 Check for Right Triangles

Determine whether 9, 12, and 16 can be sides of a right triangle.

Since the measure of the longest side is 16, let \(c = 16, a = 9, \) and \(b = 12\).

Then determine whether \(c^2 = a^2 + b^2\).

\[c^2 = a^2 + b^2\]  \hspace{1cm} \text{Pythagorean Theorem}
\[16^2 = 9^2 + 12^2\]  \hspace{1cm} \text{\(a = 9, b = 12, \) and \(c = 16\)}
\[256 = 81 + 144\]  \hspace{1cm} \text{Evaluate squares.}
\[256 \neq 225\]  \hspace{1cm} \text{Add.}

Since \(c^2 \neq a^2 + b^2\), segments with these measures cannot form a right triangle.

Check Your Progress

Determine whether each set of measures can be sides of a right triangle.

3A. 30, 40, 50  
3B. 6, 12, 18
Check Your Understanding

Example 1  
Find each missing length. If necessary, round to the nearest hundredth.

1.  
   \[ \triangle \] 3 \[ \triangle \] 4 \[ \triangle \] c \[ \triangle \]  

2.  
   \[ \triangle \] 21 \[ \triangle \] 4 \[ \triangle \] b \[ \triangle \]  

3.  
   \[ \triangle \] 19 \[ \triangle \] 6 \[ \triangle \] a \[ \triangle \]  

4.  
   \[ \triangle \] 12 \[ \triangle \] 8 \[ \triangle \] c \[ \triangle \]  

Example 2  
5. BASEBALL  
A baseball diamond is a square. The distance between consecutive bases is 90 feet.
   a. How far does a catcher have to throw the ball to get from home plate to second base?
   b. How far does a third baseman have to throw the ball to the first baseman?
   c. If the catcher is five feet behind home plate, how far does he have to throw the ball to second base?

Example 3  
Determine whether each set of measures can be sides of a right triangle.

6. 8, 12, 16  
7. 28, 45, 53  
8. 7, 24, 25  
9. 15, 25, 45

Practice and Problem Solving

Example 1  
Find each missing length. If necessary, round to the nearest hundredth.

10.  
   \[ \triangle \] 6 \[ \triangle \] 14 \[ \triangle \] c \[ \triangle \]  

11.  
   \[ \triangle \] 2 \[ \triangle \] 12 \[ \triangle \] b \[ \triangle \]  

12.  
   \[ \triangle \] 20 \[ \triangle \] \sqrt{11} \[ \triangle \] c \[ \triangle \]  

13.  
   \[ \triangle \] 9 \[ \triangle \] 31 \[ \triangle \] a \[ \triangle \]  

14.  
   \[ \triangle \] 16 \[ \triangle \] 26 \[ \triangle \] b \[ \triangle \]  

15.  
   \[ \triangle \] \sqrt{23} \[ \triangle \] \sqrt{5} \[ \triangle \] c \[ \triangle \]  

16.  
   \[ \triangle \] 7 \[ \triangle \] 25 \[ \triangle \] a \[ \triangle \]  

17.  
   \[ \triangle \] 5 \[ \triangle \] \sqrt{77} \[ \triangle \] b \[ \triangle \]  

18.  
   \[ \triangle \] \sqrt{3} \[ \triangle \] \sqrt{35} \[ \triangle \] a \[ \triangle \]  

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632 Chapter 10 Radical Functions and Geometry
**Example 2**  
Larry is buying an entertainment stand for his television. The diagonal of his television is 27 inches. The space for the television measures 20 inches by 26 inches. Will Larry’s television fit inside this stand? Explain.

**Example 3**  
Determine whether each set of measures can be sides of a right triangle. Then determine whether they form a Pythagorean triple.

20. 9, 40, 41  
21. $3, 2\sqrt{10}, \sqrt{41}$  
22. $4, \sqrt{26}, 12$

23. $\sqrt{5}, 7, 14$  
24. $8, 31.5, 32.5$  
25. $\sqrt{65}, 6\sqrt{2}, \sqrt{97}$

26. 18, 24, 30  
27. 36, 77, 85  
28. 17, 33, 98

29. **GEOMETRY** Refer to the triangle at the right.
   a. What is the length of side $a$?
   b. Find the area of the triangle.

30. **GARDENING** Khaliah wants to plant flowers in a triangular plot. She has three lengths of plastic garden edging that measure 8 inches, 15 inches, and 17 inches. Determine whether these pieces form a right triangle. Explain.

31. **LADDER** Mr. Takeo is locked out of his house. The only open window is on the second floor, which is 28 feet above the ground. There is a bush along the edge of the house, so he will have to place the neighbor’s ladder 10 feet from the house. What length of ladder does he need to reach the window?

Find the length of the hypotenuse. Round to the nearest hundredth.

32.  
33.  

34. **GEOMETRY** A rectangle has a base of 5 feet and a height of 12 feet. What is the length of the diagonal?

35. **GEOMETRY** A square has a diagonal with length of 6 meters. Find the length of the sides of the square.

36. **DOLLHOUSE** Alonso is building a dollhouse for his sister’s birthday. The house is 24 inches across and the slanted side is 16 inches long as shown. Find the height of the roof to the nearest tenth of an inch.

37. **GEOMETRY** Each side of a cube is 5 inches long. Find the length of a diagonal of the cube.
38. **TOWN SQUARES** The largest town square in the world is Tiananmen Square in Beijing, China, covering 98 acres.
   a. One square mile is 640 acres. Assuming that Tiananmen Square is a square, how many feet long is a side to the nearest foot?
   b. To the nearest foot, what is the diagonal distance across Tiananmen Square?

39. **TRUCKS** Violeta needs to construct a ramp to roll a cart of moving boxes from her garage into the back of her truck. The truck is 6 feet from the garage. The back of the truck is 36 inches above the ground. How long does the ramp have to be?

40. **GEOMETRY** A square has an area of 242 square inches. Find the length of the diagonal of the square.

41. **GEOMETRY** A rectangle has a width that is twice as long as its length and an area of 722 square inches. Find the length of the diagonal of the rectangle.

If \(c\) is the measure of the hypotenuse of a right triangle, find each missing measure. If necessary, round to the nearest hundredth.

42. \(a = x, b = x + 41, c = 85\)

43. \(a = 8, b = x, c = x + 2\)

44. \(a = 12, b = x - 2, c = x\)

45. \(a = x, b = x + 7, c = 97\)

46. \(a = x - 47, b = x, c = x + 2\)

47. \(a = x - 32, b = x - 1, c = x\)

48. **GEOMETRY** A right triangle has one leg that is 8 inches shorter than the other leg. The hypotenuse is 30 inches long. Find the length of each leg of the triangle.

49. **GEOMETRY** A rectangle has a diagonal with length of 8 centimeters. Its length is 4 centimeters longer than the width. Find the length and width of the rectangle.

**H.O.T. Problems**

50. **FIND THE ERROR** Wyatt and Dario are determining whether 36, 77, and 85 form a Pythagorean triple. Is either of them correct? Explain your reasoning.

   **Wyatt**
   \[36^2 + 77^2 = 85^2\]
   \[1296 + 5929 = 7225\]
   \[7225 = 7225\]
   yes

   **Dario**
   \[36^2 + 85^2 = 77^2\]
   \[1296 + 7225 = 5929\]
   \[9021 ≠ 5929\]
   no

51. **CHALLENGE** Find the value of \(x\) in the figure below.

   ![Diagram of a right triangle with sides 8, 14, and 2, with unknown side \(x\)]

52. **REASONING** Provide a counterexample to the statement.
   *Any two right triangles with the same hypotenuse have the same area.*

53. **OPEN ENDED** Draw a right triangle that has a hypotenuse of \(\sqrt{72}\) units.

54. **WRITING IN MATH** Explain how to determine whether three lengths could form a right triangle.
55. **GEOMETRY** Find the missing length.

\[
\begin{array}{c}
A \quad -17 \\
B \quad -12.69 \\
C \quad 12.69 \\
D \quad 17 \\
\end{array}
\]

56. What is the solution of this equation?

\[
x + 1 = \sqrt{x + 1}
\]

- F 0, 3
- H 0
- G 3
- J no solutions

57. **SHORT RESPONSE** A plumber charges $40 for the first hour of each house call plus $8 for each additional half hour he works. If the plumber works for 4 hours, how much does he charge?

58. Find the next term in the geometric sequence

\[
4, 3, 9, 27, 243, \ldots.
\]

- A 4
- B 81
- C 243
- D 64

59. Solve each equation. Check your solution. (Lesson 10-4)

- \[\sqrt{x} = 16\]
- \[\sqrt[4]{x} = 64\]
- \[\sqrt{10x} = 10\]
- \[\sqrt{x + 1} = 3 - \sqrt{x}\]

60. Simplify each expression. (Lesson 10-3)

- \(2\sqrt{3} + 5\sqrt{3}\)
- \(4\sqrt{5} - 2\sqrt{5}\)
- \(6\sqrt{7} + 2\sqrt{28}\)
- \(4\sqrt{3} + 6\sqrt{12}\)

61. **BUSINESS** The amount of money spent at West Outlet Mall in Midtown continues to increase. The total \(T(x)\) in millions of dollars can be estimated by the function \(T(x) = 12(1.12)^x\), where \(x\) is the number of years after it opened in 2005. Find the amount of sales for the mall in the years 2015, 2016, and 2017. (Lesson 9-6)

62. Describe how the graph of each function is related to the graph of \(f(x) = x^2\). (Lesson 9-3)

- \(g(x) = x^2 - 8\)
- \(h(x) = \frac{1}{4}x^2\)
- \(h(x) = -x^2 + 5\)
- \(h(x) = -x^2 - \frac{4}{3}\)

63. **ROCK CLIMBING** While rock climbing, Damaris launches a grappling hook from a height of 6 feet with an initial upward velocity of 56 feet per second. The hook just misses the stone ledge that she wants to scale. As it falls, the hook anchors on a ledge 30 feet above the ground. How long was the hook in the air? (Lesson 8-4)

64. Find each product. (Lesson 7-7)

- \((b + 8)(b + 2)\)
- \((x - 4)(x - 9)\)
- \((y + 4)(y - 8)\)
- \((8d + 3)(5d + 2)\)

65. **Skills Review** Solve each proportion. (Lesson 2-6)

- \[\frac{x}{5} = \frac{12}{3}\]
- \[\frac{12}{x} = \frac{3}{4}\]
- \[\frac{5}{4} = \frac{10}{x}\]
- \[\frac{3}{5} = \frac{12}{x + 8}\]
**The Distance and Midpoint Formulas**

**Why?**

Rescue helicopters use electronic Global Positioning Systems (GPS) to compute direct distances between two locations.

A rescue helicopter can fly 450 miles before it needs to refuel. A person needs to be flown from Washington, North Carolina, to Huntington, West Virginia. Each side of the square is 50 miles. Asheville, North Carolina, is at the origin, Huntington is at (0, 196), and Washington is at (310, 0). Can the helicopter make the trip without refueling?

**Distance Formula** The GPS system calculates direct distances by using the **Distance Formula**, which is based on the Pythagorean Theorem. You can find the distance between any two points on a coordinate plane by using the Distance Formula.

**Key Concept** **The Distance Formula**

The distance \(d\) between any two points with coordinates \((x_1, y_1)\) and \((x_2, y_2)\) is given by the following formula.

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

**EXAMPLE 1 Distance Between Two Points**

Find the distance between points at (5, 3) and (1, –2).

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

\[ = \sqrt{(1 - 5)^2 + (-2 - 3)^2} \]

\[ = \sqrt{(-4)^2 + (-5)^2} \]

\[ = \sqrt{16 + 25} \]

\[ = \sqrt{41} \text{ or about 6.4 units} \]

**Check Your Progress**

Find the distance between points with the given coordinates.

1A. (4, 2) and (–3, –1).

1B. (–7, –2) and (–5, –8)
EXAMPLE 2

Use the Distance Formula

ENTERTAINMENT  The Vaccaro Family is having a home theater system installed. The TV and the seating will be placed in opposite corners of the room. The manufacturer of the TV recommends that for the size of TV that they want, the seating should be placed at least 13 feet away. If each square is 1 foot long, is the Vaccaro’s room large enough for the TV?

The front of the TV screen is located at (1, 11), and the front of the sofa is located at (7, 2).

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

Distance Formula

\[
d = \sqrt{(7 - 1)^2 + (2 - 11)^2}
\]

\[
d = \sqrt{6^2 + (-9)^2}
\]

Simplify.

\[
d = \sqrt{117} \text{ or about 10.8 feet}
\]

No, the room is not large enough for the TV.

Check Your Progress

2. The manufacturer of the speakers recommends that they be placed at least 8 feet from the seating. If one of the speakers is being placed at (0, 9), is the Vaccaros’ family room large enough for the speakers? Explain.

When we know the distance and one of the points, we can use the Distance Formula to find the coordinates of the other point.

EXAMPLE 3

Find a Missing Coordinate

Find the possible values for \(a\) if the distance between points at (4, 7) and \((a, 3)\) are 5 units apart.

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

Distance Formula

\[
d = \sqrt{(a - 4)^2 + (3 - 7)^2}
\]

\[
d = \sqrt{(a - 4)^2 + (-4)^2}
\]

Simplify.

\[
d = \sqrt{a^2 - 8a + 32}
\]

Evaluate squares and simplify.

\[
25 = a^2 - 8a + 32
\]

Square each side.

\[
0 = a^2 - 8a + 7
\]

Subtract 25 from each side.

\[
0 = (a - 1)(a - 7)
\]

Factor.

\[
a - 1 = 0 \quad \text{or} \quad a - 7 = 0
\]

Zero Product Property

\[
a = 1 \quad \text{or} \quad a = 7
\]

Solve each equation.

Check Your Progress

3. Find the possible values of \(a\) if the distance between points at (3, \(a\)) and \((-4, 5)\) is \(\sqrt{58}\) units.
**Midpoint Formula** The point that is equidistant from both of the endpoints is called the midpoint. You can find the coordinates of the midpoint by using the Midpoint Formula.

### Key Concept

**The Midpoint Formula**

Words

The midpoint $M$ of a line segment with endpoints at $(x_1, y_1)$ and $(x_2, y_2)$ is given by

$$M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$

Model

![Diagram of midpoint formula](image)

**EXAMPLE 4** Find the Midpoint

Find the coordinates of the midpoint of the segment with endpoints at $(-1, -2)$ and $(3, -4)$.

$$M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Midpoint Formula

$$= \left( \frac{-1 + 3}{2}, \frac{-2 + (-4)}{2} \right)$$

$$(x_1, y_1) = (-1, -2) \text{ and } (x_2, y_2) = (3, -4)$$

$$= \left( \frac{2}{2}, \frac{-6}{2} \right)$$

Simplify the numerators.

$$= (1, -3)$$

Simplify.

**Check Your Progress**

Find the coordinates of the midpoint of the segment with the given endpoints.

4A. $(12, 3), (-8, 3)$

4B. $(0, 0), (5, 12)$

4C. $(6, 8), (3, 4)$

### Check Your Understanding

**Example 1**

Find the distance between points with the given coordinates.

1. $(6, -2), (12, 8)$
2. $(4, 8), (-3, -6)$
3. $(3, 0), (6, -2)$
4. $(-2, -4), (-5, -3)$

**Example 2**

5. **GOLF** Addison hit a golf ball from a tee to the coordinate $(-3, 12)$, 12 feet past the hole and 3 feet to the left. The hole is located at the coordinate $(0, 0)$. Her first putt traveled to the coordinate $(1, 2)$, 2 feet beyond the hole and 1 foot to the right.

a. How far did the ball travel on her first putt?

b. How far was her first putt from the cup?
Lesson 10-6
The Distance and Midpoint Formulas

Example 3
p. 637
Find the possible values for $a$ if the points with the given coordinates are the indicated distance apart.
6. $(-5, a), (3, 1); d = \sqrt{89}$
7. $(6, a), (5, 0); d = \sqrt{17}$
8. $(5, 8), (a, 2); d = 3\sqrt{5}$
9. $(a, 6), (-6, 2); d = 4\sqrt{10}$

Example 4
p. 638
Find the coordinates of the midpoint of the segment with the given endpoints.
10. $(5, -10), (5, 8)$
11. $(2, -2), (6, 2)$
12. $(5, 0), (0, 3)$
13. $(-4, 1), (3, -1)$
14. $(3, -17), (2, -8)$
15. $(-17, 8), (-2, 20)$

Example 1
p. 636
Find the distance between points with the given coordinates.
18. $(5, 8), (5, 7)$
19. $(6, -9), (9, -9)$
20. $(3, -3), (7, 2)$
21. $(5, 1), (0, 4)$
22. $(-5, 2), (4, -2)$
23. $(3, 5), (-6, 0)$
24. $(-7, 8), (3, 10)$
25. $(-11, 9), (3, -4)$
26. $(8, 6), (-13, -2)$
27. $(5, 2), (3, -3)$
28. $(4, 2), (5, 5)$
29. $(-3, 5), (5, -3)$

Example 2
p. 637
30. NAVIGATION  Lawana and Ken are meeting at a restaurant in a marina. Ken takes his boat, while Lawana is driving her car. Each grid on the map represents 1 mile.

a. How far did Ken travel?
b. How far did Lawana travel?
c. How many times greater is the distance that Ken traveled than the distance that Lawana traveled?

Example 3
p. 637
Find the possible values for $a$ if the points with the given coordinates are the indicated distance apart.
31. $(-9, -2), (a, 5); d = 7$
32. $(a, -6), (-5, 2); d = 10$
33. $(a, 0), (3, 1); d = \sqrt{2}$
34. $(4, a), (8, 4); d = 2\sqrt{5}$
35. $(7, 5), (-9, a); d = 2\sqrt{65}$
36. $(-2, a), (6, 1); d = 4\sqrt{5}$

Example 4
p. 638
Find the coordinates of the midpoint of the segment with the given endpoints.
37. $(0, 2), (7, 3)$
38. $(5, -2), (3, -6)$
39. $(-4, 0), (0, 14)$
40. $(10, -3), (-8, -5)$
41. $(-5, 5), (3, -3)$
42. $(-16, -7), (-4, -3)$

Find the distance between points with the given coordinates.
43. $(4, 2), \left(6, -\frac{2}{3}\right)$
44. $\left(\frac{4}{5}, -1\right), \left(2, -\frac{1}{2}\right)$
45. $\left(4\sqrt{5}, 7\right), \left(6\sqrt{5}, 1\right)$

46. GEOMETRY  Triangle ABC has vertices $A(1, 3), B(-2, 5)$, and $C(8, 8)$. Find the perimeter of the triangle. Use a calculator to estimate the perimeter to the nearest tenth.
47. GEOMETRY  Quadrilateral JKLM has vertices $J(-3, -4), K(-1, 4), L(4, 5)$, and $M(6, -5)$. Find the perimeter of the quadrilateral to the nearest tenth.
48. **TEMPERATURE** The temperature dropped from 25° to −8° over a 12-hour period beginning at 12:00 noon. The table shows the temperature readings taken at 4-hour intervals.

<table>
<thead>
<tr>
<th>Time</th>
<th>12:00 noon</th>
<th>4:00</th>
<th>8:00</th>
<th>12:00 midnight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>25°</td>
<td>14°</td>
<td>3°</td>
<td>−8°</td>
</tr>
</tbody>
</table>

a. Plot these points on a coordinate plane with time on the x-axis and temperature on the y-axis. Let x represent the number of hours, and let 12:00 noon correspond to x = 0.

b. Draw a segment to connect the points. Find the midpoint of this segment that goes through these temperatures. Interpret the meaning of the midpoint in this situation.

49. **NAVIGATION** Two cruise ships are leaving St. Lucia Island at the same time. One travels 10 miles due east and then 8 miles north. The second ship travels 12 miles due north and then 6 miles west.

   a. If St. Lucia is at the origin, how far is the first ship from St. Lucia?
   b. How far is the second ship from St. Lucia?
   c. How far apart are the ships?

50. **TOURING** Sasha is using the GPS system in her car to go from her hotel to the art museum, to a restaurant, and then to the theater. Each grid on the screen represents 500 feet. Round your answers to the nearest hundredth.

   a. How far must she travel from the hotel to the art museum?
   b. What is the distance from the art museum to the restaurant?
   c. How far is it from the restaurant to the theater?
   d. If Sasha gets a direct distance reading from the theater to her hotel, how far is it?

Find the coordinates of the midpoint of the segment with the given endpoints.

51. (4.25, 2.5), (2.5, −3)
52. \((5, \frac{-1}{2}), \left(-3, \frac{5}{2}\right)\)
53. \(\left(\frac{2}{5}, \frac{-1}{5}\right), \left(\frac{1}{3}, \frac{5}{2}\right)\)

54. **CHALLENGE** \(A(−7, 3), B(4, 0),\) and \(C(−4, 4)\) are the vertices of a triangle. Discuss two different ways to determine whether \(\triangle ABC\) is a right triangle.

55. **REASONING** Explain why there are two possible values when looking for a missing coordinate when you are given two sets of coordinates and the distance between the two points.

56. **REASONING** Determine whether the following statement is true or false. Explain your reasoning.

   *It matters which ordered pair is first when using the Distance Formula.*

57. **OPEN ENDED** Plot two points and draw the segment between them. Find the coordinates of the midpoint.

58. **WRITING IN MATH** Explain how the Midpoint Formula is related to finding the mean.
59. SHORT RESPONSE  Two sailboats leave Key Largo, Florida, at the same time. One travels 3 miles east and then 4 miles north. The other travels 8 miles south and then 6 miles west. How far apart are the boats?

60. While in Tokyo, Callie spent 560 yen for a strand of pearls. The cost of the pearls was equivalent to $35 in U.S. currency. At the time of Callie’s purchase, how many yen were equivalent to $20 in U.S. currency?
   A 109 yen
   B 320 yen
   C 980 yen
   D 2350 yen

61. SHORT RESPONSE  On a coordinate grid, \( L \) represents the location of a lighthouse, and \( B \) represents a buoy. A ship is at the midpoint of the segment connecting \( L \) and \( B \). Which coordinates best represent the ship’s position?

62. At a family reunion, Guido cut a slice of cheesecake that was about one sixteenth of the cake. If the entire cheesecake contained 4480 Calories, which is the closest to the number of Calories in Guido’s slice?
   A 280
   B 373
   C 498
   D 560

Spiral Review

If \( c \) is the measure of the hypotenuse of a right triangle, find each missing measure. If necessary, round to the nearest hundredth.  (Lesson 10-5)

63. \( a = 16, b = 63, c = ? \)
64. \( b = 3, a = \sqrt{112}, c = ? \)
65. \( c = 14, a = 9, b = ? \)
66. \( a = 6, b = 3, c = ? \)
67. \( b = \sqrt{77}, c = 12, a = ? \)
68. \( a = 4, b = \sqrt{11}, c = ? \)

69. AVIATION  The relationship between a plane’s length \( L \) and the pounds \( P \) its wings can lift is described by \( L = \sqrt{kP} \), where \( k \) is the constant of proportionality. A Boeing 747 is 232 feet long and has a takeoff weight of 870,000 pounds. Determine \( k \) for this plane to the nearest hundredth.  (Lesson 10-4)

Skills Review

Solve each proportion. If necessary, round to the nearest hundredth.  (Lesson 2-6)

70. \( \frac{4}{d} = \frac{2}{10} \)
71. \( \frac{6}{5} = \frac{f}{15} \)
72. \( \frac{20}{28} = \frac{h}{21} \)
73. \( \frac{6}{7} = \frac{7}{j} \)
74. \( \frac{16}{7} = \frac{9}{m} \)
75. \( \frac{p}{2} = \frac{45}{68} \)
Simona needs to measure the height of a Ferris wheel for a class project. Simona can measure her shadow and the shadow of the Ferris wheel. She can then use similar triangles and indirect measurement to find the height of the Ferris wheel.

**Key Concept**

**Similar Triangles**

Similar triangles have the same shape, but not necessarily the same size. The symbol \( \sim \) is used to denote that two triangles are similar. The vertices of similar triangles are written in order to show the corresponding parts.

**Example**

If \( \triangle ABC \sim \triangle DEF \), then \( m \angle A = m \angle D \), \( m \angle B = m \angle E \), \( m \angle C = m \angle F \), and \( \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{1}{2} \).

**EXAMPLE 1**

Determine Whether Two Triangles are Similar

Determine whether the pair of triangles is similar. Justify your answer.

The measure of \( \angle T \) is \( 180 - (57 + 57) \) or \( 66^\circ \).

In \( \triangle XYZ \), \( \angle X \) and \( \angle Z \) have the same measure.

Let \( x = \) the measure of \( \angle X \) and \( \angle Z \).

\[
\begin{align*}
2x + 66 &= 180 \\
2x &= 114 \\
x &= 57
\end{align*}
\]

So, \( m \angle X = 57^\circ \) and \( m \angle Z = 57^\circ \).

Since the corresponding angles have equal measures, \( \triangle XYZ \sim \triangle STQ \).

**Check Your Progress**

1. Determine whether \( \triangle ABC \) with \( m \angle A = 68 \) and \( m \angle B = m \angle C \) is similar to \( \triangle DEF \) with \( m \angle E = m \angle F = 54 \). Justify your answer.
The ratios of the lengths of the corresponding sides can also be compared to verify that two triangles are similar.

**EXAMPLE 2** Determine Whether Two Triangles are Similar

Determine whether the pair of triangles is similar. Justify your answer.

If \( \triangle VXZ \) and \( \triangle WXY \) are similar, then the measures of their corresponding sides are proportional.

\[
\frac{VX}{WX} = \frac{12}{4} = 3 \quad \frac{XZ}{XY} = \frac{15}{5} = 3 \quad \frac{VZ}{WY} = \frac{9}{3} = 3
\]

Since the corresponding sides are proportional, \( \triangle VXZ \sim \triangle WXY \).

**Check Your Progress**

2. Determine whether \( \triangle ABC \) with \( AB = 6 \), \( BC = 16 \), and \( AC = 20 \) is similar to \( \triangle JKL \) with \( JK = 3 \), \( KL = 8 \), and \( JL = 9 \). Justify your answer.

---

**Find Unknown Measures** When some of the measurements of the sides of similar triangles are known, proportions can be used to find the missing measures.

**EXAMPLE 3** Find Missing Measures

Find the missing measures for the pair of similar triangles.

\[
\frac{AB}{AC} = \frac{PQ}{PR} \quad \text{Corresponding sides of similar triangles are proportional.} \quad \frac{BC}{AC} = \frac{QR}{PR} \quad \text{Corresponding sides of similar triangles are proportional.}
\]

\[
\begin{align*}
\frac{c}{8} &= \frac{12.5}{5} \\
5c &= 100 \\
c &= 20
\end{align*}
\]

The missing measures are 20 and 17.5.

**Check Your Progress**

3A. 

3B. 

---

**ReadingMath**

**Angle Measures** The \( m \angle A \) is read as the measure of angle \( A \).
EXAMPLE 4
Indirect Measurement

**SHADOWS** Tori is 5 feet 6 inches tall, and her shadow is 2 feet 9 inches long. She is standing next to a flagpole. If the length of the shadow of the flagpole is 12 feet long, how tall is the flagpole?

**Understand** Find the height of the flagpole.

**Plan** Make a sketch of the situation.

**Solve** The Sun’s rays form similar triangles. Write a proportion that compares the heights of the objects and the lengths of their shadows.

Let \( x \) = the height of the flagpole.

\[
\frac{\text{height of flagpole}}{\text{Tori's height}} = \frac{x}{5.5} = \frac{12}{2.75}
\]

\( 2.75x = 66 \)

\( x = 24 \) The height of the flagpole is 24 feet.

**Check**

\[
\frac{24}{5.5} = \frac{12}{2.75}
\]

Substitute 24 for \( x \).

\( 4.36 = 4.36 \) ✔

**Check Your Progress**

4. **TENTS** The directions for pitching a tent include a scale drawing in which 1 inch represents 4.5 feet. In the drawing, the tent is \( 1\frac{3}{4} \) inches tall. How tall should the actual tent be?

---

**Check Your Understanding**

1. Determine whether each pair of triangles is similar. Justify your answer.

   1.
   
   \[
   \triangleABC \sim \triangleXYZ
   \]
   
ihat\( c = 4, b = 6, c = 8, x = 6 \)

   2.
   
ihat\( a = 9, y = 15, z = 21, c = 7 \)

   3. \( a = 2, b = 5, x = 10, z = 30 \)

   4. \( b = 6, c = 10, x = 5, y = 15 \)

   5. **TREES** Marla wants to know the height of the tree in her backyard. The tree casts a shadow 8 feet 6 inches long. Marla is 5 feet tall, and her shadow is 2 feet 6 inches long. How tall is the tree?
Determine whether each pair of triangles is similar. Justify your answer.

8.  

9.  

10.  

11.  

12.  

13.  

Example 3  

Find the missing measures for the pair of similar triangles if \( \triangle HKM \sim \triangle PTR \).

14.  

15.  

16.  

17.  

18.  

19.  

Example 4  

20.  

TOYS  

Diecast model cars use a scale of 1 inch : 2 feet of the real vehicle. The original vehicle has a window shaped like a right triangle. If the height of the window on the actual vehicle is 2.5 feet, what will the height of the window be on the model?

21.  

GOLF  

Beatriz is playing miniature golf on a hole like the one shown at the right. She wants to putt her ball \( U \) so that it will bank at \( T \) and travel into the hole at \( R \). Use similar triangles to find where Beatriz’s ball should strike the wall.

22.  

MAPS  

The scale on an Ohio map shows that 2.5 centimeters represents 100 miles. The distance on the map from Cleveland to Cincinnati is 5.5 centimeters. About how many miles apart are the two cities?

23.  

SCHOOL PROJECT  

For extra credit in his history class, Marquez plans to make a model of the Statue of Liberty in the scale 1 inch : 10 feet. If the height of the actual Statue of Liberty is 151 feet, what will be the height of the model?

24.  

TENNIS  

Andy is practicing his overhand. He wants to hit the ball just over the net so it will land 16 feet away from the base of the net. If Andy hits the ball 40 feet away from the net, how high does he have to hit the ball?
25 MULTIPLE REPRESENTATIONS In this problem, you will compare the ratios of corresponding sides and perimeters of similar triangles.

a. ALGEBRAIC What is the ratio of the corresponding sides of each pair of similar triangles? Record your results in the table.

<table>
<thead>
<tr>
<th>Similar Triangles</th>
<th>Ratios of Sides</th>
<th>Perimeters</th>
<th>Ratios of Perimeters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pair 1</td>
<td>smaller triangle</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>larger triangle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pair 2</td>
<td>smaller triangle</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>larger triangle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pair 3</td>
<td>smaller triangle</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>larger triangle</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. TABULAR Find the perimeter of each triangle. Then find the ratio of the perimeters for each pair of triangles. Record your results in the table.

H.O.T. Problems

26. FIND THE ERROR Kwam and Rosalinda are comparing the similar triangles. Is either of them correct? Explain.

27. CHALLENGE Triangle XYZ is similar to the two triangles formed by the perpendicular line from Z to XY, and these two triangles are similar to each other. Write three similarity statements about these triangles. Why are the triangles similar to each other?

28. REASONING Determine whether the statement is sometimes, always, or never true. Explain your reasoning.

If the measures of the sides of a triangle are multiplied by 3, then the measures of the angles of the enlarged triangle will have the same measures as the angles of the original triangle.

29. OPEN ENDED Draw and label a triangle ABC. Then draw and label a similar triangle PQR so that the area of △PQR if four times the area of △ABC. Explain your strategy.

30. WRITING IN MATH Summarize how to determine whether two triangles are similar to each other and how to find missing measures of similar triangles.
31. Find the distance between the points at (2, -4) and (-5, 8).

\[
\begin{align*}
&\text{A} \, 5 \\
&\text{B} \, 7 \\
&\text{C} \sqrt{95} \\
&\text{D} \sqrt{193}
\end{align*}
\]

32. GEOMETRY Find the value of \(a\) if the two triangles are similar.

\[
\begin{align*}
5 & \quad a \\
14 & \quad 7
\end{align*}
\]

33. Which equation represents a line with a \(y\)-intercept of -4 and a slope of 6?

\[
\begin{align*}
&\text{A} \ y = 6x - 4 \\
&\text{B} \ y = -4x + 6 \\
&\text{C} \ y = -6x - 4 \\
&\text{D} \ y = 6x + 4
\end{align*}
\]

34. SHORT RESPONSE What are the \(x\) - and \(y\)-intercepts of the function graphed below?

\[
\text{Graph}
\]

35. (0, 3), (1, 9)

36. (-2, 4), (5, 13)

37. (1, -5), (-1, -5)

38. (7, -2), (-2, 4)

39. (-6, -3), (-1, 2)

40. (-4, -3), (-7, -8)

Determine whether each set of measures can be sides of a right triangle. (Lesson 10-5)

41. 3, 4, 5

42. 8, 10, 12

43. 10, 24, 26

44. 5, 12, 13

45. 6, 9, 14

46. 4, 5, 6

47. NUTRITION The consumption of bread and cereal in the United States is increasing and can be modeled by the function \(y = 0.059x^2 - 7.423x + 362.1\), where \(y\) represents the consumption of bread and cereal in pounds per person, and \(x\) represents the number of years since 1900. If this trend continues, in what future year will the average American consume 300 pounds of bread and cereal? (Lesson 9-4)

Factor each polynomial, if possible. If the polynomial cannot be factored, write prime. (Lesson 8-6)

48. \(4k^2 - 100\)

49. \(4a^2 - 36b^2\)

50. \(x^2 + 6x - 9\)

51. \(50g^2 + 40g + 8\)

52. \(9t^3 + 66t^2 - 48t\)

53. \(20n^2 + 34n + 6\)

54. DRIVING Average speed is calculated by dividing distance by time. If the speed limit on an interstate is 65 miles per hour, how far can a person travel legally in \(1\frac{1}{2}\) hours? (Lesson 5-2)

55. \(\frac{b}{c}\)

56. \(\frac{2ab}{c}\)

57. \(\frac{ac}{-4b}\)

58. \(\frac{-3ac}{2b}\)

59. \(\frac{-2bc}{a}\)

Evaluate if \(a = 3\), \(b = -2\), and \(c = 6\). (Lesson 1-2)
You can use paper triangles to investigate the ratios of the lengths of sides of right triangles.

**Collect the Data**

**Step 1** Use a ruler and grid paper to draw several right triangles with legs in a ratio of 5:8. Include right triangles with the side lengths listed in the table below and several more right triangles similar to these three. Label the vertices of each triangle as \( A \), \( B \), and \( C \), where \( C \) is at the right angle, \( B \) is opposite the longest leg, and \( A \) is opposite the shortest leg.

**Step 2** Copy the table below. Complete the first three columns by measuring the hypotenuse (side \( AB \)) in each right triangle you created and recording its length.

**Step 3** Calculate and record the ratios in the middle two columns. Round to the nearest thousandth, if necessary.

**Step 4** Use a protractor to carefully measure angles \( A \) and \( B \) in each right triangle. Record the angle measures in the table.

<table>
<thead>
<tr>
<th>Side Lengths</th>
<th>Ratios</th>
<th>Angle Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td>side ( BC )</td>
<td>side ( AC )</td>
<td>side ( AB )</td>
</tr>
<tr>
<td>2.5</td>
<td>4</td>
<td>90°</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>90°</td>
</tr>
<tr>
<td>10</td>
<td>16</td>
<td>90°</td>
</tr>
</tbody>
</table>

**Analyze the Results**

1. Examine the measures and ratios in the table. What do you notice? Write a sentence or two to describe any patterns you see.

**Make a Conjecture**

2. For any right triangle similar to the ones you have drawn here, what will be the value of the ratio of the length of the shortest leg to the length of the longest leg?

3. If you draw a right triangle and calculate the ratio of the length of the shortest leg to the length of the hypotenuse to be approximately 0.530, what will be the measure of the larger acute angle in the right triangle?
Trigonometric Ratios

Why?

A percent grade is the ratio of how much the road rises or falls in a given horizontal distance. If a road has a percent grade of 8%, this means the road rises or falls 8 feet over a horizontal distance of 100 feet. Trigonometric ratios can be used to determine the angle that the road rises or falls.

**Trigonometric Ratios** Trigonometry is the study of relationships among the angles and sides of triangles. A trigonometric ratio is a ratio that compares the side lengths of two sides of a right triangle. The three most common trigonometric ratios, sine, cosine, and tangent, are described below.

### Key Concept

<table>
<thead>
<tr>
<th>Words</th>
<th>Symbols</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>sine of ∠A = ( \frac{\text{leg opposite } \angle A}{\text{hypotenuse}} )</td>
<td>( \sin A = \frac{a}{c} )</td>
<td><img src="https://example.com/triangle.png" alt="Triangle Diagram" /></td>
</tr>
<tr>
<td>cosine of ∠A = ( \frac{\text{leg adjacent to } \angle A}{\text{hypotenuse}} )</td>
<td>( \cos A = \frac{b}{c} )</td>
<td></td>
</tr>
<tr>
<td>tangent of ∠A = ( \frac{\text{leg opposite } \angle A}{\text{leg adjacent to } \angle A} )</td>
<td>( \tan A = \frac{a}{b} )</td>
<td></td>
</tr>
</tbody>
</table>

Opposite, adjacent, and hypotenuse are abbreviated opp, adj, and hyp, respectively.

### Example 1

Find the values of the three trigonometric ratios for angle A.

**Step 1** Use the Pythagorean Theorem to find AC.

\[ a^2 + b^2 = c^2 \quad \text{Pythagorean Theorem} \]

\[ 9^2 + b^2 = 15^2 \]

\[ 81 + b^2 = 225 \quad \text{Simplify.} \]

\[ b^2 = 144 \quad \text{Subtract 81 from each side.} \]

\[ b = 12 \quad \text{Take the square root of each side.} \]

**Step 2** Use the side lengths to write the trigonometric ratios.

\[ \sin A = \frac{\text{opp}}{\text{hyp}} = \frac{9}{15} = \frac{3}{5} \]

\[ \cos A = \frac{\text{adj}}{\text{hyp}} = \frac{12}{15} = \frac{4}{5} \]

\[ \tan A = \frac{\text{opp}}{\text{adj}} = \frac{9}{12} = \frac{3}{4} \]
EXAMPLE 2  Use a Calculator to Evaluate Expressions

Use a calculator to find \( \cos 42^\circ \) to the nearest ten-thousandth.

**KEYSTROKES:**
\[
\text{cos} \ 42 \ \text{ENTER}
\]

Rounded to the nearest ten-thousandth, \( \cos 42^\circ \approx 0.7431 \).

**Check Your Progress**

2A. \( \sin 31^\circ \)  
2B. \( \tan 76^\circ \)  
2C. \( \cos 55^\circ \)

Use Trigonometric Ratios  When you find all unknown measures of the sides and angles of a right triangle, you are solving the triangle. You can find the missing measures if you know the measure of two sides of the triangle or the measure of one side and the measure of one acute angle.

EXAMPLE 3  Solve a Triangle

Solve the right triangle. Round each side length to the nearest tenth.

**Step 1** Find the measure of \( \angle A \).  \( 180^\circ - (90^\circ + 41^\circ) = 49^\circ \)

The measure of \( \angle A = 49^\circ \).

**Step 2** Find \( a \). Since you are given the measure of the side opposite \( \angle B \) and are finding the measure of the side adjacent to \( \angle B \), use the tangent ratio.

\[
\tan 41^\circ = \frac{6}{a}
\]

Definition of tangent

\[
a \tan 41^\circ = 6
\]

Multiply each side by \( a \).

\[
a = \frac{6}{\tan 41^\circ}, \text{ or about } 6.9
\]

Divide each side by \( \tan 41^\circ \). Use a calculator.

So the measure of \( a \) or \( BC \) is about 6.9.

**Step 3** Find \( c \). Since you are given the measure of the side opposite \( \angle B \) and are finding the measure of the hypotenuse, use the sine ratio.

\[
\sin 41^\circ = \frac{6}{c}
\]

Definition of sine

\[
c \sin 41^\circ = 6
\]

Multiply each side by \( c \).

\[
c = \frac{6}{\sin 41^\circ}, \text{ or about } 9.1
\]

Divide each side by \( \sin 41^\circ \). Use a calculator.

So the measure of \( c \) or \( AB \) is about 9.1.

**Check Your Progress**

3A.  
3B.  

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EXAMPLE 4  Find a Missing Side Length

EXERCISE  A personal trainer sets the incline on a treadmill to $10^\circ$. The walking surface of the treadmill is 5 feet long. Approximately how many inches did the trainer raise the end of the treadmill from the floor?

Solution:

\[
\sin 10^\circ = \frac{h}{5} \quad \text{Definition of sine}
\]

\[
5 \cdot \sin 10^\circ = h \quad \text{Multiply each side by 5.}
\]

\[
0.87 \approx h \quad \text{Use a calculator.}
\]

The value of $h$ is in feet. Multiply 0.87 by 12 to convert feet to inches.

The trainer raised the treadmill about 10.4 inches.

Check Your Progress

4. SKATEBOARDING  The angle that a skateboarding ramp forms with the ground is $25^\circ$ and the height of the ramp is 6 feet. Determine the length of the ramp.

A trigonometric function has a rule given by a trigonometric ratio. If you know the sine, cosine, or tangent of an acute angle, you can use the inverse of the trigonometric function to find the measure of the angle.

### Key Concept

**Inverse Trigonometric Functions**

<table>
<thead>
<tr>
<th>Words</th>
<th>If $\angle A$ is an acute angle and the sine of $A$ is $x$, then the inverse sine of $x$ is the measure of $\angle A$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbols</td>
<td>If $\sin A = x$, then $\sin^{-1} x = m\angle A$.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Words</th>
<th>If $\angle A$ is an acute angle and the cosine of $A$ is $x$, then the inverse cosine of $x$ is the measure of $\angle A$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbols</td>
<td>If $\cos A = x$, then $\cos^{-1} x = m\angle A$.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Words</th>
<th>If $\angle A$ is an acute angle and the tangent of $A$ is $x$, then the inverse tangent of $x$ is the measure of $\angle A$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbols</td>
<td>If $\tan A = x$, then $\tan^{-1} x = m\angle A$.</td>
</tr>
</tbody>
</table>

EXAMPLE 5  Find a Missing Angle Measure

Find $m\angle Y$ to the nearest degree.

You know the measure of the side adjacent to $\angle Y$ and the measure of the hypotenuse. Use the cosine ratio.

\[
\cos Y = \frac{8}{19} \quad \text{Definition of cosine}
\]

Use a calculator and the $[\cos^{-1}]$ function to find the measure of the angle.

**KEYSTROKES:**

\[ \text{2nd} [\cos^{-1}] 8 \div 19 \quad \text{ENTER} \]

65.098937

So, $m\angle Y = 65$.

Check Your Progress

5. Find $m\angle X$ to the nearest degree if $XY = 14$ and $YZ = 5$. 

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Check Your Understanding

Example 1  p. 649
Find the values of the three trigonometric ratios for angle $A$.

1. $\triangle ABC$ with sides $a = 25$, $b = 7$, and $c = 24$.
2. $\triangle ABC$ with sides $a = 3$, $b = 4$, and $c = 5$.
3. $\triangle ABC$ with sides $a = 12$, $b = 5$, and $c = 13$.
4. $\triangle ABC$ with sides $a = 85$, $b = 36$, and $c = 77$.

Example 2  p. 650
Use a calculator to find the value of each trigonometric ratio to the nearest ten-thousandth.

5. $\sin 37^\circ$  
6. $\cos 23^\circ$  
7. $\tan 14^\circ$  
8. $\cos 82^\circ$

Example 3  p. 650
Solve each right triangle. Round each side length to the nearest tenth.

9. $\triangle XYZ$ with angle $Z = 39^\circ$ and side $y = 7$.
10. $\triangle EFD$ with angle $E = 24^\circ$ and side $d = 14$.
11. $\triangle PQR$ with angle $R = 30^\circ$ and side $r = 5$.
12. $\triangle WXY$ with angle $X = 60^\circ$ and side $x = 9$.

Example 4  p. 651
13. **SNOWBOARDING** A hill used for snowboarding has a vertical drop of 3500 feet. The angle the run makes with the ground is $18^\circ$. Estimate the length of the run of the hill.

Example 5  p. 651
Find $m\angle X$ for each right triangle to the nearest degree.

14. $\triangle XYZ$ with sides $x = 12$, $y = 8$, and $z = 10$.
15. $\triangle XYZ$ with sides $x = 6$, $y = 15$, and $z = 17$.
16. $\triangle XYZ$ with sides $x = 3$, $y = 7$, and $z = 5$.
17. $\triangle XYZ$ with sides $x = 24$, $y = 13$, and $z = 25$.
Example 1  p. 649
Find the values of the three trigonometric ratios for angle $B$.

18. \[
\begin{array}{c}
A \quad 17 \\
B \quad 8 \\
C
\end{array}
\]

19. \[
\begin{array}{c}
A \quad 12 \\
B \quad 5 \\
C
\end{array}
\]

20. \[
\begin{array}{c}
A \quad 16 \\
B \quad 4 \\
C
\end{array}
\]

Example 2  p. 650
Use a calculator to find the value of each trigonometric ratio to the nearest ten-thousandth.

21. $\tan 2^\circ$  
22. $\sin 89^\circ$  
23. $\cos 44^\circ$  
24. $\tan 45^\circ$

25. $\sin 73^\circ$  
26. $\cos 90^\circ$  
27. $\sin 30^\circ$  
28. $\tan 60^\circ$

Example 3  p. 650
Solve each right triangle. Round each side length to the nearest tenth.

29. \[
\begin{array}{c}
X \quad 47^\circ \\
Y \quad 16 \\
Z
\end{array}
\]

30. \[
\begin{array}{c}
J \quad 68^\circ \\
K \quad 9 \\
L
\end{array}
\]

31. \[
\begin{array}{c}
Q \quad 14^\circ \\
R \quad 7 \\
P
\end{array}
\]

32. \[
\begin{array}{c}
A \quad 34^\circ \\
B \quad 18 \\
C
\end{array}
\]

33. \[
\begin{array}{c}
W \quad 51^\circ \\
U \quad 14 \\
V
\end{array}
\]

34. \[
\begin{array}{c}
D \quad 29^\circ \\
E \quad 5 \\
F
\end{array}
\]

Example 4  p. 651
35. ESCALATORS  At a local mall, an escalator is 110 feet long. The angle the escalator makes with the ground is $29^\circ$. Find the height reached by the escalator.

Example 5  p. 651
Find $m\angle J$ for each right triangle to the nearest degree.

36. \[
\begin{array}{c}
L \quad 10 \\
J \quad 24 \\
K
\end{array}
\]

37. \[
\begin{array}{c}
J \quad 17 \\
K \quad 15 \\
L
\end{array}
\]

38. \[
\begin{array}{c}
J \quad 14 \\
K \quad 23 \\
L
\end{array}
\]

39. \[
\begin{array}{c}
K \quad 6 \\
L \quad 10 \\
J
\end{array}
\]

40. \[
\begin{array}{c}
J \quad 16 \\
K \quad 5 \\
L
\end{array}
\]

41. \[
\begin{array}{c}
J \quad 11 \\
K \quad 17 \\
L
\end{array}
\]

42. MONUMENTS  The Lincoln Memorial building measures 204 feet long, 134 feet wide, and 99 feet tall. Chloe is looking at the top of the monument at an angle of $55^\circ$. How far away is she standing from the monument?
43. **AIRPLANES**  Ella looks down at a city from an airplane window. The airplane is 5000 feet in the air, and she looks down at an angle of 8°. Determine the horizontal distance to the city.

44. **FORESTS**  A forest ranger estimates the height of a tree is about 175 feet. If the forest ranger is standing 100 feet from the base of the tree, what is the measure of the angle formed between the ground and the top of the tree?

Suppose ∠A is an acute angle of right triangle ABC.

45. Find sin A and tan A if cos A = \(\frac{3}{4}\).
46. Find tan A and cos A if sin A = \(\frac{2}{7}\).
47. Find cos A and tan A if sin A = \(\frac{1}{4}\).
48. Find sin A and cos A if tan A = \(\frac{5}{3}\).

49. **SUBMARINES**  A submarine descends into the ocean at an angle of 10° and travels 3 miles. How far beneath the surface of the water has the submarine reached?

50. **MULTIPLE REPRESENTATIONS**  In this problem, you will explore a relationship between the sine and cosine functions.

![Image of triangles with labels A, B, C, J, K, L, X, Y, Z and trigonometric ratios]

**a. TABULAR**  Copy and complete the table using the triangles shown above.

<table>
<thead>
<tr>
<th>Triangle</th>
<th>Trigonometric Ratios</th>
<th>sin²</th>
<th>cos²</th>
<th>sin² + cos² =</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABC</td>
<td>(\sin A = )</td>
<td>(\cos A = )</td>
<td>(\sin^2 A = )</td>
<td>(\cos^2 A = )</td>
</tr>
<tr>
<td>JKL</td>
<td>(\sin J = )</td>
<td>(\cos J = )</td>
<td>(\sin^2 J = )</td>
<td>(\cos^2 J = )</td>
</tr>
<tr>
<td>XYZ</td>
<td>(\sin X = )</td>
<td>(\cos X = )</td>
<td>(\sin^2 X = )</td>
<td>(\cos^2 X = )</td>
</tr>
</tbody>
</table>

**b. VERBAL**  Make a conjecture about the sum of the squares of the sine and cosine functions of an acute angle in a right triangle.

**H.O.T. Problems**  Use Higher-Order Thinking Skills

51. **CHALLENGE**  Solve the triangle shown.
52. **REASONING**  Use the definitions of the sine and cosine ratios to define the tangent ratio.
53. **OPEN ENDED**  Write a problem that uses the cosine ratio to find the measure of an unknown angle in a triangle. Then solve the problem.
54. **REASONING**  The sine and cosine of an acute angle in a right triangle are equal. What can you conclude about the triangle?
55. **WRITING IN MATH**  Explain how to use trigonometric ratios to find the length of a missing side of a right triangle.

The tallest living tree is a redwood called Hyperion. It measures 378.1 feet.

There are about 135 redwood trees that are taller than 350 feet.

Source: SFGate
56. Which graph below represents the solution set for \(-2 \leq x \leq 4\)?

A

B

C

D

57. **Probability** A bin contains colored chips as shown in the table. What is the probability that a green chip is chosen?

<table>
<thead>
<tr>
<th>Color</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>yellow</td>
<td>7</td>
</tr>
<tr>
<td>blue</td>
<td>9</td>
</tr>
<tr>
<td>orange</td>
<td>3</td>
</tr>
<tr>
<td>green</td>
<td>5</td>
</tr>
<tr>
<td>red</td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>F</th>
<th>0.2</th>
<th>H</th>
<th>0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>0.5</td>
<td>J</td>
<td>0.8</td>
</tr>
</tbody>
</table>

58. In the graph, for what value(s) of \(x\) is \(y = 0\)?

A 0
B \(-1\)
C 1
D \(1 \text{ and } -1\)

59. **Extended Response** A 16-foot ladder is placed against the side of a house so that the bottom of the ladder is 8 feet from the base of the house.

a. If the bottom of the ladder is moved closer to the base of the house, does the height reached by the ladder increase or decrease?

b. What conclusion can you make about the distance between the bottom of the ladder and the base of the house and the height reached by the ladder?

c. How high does the ladder reach if the ladder is 3 feet from the base of the house?

---

**Spiral Review**

For each set of measures given, find the measures of the missing sides if \(\triangle ABC \sim \triangle DFH\). (Lesson 10-7)

60. \(a = 16, b = 12, c = 8, f = 6\)
61. \(d = 9, f = 6, h = 3, b = 18\)
62. \(a = 42, b = 21, h = 11, f = 14\)
63. \(c = 22.5, b = 20, h = 9, d = 2\)

Find the coordinates of the midpoint of the segment with the given endpoints. (Lesson 10-6)

64. \((5, 3), (11, 9)\)
65. \((8, 2), (6, 4)\)
66. \((-1, 7), (13, -3)\)

67. **Sales** A salesperson is paid $32,000 a year plus 5% of the amount in sales made. What is the amount of sales needed to have an annual income greater than $45,000? (Lesson 5-3)

---

**Skills Review**

Solve each proportion. (Lesson 2-6)

68. \(\frac{8}{9} = \frac{6}{z}\)
69. \(\frac{p}{6} = \frac{4}{3}\)
70. \(\frac{0.3}{r} = \frac{0.9}{1.7}\)
71. \(\frac{0.6}{1.1} = \frac{y}{8.47}\)
Key Vocabulary
- conjugate (p. 614)
- reverse (p. 631)
- cosine (p. 649)
- Distance Formula (p. 636)
- extraneous solutions (p. 625)
- hypotenuse (p. 630)
- inverse cosine (p. 651)
- inverse sine (p. 651)
- inverse tangent (p. 651)
- legs (p. 630)
- midpoint (p. 638)
- Midpoint Formula (p. 638)
- Pythagorean triple (p. 631)
- radical equations (p. 624)
- radical expression (p. 612)
- radical function (p. 612)
- radicand (p. 605)
- rationalizing the denominator (p. 614)
- similar triangles (p. 642)
- sine (p. 649)
- solving the triangle (p. 650)
- square root function (p. 605)
- tangent (p. 649)
- trigonometric function (p. 649)
- trigonometric ratio (p. 649)
- trigonometry (p. 649)

Vocabulary Check
State whether each sentence is true or false. If false, replace the underlined word, phrase, expression, or number to make a true sentence.

1. A triangle with sides having measures of 3, 4, and 6 is a right triangle.
2. Two triangles are congruent if corresponding angles are congruent.
3. The binomials \(2 + \sqrt{5}\) and \(2 - \sqrt{5}\) are conjugates.
4. In the expression \(-5\sqrt{2}\), the radicand is \(-2\).
5. The shortest side of a right triangle is the hypotenuse.
6. The cosine of an angle is found by dividing the measure of the side opposite of the angle by the hypotenuse.
7. The domain of the function \(y = \sqrt{x}\) is \([x | x \leq 0]\).
8. After the first step in solving \(\sqrt{2x + 4} = x + 5\), you would have \(2x + 4 = x^2 + 10x + 25\).
9. The converse of the Pythagorean Theorem is true.
10. The range of the function \(y = \sqrt{x}\) is \([y | y > 0]\).
Lesson-by-Lesson Review

10-1 Radical Functions (pp. 605–610)

Graph each function. Compare to the parent graph. State the domain and range.

11. \( y = \sqrt{x} + 3 \)
12. \( y = \sqrt{x} - 2 \)
13. \( y = -5\sqrt{x} \)
14. \( y = \sqrt{x} - 6 \)
15. \( y = \sqrt{x - 1} \)
16. \( y = \sqrt{x} + 5 \)

17. **GEOMETRY** The function \( s = \sqrt{A} \) can be used to find the length of a side of a square given its area. Use this function to determine the length of a side of a square with an area of 90 square inches. Round to the nearest tenth if necessary.

EXAMPLE 1

Graph \( y = -3\sqrt{x} \). Compare to the parent graph. State the domain and range.

Make a table. Choose nonnegative values for \( x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>0</td>
<td>-3</td>
<td>( \approx -4.2 )</td>
<td>( \approx -5.2 )</td>
<td>-6</td>
</tr>
</tbody>
</table>

Plot points and draw a smooth curve.

The graph is a dilation of \( y = \sqrt{x} \) and is reflected across the \( x \)-axis.

The domain is all positive \( x \) values or \( \{ x \mid x \geq 0 \} \).

The range is all negative \( y \) values or \( \{ y \mid y \leq 0 \} \).

10-2 Simplifying Radical Expressions (pp. 612–617)

Simplify.

18. \( \sqrt{36x^2y^7} \)
19. \( \sqrt{20ab^3} \)
20. \( \sqrt{3} \cdot \sqrt{6} \)
21. \( 2\sqrt{3} \cdot 3\sqrt{12} \)
22. \( (4 - \sqrt{5})^2 \)
23. \( (1 + \sqrt{2})^2 \)
24. \( \sqrt{\frac{50}{a^2}} \)
25. \( \sqrt{\frac{2}{5}} \cdot \sqrt{\frac{3}{4}} \)
26. \( \frac{3}{2 - \sqrt{5}} \)
27. \( \frac{5}{\sqrt{7} + 6} \)
28. **WEATHER** To estimate how long a thunderstorm will last, use the formula \( t = \sqrt{\frac{d^3}{216}} \) where \( t \) is the time in hours and \( d \) is the diameter of the storm in miles. A storm is 10 miles in diameter. How long will it last?

EXAMPLE 2

Simplify \( \frac{2}{4 + \sqrt{3}} \).

\[
\frac{2}{4 + \sqrt{3}} = \frac{2}{4 + \sqrt{3}} \cdot \frac{4 - \sqrt{3}}{4 - \sqrt{3}}
\]

Rationalize the denominator.

\[
= \frac{2(4) - 2\sqrt{3}}{4^2 - (\sqrt{3})^2} = \frac{8 - 2\sqrt{3}}{16 - 3} = \frac{8 - 2\sqrt{3}}{13}
\]

Simplify.

\[
\frac{2}{4 + \sqrt{3}} = \frac{2(4) - 2\sqrt{3}}{4^2 - (\sqrt{3})^2} = \frac{8 - 2\sqrt{3}}{13}
\]
10-3 Operations with Radical Expressions (pp. 619–623)

Simplify each expression.

29. \( \sqrt{6} - \sqrt{54} + 3\sqrt{12} + 5\sqrt{3} \)
30. \(2\sqrt{6} - \sqrt{48}\)
31. \(4\sqrt{3x} - 3\sqrt{3x} + 3\sqrt{3x}\)
32. \(\sqrt{50} + \sqrt{75}\)
33. \(\sqrt{2(5 + 3\sqrt{3})}\)
34. \((2\sqrt{3} - \sqrt{5})(\sqrt{10} + 4\sqrt{6})\)
35. \((6\sqrt{5} + 2)(4\sqrt{2} + \sqrt{3})\)

36. **MOTION** The velocity of a dropped object can be found using \(v = \sqrt{2gd}\), where \(v\) is the velocity in feet per second, \(g\) is the acceleration due to gravity, and \(d\) is the distance in feet the object drops. Find the speed of a penny when it hits the ground, after being dropped from a 984-foot building. Use 32 feet per second squared for \(g\).

**EXAMPLE 3**

Simplify \(2\sqrt{6} - \sqrt{24}\).

\[
2\sqrt{6} - \sqrt{24} = 2\sqrt{6} - \sqrt{4 \cdot 6} = 2\sqrt{6} - 2\sqrt{6} = 0
\]

**EXAMPLE 4**

Simplify \((\sqrt{3} - \sqrt{2})(\sqrt{3} + 2\sqrt{2})\).

\[
(\sqrt{3} - \sqrt{2})(\sqrt{3} + 2\sqrt{2}) = (\sqrt{3})(\sqrt{3}) + (\sqrt{3})(2\sqrt{2}) + (-\sqrt{2})(\sqrt{3}) + (\sqrt{2})(2\sqrt{2})
\]

\[
= 3 + 2\sqrt{6} - \sqrt{6} + 4 = 7 + \sqrt{6}
\]

10-4 Radical Equations (pp. 624–628)

Solve each equation. Check your solution.

37. \(10 + 2\sqrt{x} = 0\)
38. \(\sqrt{5} - 4x - 6 = 7\)
39. \(\sqrt{a} + 4 = 6\)
40. \(\sqrt{3x} = 2\)
41. \(\sqrt{x} + 4 = x - 8\)
42. \(\sqrt{3x} - 14 + x = 6\)

43. **FREE FALL** Assuming no air resistance, the time \(t\) in seconds that it takes an object to fall \(h\) feet can be determined by \(t = \sqrt{\frac{h}{4}}\). If a skydiver jumps from an airplane and free falls for 10 seconds before opening the parachute, how many feet does the skydiver free fall?

**EXAMPLE 5**

Solve \(\sqrt{7x} + 4 - 18 = 5\).

\[
\sqrt{7x} + 4 - 18 = 5
\]

\[
\sqrt{7x} + 4 = 5 + 18
\]

\[
\sqrt{7x} + 4 = 23
\]

\[
(\sqrt{7x} + 4)^2 = 23^2
\]

\[
7x + 4 = 529
\]

\[
7x = 525
\]

\[
x = 75
\]
**Determine whether each set of measures can be sides of a right triangle.**

<table>
<thead>
<tr>
<th>Set of Measures</th>
<th>44. 6, 8, 10</th>
<th>45. 3, 4, 5</th>
<th>46. 12, 16, 21</th>
<th>47. 10, 12, 15</th>
<th>48. 2, 3, 4</th>
<th>49. 7, 24, 25</th>
<th>50. 5, 12, 13</th>
<th>51. 15, 19, 23</th>
</tr>
</thead>
</table>

**EXAMPLE 6**

Determine whether the set of measures 12, 16, and 20 can be sides of a right triangle.

\[
\begin{align*}
12^2 + 16^2 & = 20^2 \\
144 + 256 & = 400 \\
400 & = 400 \\
\end{align*}
\]

Multiply.

Add. These side measures form a right triangle.

**EXAMPLE 7**

Find the distance between \((-3, 5)\) and \((-5, -3)\) and the midpoint of the segment with those endpoints. Round to the nearest tenth if necessary.

\[
\begin{align*}
d & = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
& = \sqrt{[-3 - (-5)]^2 + [5 - (-3)]^2} \\
& = \sqrt{2^2 + 8^2} \\
& = \sqrt{4 + 64} \\
& \approx 8.2 \\
M & = \left(\frac{-3 + (-5)}{2}, \frac{5 + (-3)}{2}\right) \\
& = (-4, 1)
\end{align*}
\]
10-7 Similar Triangles (pp. 642–647)

Find the missing measures for the pair of similar triangles if \( \triangle ABC \sim \triangle DFG \).

60. \( a = 3, b = 4, c = 5, d = 12 \)
61. \( a = 3, b = 4, c = 5, d = 4.5 \)
62. \( a = 4, b = 8, c = 11, e = 4 \)
63. \( a = 5, b = 7, c = 9, f = 18 \)

64. MODELS Kristin is making a model of the artwork shown in the scale of 1 inch = 2 feet. If the height of the artwork is 10 feet, what will the height of the model be?

EXAMPLE 8

Find the missing measure for the pair of triangles if \( \triangle ABC \sim \triangle DFG \).

\[
\frac{AB}{DF} = \frac{BC}{FG}
\]

Corresponding sides of similar triangles are proportional.

\[
\frac{6}{g} = \frac{3}{9} \quad AB = 6, \quad BC = 3, \quad DF = g, \quad \text{and} \quad FG = 9
\]

Find the cross products and simplify.

10-8 Trigonometric Ratios (pp. 649–655)

Find the values of the three trigonometric ratios for angle \( A \).

65. \( \sin A = \frac{5}{13}, \quad \cos A = \frac{12}{13}, \quad \tan A = \frac{5}{12} \)
66. \( \sin A = \frac{3}{5}, \quad \cos A = \frac{4}{5}, \quad \tan A = \frac{3}{4} \)

67. RAMPS A ramp is at a 30° angle from the ground. The height of the ramp is 3 feet. How long is the ramp?

EXAMPLE 9

Find the values of the three trigonometric ratios for angle \( A \).

Find the hypotenuse: \( c^2 = 10^2 + 24^2 \), so \( c = 26 \).

\[
\sin A = \frac{\text{leg opposite } \angle A}{\text{hypotenuse}} = \frac{24}{26} = \frac{12}{13}
\]

\[
\cos A = \frac{\text{leg adjacent } \angle A}{\text{hypotenuse}} = \frac{10}{26} = \frac{5}{13}
\]

\[
\tan A = \frac{\text{leg opposite } \angle A}{\text{leg adjacent } \angle A} = \frac{24}{10} = \frac{12}{5}
\]
Graph each function, and compare to the parent graph. State the domain and range.

1. \( y = -\sqrt{x} \)  
2. \( y = \frac{1}{4}\sqrt{x} \)  
3. \( y = \sqrt{x} + 5 \)  
4. \( y = \sqrt{x + 4} \)

5. **GEOMETRY** The length of the side of a square is given by the function \( s = \sqrt{A} \), where \( A \) is the area of the square. What is the perimeter of a square that has an area of 64 square inches?
   A 64 inches  
   B 8 inches  
   C 32 inches  
   D 16 inches

Simplify each expression.

6. \( 5\sqrt{36} \)  
7. \( \frac{3}{1 - \sqrt{2}} \)  
8. \( 2\sqrt{3} + 7\sqrt{3} \)  
9. \( 3\sqrt{6}(5\sqrt{2}) \)

10. **GEOMETRY** Find the area of the rectangle.

\[
\begin{array}{c}
2\sqrt{14} \\
\sqrt{7}
\end{array}
\]

F 14\sqrt{2}  
G 14  
H 98\sqrt{2}  
J 7\sqrt{2}

Solve each equation. Check your solution.

11. \( \sqrt{10x} = 20 \)  
12. \( \sqrt{4x - 3} = 6 - x \)

13. **PACKAGING** A cylindrical container of chocolate drink mix has a volume of about 162 in\(^3\). The radius of the container can be found by using the formula \( r = \sqrt{\frac{V}{\pi h}} \), where \( r \) is the radius and \( h \) is the height. If the height is 8.25 inches, find the radius of the container.

Find each missing length. If necessary, round to the nearest tenth.

14. \( 6 \)  
15. \( 4 \)

Find the distance between points with the given coordinates.

16. \((2, 3), (3, 5)\)  
17. \((-3, 4), (-2, -3)\)  
18. \((-1, -1), (3, 2)\)  
19. \((-4, -6), (-7, 1)\)

Find the coordinates of the midpoint of the segment with the given endpoints.

20. \((2, 3), (3, 5)\)  
21. \((-3, 4), (-2, -3)\)  
22. \((-1, -1), (3, 2)\)  
23. \((-4, -8), (10, -6)\)

24. **PIZZA DELIVERY** The Pizza Place delivers pizza to any location within a radius of 5 miles from the store for free. A pizza delivery person drives 32 blocks north and then 45 blocks east to deliver a pizza. In this city, there are about 6 blocks per half mile.
   a. Should there be a charge for delivery? Explain.
   b. Describe two delivery situations that would result in about 5 miles.

25. Find the missing measures if \( \triangle ABC \sim \triangle XYZ \).

26. Find the values of the three trigonometric ratios for angle \( A \).

27. Find \( m\angle X \) to the nearest degree.
Draw a Picture

Sometimes it is easier to visualize how to solve a problem if you draw a picture first. You can sketch your picture on scrap paper or in your test booklet (if allowed). Be careful not make any marks on your answer sheet other than your answers.

**Strategies for Drawing a Picture**

**Step 1**
Read the problem statement carefully.
Ask yourself:
- What am I being asked to solve?
- What information is given in the problem?
- What is the unknown quantity for which I need to solve?

**Step 2**
Sketch and label your picture.
- Draw your picture as clearly and accurately as possible.
- Label the picture carefully. Be sure to include all of the information given in the problem statement.

**Step 3**
Solve the problem.
- Use your picture to help you model the problem situation with an equation.
  Then solve the equation.
- Check your answer to make sure it is reasonable.

**EXAMPLE**
Read the problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

One sunny day, a church steeple casts a shadow that is 24 feet 3 inches long. At the same time, Nicole casts a shadow that is 1 foot 9 inches long. If Nicole is 5 feet 3 inches tall, what is the height of the steeple?
Read each problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

1. A building casts a 15-foot shadow, while a billboard casts a 4.5-foot shadow. If the billboard is 26 feet high, what is the height of the building? Round to the nearest tenth if necessary.

2. Jamey places a mirror on the ground at a distance of 56 feet from the base of a water tower. When he stands at a distance of 6 feet from the mirror, he can see the top of the water tower in the mirror’s reflection. If Jamey is 5 feet 6 inches tall, what is the height of the water tower? Express your answer in feet and inches.
Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher on a sheet of paper.

1. What is the equation of the square root function graphed below?

![Graph of a square root function]

A\( y = -2\sqrt{x} + 1 \)
B\( y = -2\sqrt{x} + 3 \)
C\( y = 2\sqrt{x} + 3 \)
D\( y = 2\sqrt{x} + 1 \)

2. Simplify \( \frac{1}{4 + \sqrt{2}} \).

A\( \frac{4 + \sqrt{2}}{14} \)
B\( \frac{2 - \sqrt{2}}{7} \)
C\( \frac{4 - \sqrt{2}}{14} \)
D\( \frac{2 + \sqrt{2}}{7} \)

3. What is the area of the triangle below?

![Triangular area]

A\( 3\sqrt{2} + 10\sqrt{5} \)
B\( 17 + 5\sqrt{10} \)
C\( 12\sqrt{2} + 8\sqrt{5} \)
D\( 8.5 + 2.5\sqrt{10} \)

4. The formula for the slant height \( c \) of a cone is \( c = \sqrt{h^2 + r^2} \), where \( h \) is the height of the cone and \( r \) is the radius of its base. What is the radius of the cone below? Round to the nearest tenth.

![Cone diagram]

A 4.9  C 9.8
B 6.3  D 10.2

5. Which of the following sets of measures could not be the sides of a right triangle?

A \((12, 16, 24)\)  C \((24, 45, 51)\)
B \((10, 24, 26)\)  D \((18, 24, 30)\)

6. Which of the following is an equation of the line perpendicular to \(4x - 2y = 6\) and passing through \((4, -4)\)?

A\( y = \frac{3}{4}x + 3 \)
B\( y = \frac{3}{4}x - 1 \)
C\( y = \frac{1}{2}x - 4 \)
D\( y = \frac{1}{2}x - 2 \)

7. The scale on a map shows that 1.5 centimeters is equivalent to 40 miles. If the distance on the map between two cities is 8 centimeters, about how many miles apart are the cities?

A 178 miles  B 213 miles  C 224 miles  D 275 miles

Test-Taking Tip

Question 4 Substitute for \( c \) and \( h \) in the formula. Then solve for \( r \).
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

8. **GRIDDED RESPONSE** How many times does the graph of \( y = x^2 - 4x + 10 \) cross the x-axis?

9. Factor \( 2x^4 - 32 \) completely.

10. **GRIDDED RESPONSE** In football, a field goal is worth 3 points, and the extra point after a touchdown is worth 1 point. During the 2006 season, John Kasay of the Carolina Panthers scored a total of 100 points for his team by making a total of 52 field goals and extra points. How many field goals did he make?

11. Shannon bought a satellite radio and a subscription to satellite radio. What is the total cost for his first year of service?

<table>
<thead>
<tr>
<th>Item</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>radio</td>
<td>$39.99</td>
</tr>
<tr>
<td>subscription</td>
<td>$11.99 per month</td>
</tr>
</tbody>
</table>

12. **GRIDDED RESPONSE** The distance required for a car to stop is directly proportional to the square of its velocity. If a car can stop in 242 meters at 22 kilometers per hour, how many meters are needed to stop at 30 kilometers per hour?

13. The highest point in Kentucky is at an elevation of 4145 feet above sea level. The lowest point in the state is at an elevation of 257 feet above sea level. Which of the following inequalities best describes the elevations in the state of Kentucky?

14. Simplify the expression below. Show your work.

\[
\left( \frac{-2r^2 - 2q^2 - 2}{5r^4q^2t^{-3}} \right)
\]

15. **GRIDDED RESPONSE** For the first home basketball game, 652 tickets were sold for a total revenue of $5216. If each ticket costs the same, how much is the cost per ticket? State your answer in dollars.

**Extended Response**

Record your answers on a sheet of paper. Show your work.

16. Karen is making a map of her hometown using a coordinate grid. The scale of her map is 1 unit = 2.5 miles.

- a. What is the actual distance between Karen’s school and the park? Round to the nearest tenth of a mile if necessary.
- b. Suppose Karen’s house is located midway between the mall and the school. What coordinates represent her house? Show your work.

**Need Extra Help?**

<table>
<thead>
<tr>
<th>If you missed Question...</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
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<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Go to Lesson or Page...</td>
<td>10-1</td>
<td>10-2</td>
<td>10-3</td>
<td>10-4</td>
<td>10-5</td>
<td>4-4</td>
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