Why?

The Very Large Array is an arrangement of 27 radio antennas in a Y pattern. The data the antennas collect is used by astronomers around the world to study the planets and stars. Astrophysicists use and apply properties of exponents to model the distance and orbit of celestial bodies.

Then

In Chapter 1, you performed operations on expressions with exponents.

Now

In Chapter 7, you will:
- Simplify expressions involving monomials.
- Use scientific notation.
- Find degrees of polynomials, write polynomials in standard form, and add, subtract, and multiply polynomials.

KY Program of Studies

HS-NPO-S-N03 Students will multiply and divide numbers expressed in scientific notation.
HS-AT-S-VEO6 Students will add, subtract and multiply polynomials.

In Chapter 7, you will:
- Simplify expressions involving monomials.
- Use scientific notation.
- Find degrees of polynomials, write polynomials in standard form, and add, subtract, and multiply polynomials.
Get Ready for Chapter 7

Diagnose Readiness You have two options for checking Prerequisite Skills.

Text Option

Take the Quick Check below. Refer to the Quick Review for help.

Write each expression using exponents. (Lesson 1-1)

1. \(4 \cdot 4 \cdot 4 \cdot 4 \cdot 4\)
2. \(y \cdot y \cdot y\)
3. \(6 \cdot 6\)
4. \(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2\)
5. \(b \cdot b \cdot b \cdot b \cdot b\)
6. \(m \cdot m \cdot m \cdot p \cdot p \cdot p \cdot p \cdot p \cdot p\)
7. \(\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}\)
8. \(\frac{x}{y} \cdot \frac{x}{y} \cdot \frac{x}{y} \cdot \frac{w}{z} \cdot \frac{w}{z}\)

Evaluate each expression. (Lesson 1-2)

9. \(2^3\)
10. \((-5)^2\)
11. \(3^3\)
12. \((-4)^3\)
13. \(\left(\frac{2}{3}\right)^2\)
14. \(\left(\frac{1}{2}\right)^4\)
15. SCHOOL The probability of guessing correctly on 5 true-false questions is \(\frac{1}{2}\)^5. Express this probability as a fraction without exponents.

Find the area or volume of each figure. (Lessons 0-8 and 0-9)

16. A circle with a radius of 2 m.
17. A rectangular prism with dimensions 5 cm by 3 cm by 7 cm.
18. PHOTOGRAPHY A photo is 4 inches by 6 inches. What is the area of the photo?

EXAMPLE 1

Write \(5 \cdot 5 \cdot 5 + x \cdot x \cdot x\) using exponents.

4 factors of 5 is \(5^4\).

3 factors of \(x\) is \(x^3\).

So, \(5 \cdot 5 \cdot 5 + x \cdot x \cdot x = 5^4 + x^3\).

EXAMPLE 2

Evaluate \(\left(\frac{5}{7}\right)^2\).

\(\left(\frac{5}{7}\right)^2 = \frac{5^2}{7^2} = \frac{25}{49}\) Power of a Quotient

Simplify.

EXAMPLE 3

Find the volume of the figure.

\(V = \ell \cdot w \cdot h\)

\(V = 5 \cdot 5 \cdot 5 \text{ or } 125\)

\(\ell = 5, w = 5,\) and \(h = 5\)

The volume is 125 cubic feet.

Online Option

Take a self-check Chapter Readiness Quiz at glencoe.com.
Get Started on Chapter 7

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 7. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

**New Vocabulary**

<table>
<thead>
<tr>
<th>English</th>
<th>Español</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>constante</td>
</tr>
<tr>
<td>monomial</td>
<td>monomio</td>
</tr>
<tr>
<td>negative exponent</td>
<td>exponente negativo</td>
</tr>
<tr>
<td>zero exponents</td>
<td>cero exponente</td>
</tr>
<tr>
<td>order of magnitude</td>
<td>ordenar de magnitud</td>
</tr>
<tr>
<td>scientific notation</td>
<td>notación científica</td>
</tr>
<tr>
<td>binomial</td>
<td>binomio</td>
</tr>
<tr>
<td>degree of a monomial</td>
<td>grado de un monomio</td>
</tr>
<tr>
<td>degree of a polynomial</td>
<td>grado de un polinomio</td>
</tr>
<tr>
<td>polynomial</td>
<td>polinomio</td>
</tr>
<tr>
<td>trinomial</td>
<td>trinomio</td>
</tr>
<tr>
<td>leading coefficient</td>
<td>coeficiente líder</td>
</tr>
<tr>
<td>standard form of a polynomial</td>
<td>forma estándar de polinomio</td>
</tr>
<tr>
<td>FOIL method</td>
<td>método foil</td>
</tr>
<tr>
<td>quadratic expression</td>
<td>expresión cuadrática</td>
</tr>
</tbody>
</table>

**Distributive Property** • p. 23 • Propiedad distributiva
For any numbers $a$, $b$, and $c$, $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$.

**Exponent** • p. 5 • exponente
In an expression of the form $x^n$, the exponent is $n$. It indicates the number of times $x$ is used as a factor.

$x^n = x \times x \times x \times \ldots \times x$

### KY Math Online glencoe.com
- Study the chapter online
- Explore Math in Motion
- Get extra help from your own Personal Tutor
- Use Extra Examples for additional help
- Take a Self-Check Quiz
- Review Vocabulary in fun ways
Many formulas contain monomials. For example, the formula for the horsepower of a car is \( H = w \left( \frac{v}{234} \right)^3 \). \( H \) represents the horsepower produced by the engine, \( w \) equals the weight of the car with passengers, and \( v \) is the velocity of the car at the end of a quarter of a mile. As the velocity increases, the horsepower increases.

**Monomials** A monomial is a number, a variable, or the product of a number and one or more variables with nonnegative integer exponents. It has only one term. In the formula to calculate the horsepower of a car, the term \( w \left( \frac{v}{234} \right)^3 \) is a monomial.

An expression that involves division by a variable, like \( \frac{ab}{c} \), is not a monomial.

A constant is a monomial that is a real number. The monomial \( 3x \) is an example of a linear expression since the exponent of \( x \) is 1. The monomial \( 2x^2 \) is a nonlinear expression since the exponent is a positive number other than 1.
By applying the definition of a power, you can find the product of powers. Look for a pattern in the exponents.

<table>
<thead>
<tr>
<th>2 factors</th>
<th>4 factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2^2 \cdot 4^2 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$</td>
<td>$4^3 \cdot 4^2 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$</td>
</tr>
<tr>
<td>$2 + 4 = 6$ factors</td>
<td>$3 + 2 = 5$ factors</td>
</tr>
</tbody>
</table>

These examples demonstrate the property for the product of powers.

**Key Concept**

**Product of Powers**

**Words**
To multiply two powers that have the same base, add their exponents.

**Symbols**
For any real number $a$ and any integers $m$ and $p$, $a^m \cdot a^p = a^{m+p}$.

**Examples**
$b^3 \cdot b^5 = b^{3+5}$ or $b^8$
$g^4 \cdot g^6 = g^{4+6}$ or $g^{10}$

**EXAMPLE 2**

**Product of Powers**

Simplify each expression.

a. $(6n^3)(2n^7)$

$(6n^3)(2n^7) = (6 \cdot 2)(n^3 \cdot n^7)$

$= (6 \cdot 2)(n^{3+7})$

$= 12n^{10}$

b. $(3pt^3)(p^3t^4)$

$(3pt^3)(p^3t^4) = (3 \cdot 1)(p \cdot p^3)(t^3 \cdot t^4)$

$= (3 \cdot 1)(p^{1+3})(t^{3+4})$

$= 3p^4t^7$

**Check Your Progress**

2A. $(3y^4)(7y^3)$

2B. $(-4r^2x^3)(-6r^5x^2t)$

We can use the Product of Powers Property to find the power of a power. In the following examples, look for a pattern in the exponents.

$3^2 = (3^2)(3^2)(3^2)(3^2)$

$= 3^{2+2+2+2}$

$= 3^8$

$r^4 = (r^4)(r^4)(r^4)$

$= r^{4+4+4}$

$= r^{12}$

These examples demonstrate the property for the power of a power.
EXAMPLE 3  Power of a Power

Simplify \((2^3)^2\)^4.
\[
\left( (2^3)^2 \right)^4 = (2^6)^4
\]
\[
= 2^{24} \text{ or } 16,777,216
\]

Check Your Progress

Simplify each expression.

3A. \(\left( (2^2)^2 \right)^4\)
3B. \(\left( (3^3)^2 \right)^2\)

We can use the Product of Powers Property and the Power of a Power Property to find the power of a product. In the following examples, look for a pattern in the exponents.

\[
(tw)^3 = (tw)(tw)(tw) = (t \cdot t \cdot t)(w \cdot w \cdot w) = t^3w^3
\]

\[
(2yz^2)^3 = (2yz^2)(2yz^2)(2yz^2) = (2 \cdot 2 \cdot 2)(y \cdot y \cdot y)(z^2 \cdot z^2 \cdot z^2) = 2^3y^3z^6 \text{ or } 8y^3z^6
\]

These examples demonstrate the property for the power of a product.

Key Concept  Power of a Product

Words  To find the power of a product, find the power of each factor and multiply.

Symbols  For any real number \(a\) and any integer \(m\), \(\left( ab \right)^m = a^m b^m\).

Example  \((-2xy^3)^5 = (-2)^5x^5y^{15}\) or \(-32x^5y^{15}\)

EXAMPLE 4  Power of a Product

GEOMETRY  Express the area of the circle as a monomial.

Area = \(\pi r^2\)
\[
= \pi(2xy^2)^2
\]
\[
= \pi(2^2x^2y^4)
\]
\[
= 4x^2y^4\pi
\]

The area of the circle is \(4x^2y^4\pi\) square units.

Check Your Progress

4A. Express the area of a square with sides of length \(3xy^2\) as a monomial.
4B. Express the area of a triangle with a height of \(4a\) and a base of \(5ab^2\) as a monomial.
Simplify Expressions  We can combine and use these properties to simplify expressions involving monomials.

**Key Concept**  **Simplify Expressions**

To simplify a monomial expression, write an equivalent expression in which:
- each base appears exactly once,
- there are no powers of powers, and
- all fractions are in simplest form.

**EXAMPLE 5**  **Simplify Expressions**

Simplify $(3xy)^2[(-2y)^2]^3$.

$(3xy)^2[(-2y)^2]^3 = (3xy)^2(-2y)^6$

$= (3)^2x^2(y)^2(-2)^6y^6$

$= 9x^2y^8(64)y^6$

$= 9(64)x^2 \cdot y^8 \cdot y^6$

$= 576x^2y^{14}$

**Check Your Progress**

5. Simplify $\left(\frac{1}{2}a^2b^2\right)^3\left((-4b)^2\right)^2$.

**Check Your Understanding**

1. Determine whether each expression is a monomial. Write yes or no. Explain your reasoning.
   - 1. 15
   - 2. $2 - 3a$
   - 3. $\frac{5c}{d}$
   - 4. $-15g^2$
   - 5. $\frac{r}{2}$
   - 6. $7b + 9$
   - 7. $k(k^3)$
   - 8. $m^4(m^2)$
   - 9. $2q^2(9q^4)$
   - 10. $(5u^4v)(7u^4v^3)$
   - 11. $[(3^2)^2]^2$
   - 12. $(xy^4)^6$
   - 13. $(4a^4b^9c)^2$
   - 14. $(-2f^2g^3h^2)^3$
   - 15. $(-3p^5q^6)^4$  

16. **GEOMETRY**  The formula for the surface area of a cube is $SA = 6s^2$, where $SA$ is the surface area and $s$ is the length of any side.
   a. Express the surface area of the cube as a monomial.
   b. What is the surface area of the cube if $a = 3$ and $b = 4$?

17. Simplify each expression.
   - 17. $(5x^2y)(2xy^2z)^3(4xyz)$
   - 18. $(-3d^2f^3g)^2[(-3d^2g)^3]^2$
   - 19. $(-2g^3h)(-3g^4)^2(-gh)^2$
   - 20. $(-7ab^4c)^3[(2a^2c)^2]^3$
Example 1  
\[ \text{p. 401} \] 
Determine whether each expression is a monomial. Write yes or no. Explain your reasoning.

21. \(122\)  
22. \(3a^4\)  
23. \(2c + 2\)  
24. \(-\frac{2a}{3b}\)  
25. \(\frac{5k}{10}\)  
26. \(6m + 3n\)

Examples 2 and 3  
\[ \text{pp. 402–403} \]
Simplify each expression.

27. \((q^3)(2q^4)\)  
28. \((-2u^2)(6u^6)\)  
29. \((9w^2-x^8)(w^6x^4)\)  
30. \((y^6z^3)(6y^4z^2)\)  
31. \((b^8c^6d^3)(7b^6c^2d)\)  
32. \((14g^2h^2)(-3g^4h^2)\)  
33. \((i^5k^7)^4\)  
34. \((n^3p)^4\)  
35. \(\left[(2^2)^2\right]^2\)  
36. \(\left[(3^2)^3\right]^4\)  
37. \(\left[(4r^2i)^3\right]^2\)  
38. \(\left[(-2xy^2)^3\right]^2\)

Example 4  
\[ \text{p. 403} \]
GEOMETRY  
Express the area of each triangle as a monomial.

39.  
40.  

Example 5  
\[ \text{p. 404} \]
Simplify each expression.

41. \((2a^3)^4(a^3)^3\)  
42. \((c^3)^2(-3c^5)^2\)  
43. \((2g^4h^3)[(-2g^4h)^2]^2\)  
44. \((5k^2m)^3[4km^4]^2\)  
45. \((p^5r^3)(-7p^3r^4)(6pr^3)\)  
46. \((5x^2y)^2(2xy^2z)^3(4xyz)\)  
47. \((5a^2b^3c^4)(6a^3b^4c^2)\)  
48. \((10xy^5z^3)(3x^4y^6z^3)\)  
49. \((0.5x^3)^2\)  
50. \((0.4h^3)^3\)  
51. \(\left(-\frac{3}{4}\right)^3\)  
52. \(\left(-\frac{3}{4}\right)^2\)  
53. \((8y^3)(-3x^2y^2)(\frac{3}{8}xy^4)\)  
54. \((\frac{4}{7}m)^2(49m)(17p)(\frac{1}{3}p^5)\)  
55. \((-3r^3w^3)^2(2rw)^2(-3r^2)^3(4rw^2)^3(2r^2w^3)^4\)  
56. \((3ab^2c)^2(-2a^2b^4)^2(a^4c^2)^3(a^2b^4c^5)^2(2a^3b^2c^4)^3\)

57. SAVINGS  
Cleavon is saving money in a bank account that offers him 3% simple interest. The formula for computing simple interest is \(I = Prt\), where \(I\) is the interest earned, \(P\) represents the principal that he put into the account, \(r\) is the interest rate (in decimal form), and \(t\) represents time in years.

a. Cleavon makes a deposit of \(\$2c\) and leaves it for 2 years. Write a monomial that represents the interest earned.

b. If \(c\) represents a birthday gift of \(\$250\), how much will Cleavon have in this account after 2 years?

GEOMETRY  
Express the volume of each solid as a monomial.

58.  
59.  
60.  

84% of teens have some money saved. The average teen has \$1044 saved.  
Source: Charles Schwab Teens & Money Survey
61. **PACKAGING** For a commercial art class, Aiko must design a new container for individually wrapped pieces of candy. The shape that she chose for her container is a cylinder. The formula for the volume of a cylinder is \( V = \pi r^2 h \).

a. The radius that Aiko would like to use is \( 2p^3 \), and the height is \( 4p^3 \). Write a monomial that represents the volume of her container.

b. Make a table of values for five possible radius widths and heights of Aiko’s container if the volume is to remain the same.

c. What is the volume of Aiko’s container if the height is doubled?

62. **ENERGY** Matter can be converted completely into energy by using the formula referred to in the Math History Link. Energy is measured in joules, mass in kilograms, and the speed of light is about 300 million meters per second.

a. Convert 3 kilograms of gasoline completely into energy.

b. What is the energy if the amount of gasoline is doubled?

63. **MULTIPLE REPRESENTATIONS** In this problem, you will explore exponents.

a. **TABULAR** Copy and use a calculator to complete the table showing powers of 3.

<table>
<thead>
<tr>
<th>Power</th>
<th>3⁰</th>
<th>3¹</th>
<th>3²</th>
<th>3³</th>
<th>3⁻¹</th>
<th>3⁻²</th>
<th>3⁻³</th>
<th>3⁻⁴</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{9} )</td>
<td>( \frac{1}{27} )</td>
<td>( \frac{1}{81} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. **ANALYTICAL** What do you notice about each power and its value?

c. **ANALYTICAL** What do you think the values of \( 5^0 \) and \( 5^{-1} \) are? Verify your conjecture using a calculator.

d. **ANALYTICAL** Complete: For any nonzero number \( a \) and any integer \( n \), \( a^{-n} = \text{____}. \)

e. **VERBAL** Describe the value of a number raised to the zero power.

---

**H.O.T. Problems**

64. **CHALLENGE** For any real nonzero numbers \( a \) and \( b \) and any integers \( m \) and \( t \), simplify the expression \( \left( \frac{a^m}{b^t} \right)^{2t} \) and describe each step.

65. **REASONING** Copy the table below.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Related Expression</th>
<th>Power of ( x )</th>
<th>Linear or Nonlinear</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = x )</td>
<td>( x^1 )</td>
<td>( 1 )</td>
<td>Linear</td>
</tr>
<tr>
<td>( y = x^2 )</td>
<td>( x^2 )</td>
<td>( 2 )</td>
<td>Nonlinear</td>
</tr>
<tr>
<td>( y = x^3 )</td>
<td>( x^3 )</td>
<td>( 3 )</td>
<td>Nonlinear</td>
</tr>
</tbody>
</table>

a. For each equation, write the related expression and record the power of \( x \).

b. Graph each equation using a graphing calculator.

c. Classify each graph as linear or nonlinear.

d. Explain how to determine whether an equation, or its related expression, is linear or nonlinear without graphing.

66. **OPEN ENDED** Write three different expressions that can be simplified to \( x^6 \).

67. **WRITING IN MATH** Write two other formulas that have monomial expressions in them. Explain how each is used in a real-world situation.
68. Which of the following is not a monomial?

A $-6xy$  
B $\frac{1}{2}a^2$  
C $-0.5b^{-3}$  
D $5gh^4$

69. GEOMETRY The accompanying diagram shows the transformation of $\triangle XYZ$ to $\triangle X'Y'Z'$.

This transformation is an example of a

F dilation
G line reflection
H rotation
J translation

70. CARS In 1994, the average price of a new domestic car was $16,930. In 2002, the average price was $19,126. Based on a linear model, what is the predicted average price for 2010?

A $22,969$
B $21,322$
C $20,773$
D $18,577$

71. SHORT RESPONSE If the graph of a line has a positive slope and a negative $y$-intercept, what happens to the $x$-intercept if the slope and the $y$-intercept are both doubled?

72. $y < 4x$
2x + 3y ≥ -21

73. $y ≥ 2$

2y + 2x ≤ 4

74. $y > -2x - 1$

2y ≤ 3x + 2

2x + 12y < -6

75. $3x + 2y < 10$

76. \[
\begin{bmatrix}
2 & 5 & 3 \\
-5 & -1 & 10 \\
4 & -4 & 0
\end{bmatrix} + \begin{bmatrix}
-8 & 2 & -6 \\
3 & 6 & -1 \\
-6 & -10 & 6
\end{bmatrix}
\]

77. \[
\begin{bmatrix}
11 & 0 & 7 \\
8 & 11 & -10
\end{bmatrix} - \begin{bmatrix}
-3 & 0 & 4 \\
6 & -3 & 4
\end{bmatrix}
\]

78. \[
\begin{bmatrix}
-5 & 2 & -11 \\
2 & -2 & 1
\end{bmatrix} + \begin{bmatrix}
2 & 5 \\
3 & -9
\end{bmatrix}
\]

79. \[
\begin{bmatrix}
-5 & -7 \\
-1 & 11 & 1 \\
6 & -3 & 4
\end{bmatrix} + \begin{bmatrix}
-4 & 0 & -9 \\
12 & -12 & 8
\end{bmatrix}
\]

80. BABYSITTING Alexis charges $10 plus $4 per hour to babysit. Alexis needs at least $40 more to buy a television for which she is saving. Write an inequality for this situation. Will she be able to get her television if she babysits for 5 hours? (Lesson 5-6)

81. $-64 ÷ -8$
82. $-78 ÷ 1.3$
83. $42.3 ÷ -6$
84. $-23.94 ÷ 10.5$
85. $-32.5 ÷ -2.5$
86. $-98.44 ÷ 4.6$
Then
You multiplied monomials. (Lesson 7-1)

Now
- Find the quotient of two monomials.
- Simplify expressions containing negative and zero exponents.

KY Program of Studies
HS-NPO-S-NO4 Students will apply absolute value, integer exponents, roots and factorials to solve problems.
HS-AT-S-VEO5 Students will understand the properties of integer exponents and apply these properties to simplify algebraic expressions.

New Vocabulary
zero exponent
negative exponent
order of magnitude

Why?
The tallest redwood tree is 112 or about $10^2$ meters tall. The average height of a woman in the United States is 1.62 meters. The closest power of ten to 1.62 is $10^0$, so a woman is about $10^0$ meters tall. The ratio of the tree’s height to the woman’s height is $\frac{10^2}{10^0}$ or $10^2$. This means the tallest redwood tree is approximately 100 times as tall as the average woman.

Quotients of Monomials We can use the principles for reducing fractions to find quotients of monomials like $\frac{10^2}{10^0}$. In the following examples, look for a pattern in the exponents.

\[
\frac{2^7}{2^4} = \frac{1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{1 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^3
\]

\[
\frac{t^4}{t^3} = \frac{1 \cdot t \cdot t \cdot t \cdot t}{1 \cdot t \cdot t \cdot t} = t
\]

These examples demonstrate the Quotient of Powers Rule.

Key Concept
Quotient of Powers
Words To divide two powers with the same base, subtract the exponents.

Symbols For any nonzero number $a$, and any integers $m$ and $p$, $\frac{a^m}{a^p} = a^{m-p}$.

Examples $\frac{c^{11}}{c^8} = c^{11-8} = c^3$ or $\frac{r^5}{r^2} = r^{5-2} = r^3$

EXAMPLE 1 Quotient of Powers
Simplify $\frac{g^3h^5}{gh^2}$. Assume that no denominator equals zero.

\[
\frac{g^3h^5}{gh^2} = \left(\frac{g^3}{g}\right) \left(\frac{h^5}{h^2}\right) = (g^{3-1})(h^{5-2}) = g^2h^3
\]

Check Your Progress
Simplify each expression. Assume that no denominator equals zero.

1A. $\frac{x^3y^4}{x^2y}$

1B. $\frac{k^7m^{10}p}{k^5m^8p}$
We can use the Product of Powers Rule to find the powers of quotients for monomials. In the following example, look for a pattern in the exponents.

\[
(\frac{3}{4})^3 = (\frac{3}{4})^3 \cdot (\frac{3}{4})^3 = \frac{3^3}{4^3}
\]

\[
(\frac{c}{d})^2 = (\frac{c}{d})^2 \cdot (\frac{c}{d})^2 = \frac{c^2}{d^2}
\]

**Key Concept**

**Power of a Quotient**

**Words** To find the power of a quotient, find the power of the numerator and the power of the denominator.

**Symbols** For any real numbers \(a\) and \(b\), \(\neq 0\), and any integer \(m\), \((\frac{a}{b})^m = \frac{a^m}{b^m}\).

**Examples**

\[
\left(\frac{3}{5}\right)^4 = \frac{3^4}{5^4}
\]

\[
\left(\frac{c}{d}\right)^5 = \frac{c^5}{d^5}
\]

**EXAMPLE 2**

**Power of a Quotient**

Simplify \(\left(\frac{3p^3}{7}\right)^2\).

\[
\left(\frac{3p^3}{7}\right)^2 = \left(\frac{3p^3}{7}\right)^2 \cdot \left(\frac{3p^3}{7}\right)^2 = \frac{3^2(p^3)^2}{7^2} \quad \text{Power of a Quotient}
\]

\[
= \frac{9p^6}{49} \quad \text{Power of a Product}
\]

**Check Your Progress**

Simplify each expression.

2A. \(\left(\frac{3x^4}{4}\right)^3\)

2B. \(\left(\frac{5x^5 y^6}{6}\right)^2\)

2C. \(\left(\frac{2y^2}{3z^3}\right)^2\)

2D. \(\left(\frac{4x^3 y^4}{5}\right)^3\)

A calculator can be used to explore expressions with 0 as the exponent. There are two methods to explain why a calculator gives a value of 1 for \(3^0\).

**Method 1**

\[
\frac{3^5}{3^5} = 3^5 - 3^5 = 3^0 \quad \text{Quotient of Powers}
\]

Since \(\frac{3^5}{3^5}\) can only have one value, we can conclude that \(3^0 = 1\).
Zero Exponent Property

Words Any nonzero number raised to the zero power is equal to 1.

Symbols For any nonzero number $a$, $a^0 = 1$.

Examples $15^0 = 1$ \quad \frac{b}{c}^0 = 1 \quad \left(\frac{2}{7}\right)^0 = 1$

Example 3

Zero Exponent

Simplify each expression. Assume that no denominator equals zero.

a. \[ \left(\frac{4n^2q^5r^2}{9n^3q^7r}\right)^0 = 1 \quad a^0 = 1 \]

b. \[ \frac{x^5y^0}{x^3} = x^2 \quad a^0 = 1 \]

Check Your Progress

3A. \[ \frac{b4c^2d^0}{b^2c} \]

3B. \[ \left(\frac{2f^4g^7h^3}{15f^3g^9h^6}\right)^0 \]

Negative Exponents

To investigate the meaning of a negative exponent, we can simplify expressions like $\frac{c^2}{c^5}$ using two methods.

Method 1

\[
\frac{c^2}{c^5} = c^2 \cdot c^{-3} = c^{-3}
\]

Method 2

\[
\frac{c^2}{c^5} = \frac{c^2}{c^5} = \frac{c}{c} \cdot \frac{c}{c} \cdot \frac{c}{c} \cdot \frac{c}{c} \cdot \frac{c}{c} = \frac{1}{c^3}
\]

Since $\frac{c^2}{c^5}$ can only have one value, we can conclude that $c^{-3} = \frac{1}{c^3}$.

Key Concept

Negative Exponent Property

Words For any nonzero number $a$ and any integer $n$, $a^{-n}$ is the reciprocal of $a^n$. Also, the reciprocal of $a^{-n} = a^n$.

Symbols For any nonzero number $a$ and any integer $n$, $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$.

Examples $2^{-4} = \frac{1}{2^4} = \frac{1}{16}$ \quad \frac{1}{j^{-4}} = j^4$
An expression is considered simplified when it contains only positive exponents, each base appears exactly once, there are no powers of powers, and all fractions are in simplest form.

**EXAMPLE 4**  
**Negative Exponents**

Simplify each expression. Assume that no denominator equals zero.

**a.** \( \frac{n^{-5} p^4}{r^{-2}} \)

\[
\frac{n^{-5} p^4}{r^{-2}} = \left( \frac{n^{-5}}{1} \right) \left( \frac{p^4}{1} \right) \left( \frac{r^2}{1} \right)
\]

Write as a product of fractions.

\[
= \left( \frac{1}{n^5} \right) \left( \frac{p^4}{1} \right) \left( \frac{r^2}{1} \right)
\]

\[
= \frac{p^4 r^2}{n^5}
\]

Multiply.

**b.** \( \frac{5y^{-3} t^4}{20r^2 t^7 u^{-5}} \)

\[
\frac{5y^{-3} t^4}{20r^2 t^7 u^{-5}} = \left( \frac{5}{20} \right) \left( \frac{y^{-3}}{r^2} \right) \left( \frac{t^4}{t^7} \right) \left( \frac{1}{u^{-5}} \right)
\]

Group powers with the same base.

Quotient of Powers and Negative Exponents Property

Simplify.

Negative Exponent Property

Multiply.

\[
= \frac{1}{4} y^{-5} t^{-3} u^5
\]

\[
= \frac{1}{4} \left( \frac{1}{r^5 t^3} \right) (u^5)
\]

\[
= \frac{u^5}{4r^5 t^3}
\]

**c.** \( \frac{2a^2 b^3 c^{-5}}{10a^{-3} b^{-1} c^{-4}} \)

\[
\frac{2a^2 b^3 c^{-5}}{10a^{-3} b^{-1} c^{-4}} = \left( \frac{2}{10} \right) \left( \frac{a^2}{a^{-3}} \right) \left( \frac{b^3}{b^{-1}} \right) \left( \frac{c^{-5}}{c^{-4}} \right)
\]

Group powers with the same base.

Quotient of Powers and Negative Exponents Property

Simplify.

Negative Exponent Property

Multiply.

\[
= \frac{1}{5} \left( a^{2-(-3)} \right) \left( b^{3-(-1)} \right) \left( c^{-5-(-4)} \right)
\]

\[
= \frac{1}{5} a^5 b^4 c^{-1}
\]

\[
= \frac{1}{5} \left( a^5 \right) \left( b^4 \right) \left( \frac{1}{c} \right)
\]

\[
= \frac{5a^5 b^4}{5c}
\]

**Check Your Progress**

Simplify each expression. Assume that no denominator equals zero.

4A. \( \frac{v^{-3} w x^2}{w y^{-6}} \)

4B. \( \frac{32a^{-8} b^3 c^{-4}}{4a^3 b^5 c^{-2}} \)

4C. \( \frac{5f^{-3} k^2 m^{-6}}{25k^{-4} m^{-2}} \)

Order of magnitude is used to compare measures and to estimate and perform rough calculations. The order of magnitude of a quantity is the number rounded to the nearest power of 10. For example, the power of 10 closest to 95,000,000,000 is \(10^{11}\), or 100,000,000,000. So the order of magnitude of 95,000,000,000 is \(10^{11}\).
Example 5

Apply Properties of Exponents

**HEIGHT** Suppose the average height of a man is about 1.7 meters, and the average height of an ant is 0.0008 meter. Determine the order of magnitude of the height of a man and the height of an ant. How many orders of magnitude as tall is a man as an ant?

**Understand** We must find the order of magnitude of the heights of the man and ant. Then find the ratio of the man’s height to the ant’s height.

**Plan** Round each height to the nearest power of ten. Then find the ratio of the height of the man to the height of the ant.

**Solve** The average height of a man is close to 1 meter. So, the order of magnitude is $10^0$ meter. The average height of an ant is about 0.001 meter. So, the order of magnitude is $10^{-3}$ meters.

The ratio of the height of a man to the height of an ant is

$$\frac{10^0}{10^{-3}} = \frac{10^3}{10^{-3}} = 10^3 - (-3) = 10^3$$

**Check** The ratio of the man’s height to the ant’s height is

$$\frac{1.7}{0.0008} = 2125.$$ The order of magnitude of 2125 is $10^3$. ✓

**Check Your Progress**

5. **ASTRONOMY** The order of magnitude of the mass of Earth is about $10^{27}$. The order of magnitude of the Milky Way galaxy is about $10^{44}$. How many orders of magnitude as big is the Milky Way galaxy as Earth?

---

**Check Your Understanding**

Simplify each expression. Assume that no denominator equals zero.

1. \( \frac{t^5u^4}{t^2u} \)
2. \( \frac{a^6b^4c^{10}}{a^3b^2c} \)
3. \( \frac{m^6y^5p^3}{m^5r^7p^3} \)
4. \( \frac{b^4c^6y^8}{b^4c^3y^5} \)
5. \( \frac{8y^2m}{h^5g^7} \)
6. \( \frac{x^4t^3v^2}{t^2v^2} \)
7. \( \frac{x^3y^2z^6}{z^5x^2y} \)
8. \( \frac{n^4q^4w^6}{q^2m^3w} \)
9. \( \left( \frac{2a^3b^5}{3} \right)^2 \)
10. \( \frac{r^3v^{-2}}{t^{-7}} \)
11. \( \left( \frac{2c^3d^5}{5g^2} \right)^5 \)
12. \( \left( \frac{3xy^4z^2}{x^3yz^4} \right)^0 \)
13. \( \left( \frac{3p^4q^4}{32p^3q^5} \right)^0 \)
14. \( \frac{4x^2v^0l^5}{2nl^3} \)
15. \( \frac{-3y^2z^6}{h^{-4}} \)
16. \( \frac{-8x^2y^8z^{-5}}{12x^4y^{-7}z^7} \)
17. \( \frac{2a^3b^{-7}c^{10}}{6a^{-3}b^2c^{-3}} \)

---

Example 5

18. **GROSS DOMESTIC PRODUCT** The gross domestic product (GDP) for the United States in 2006 was $13.06 trillion, and the GDP per person was $43,800. Use order of magnitude to approximate the population of the United States in 2006.
**Example 5**

**INTERNET** In a recent year, there were approximately 3.95 million Internet hosts. Suppose there were 208 million Internet users. Determine the order of magnitude for the Internet hosts and Internet users. Using the orders of magnitude, how many Internet users were there compared to Internet hosts?

**PROBABILITY** The probability of rolling a die and getting an even number is $\frac{1}{2}$.

If you roll the die twice, the probability of an even number is $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ or $\left(\frac{1}{2}\right)^2$.

a. Write an expression to represent the probability of rolling a die $d$ times and getting an even number.

b. Write the expression as a power of 2.

**Simplify each expression. Assume that no denominator equals zero.**

57. **COMPUTERS** In 1993, the processing speed of a desktop computer was about $10^8$ instructions per second. By 2004, it had increased to $10^{10}$ instructions per second. The newer computer is how many times as fast as the older one?
58. **ASTRONOMY** The brightness of a star is measured in magnitudes. The lower the magnitude number, the brighter the star. A magnitude 9 star is 2.51 times as bright as a magnitude 10 star. A magnitude 8 star is $2.51 \cdot 2.51 = 2.51^2$ times as bright as a magnitude 10 star.

a. How many times as bright is a magnitude 3 star as a magnitude 10 star?

b. Write an expression to represent the brightness of a star of magnitude $m$ compared to a magnitude 10 star.

c. Magnitudes can be measured in negative numbers. Does your expression hold true? Give an example or counterexample.

59. **PROBABILITY** The probability of rolling a die and getting a 3 is $\frac{1}{6}$. If you roll the die twice, the probability of getting a 3 is $\frac{1}{6} \cdot \frac{1}{6}$ or $(\frac{1}{6})^2$.

a. Write an expression to represent the probability of rolling a die $d$ times and getting a 3.

b. Write the expression as a power of 6.

60. **MULTIPLE REPRESENTATIONS** To find the area of a circle, use $A = \pi r^2$. The formula for the area of a square is $A = s^2$.

a. **ALGEBRAIC** Find the ratio of the area of the circle to the area of the square.

b. **ALGEBRAIC** If the radius of the circle is $2r$, find the ratio of the area of the circle to the square.

c. **TABULAR** Copy and complete the table.

<table>
<thead>
<tr>
<th>Radius</th>
<th>Area of Circle</th>
<th>Area of Square</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2r$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3r$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$4r$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$5r$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$6r$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

d. **ANALYTICAL** What conclusion can be drawn from this?

H.O.T. Problems

61. **REASONING** Determine whether $x^y - x^2 = x^{y-2}$ is sometimes, always, or never true. Explain your reasoning.

62. **OPEN ENDED** Name two monomials with a quotient of $24a^2b^3$.

63. **CHALLENGE** Use the Quotient of Powers Property to explain why $x^{-n} = \frac{1}{x^n}$.

64. **REASONING** Write a convincing argument to show why $3^0 = 1$ using the following pattern: $3^5 = 243$, $3^4 = 81$, $3^3 = 27$, $3^2 = 9$.

65. **WRITING IN MATH** Explain how to use the Quotient of Powers property and the Power of a Quotient property.
66. **GEOMETRY** In the figure, the lengths are given in meters. What is the perimeter of the figure in meters?

A  $40x$
B  $80x$
C  $160x$
D  $400x$

67. In researching her science project, Leigh learned that light travels at a constant rate and that it takes 500 seconds for light to travel the 93 million miles from the Sun to Earth. Mars is 142 million miles from the Sun. About how many seconds will it take for light to travel from the Sun to Mars?

F  235 seconds
G  327 seconds
H  642 seconds
J  763 seconds

68. **EXTENDED RESPONSE** Jessie and Jonas are playing a game using the spinners below. Each spinner is equally likely to stop on any of the four numbers. In the game, a player spins both spinners and calculates the product of the two numbers on which the spinners have stopped.

[Diagram of spinners]

a. What product has the greatest probability of occurring?

b. What is the probability of that product occurring?

69. Simplify $(4 - 2 \cdot 5 \cdot 6)^3$.

A  $\frac{1}{64}$
C  320
B  64
D  1024

**Spiral Review**

70. **GEOLOGY** The seismic waves of a magnitude 6 earthquake are $10^2$ times as great as a magnitude 4 earthquake. The seismic waves of a magnitude 4 earthquake are 10 times as great as a magnitude 3 earthquake. How many times as great are the seismic waves of a magnitude 6 earthquake as those of a magnitude 3 earthquake? (Lesson 7-1)

Solve each system of inequalities by graphing. (Lesson 6-8)

71. $y \geq 1$
   $x < -1$
72. $y \geq -3$
   $y - x < 1$
73. $y < 3x + 2$
   $y \geq -2x + 4$
74. $y - 2x < 2$
   $y - 2x > 4$

Solve each inequality. Check your solution. (Lesson 5-3)

75. $5(2h - 6) > 4h$
76. $22 \geq 4(b - 8) + 10$
77. $5(u - 8) \leq 3(u + 10)$
78. $8 + t \leq 3(t + 4) + 2$
79. $9n + 3(1 - 6n) \leq 21$
80. $-6(b + 5) > 3(b - 5)$

81. **GRADES** In a high school science class, a test is worth three times as much as a quiz. If a student has test grades of 85 and 92 and quiz grades of 82, 75, and 95, what is the student’s average grade? (Lesson 2-9)

**Skills Review**

Evaluate each expression. (Lesson 1-1)

82. $9^2$
83. $11^2$
84. $10^6$
85. $10^4$
86. $3^5$
87. $5^3$
88. $12^3$
89. $4^6$
Then
You found products and quotients of monomials. (Lessons 7-1 and 7-2)

Now
- Express numbers in scientific notation.
- Find products and quotients of numbers expressed in scientific notation.

Scientific Notation Very large and very small numbers such as $20 million can be cumbersome to use in calculations. For this reason, numbers are often expressed in scientific notation. A number written in scientific notation is of the form $a \times 10^n$, where $1 \leq a < 10$ and $n$ is an integer.

**Key Concept**

**Standard Form to Scientific Notation**

**Step 1** Move the decimal point until it is to the right of the first nonzero digit. The result is a real number $a$.

**Step 2** Note the number of places $n$ and the direction that you moved the decimal point.

**Step 3** If the decimal point is moved left, write the number as $a \times 10^n$. If the decimal point is moved right, write the number as $a \times 10^{-n}$.

**Step 4** Remove the extra zeros.

**EXAMPLE 1**

Express each number in scientific notation.

a. 201,000,000
   - **Step 1** $201,000,000 \rightarrow 2.01000000 \quad a = 2.01000000$
   - **Step 2** The decimal point moved 8 places to the left, so $n = 8$.
   - **Step 3** $201,000,000 = 2.01000000 \times 10^8$
   - **Step 4** $2.01 \times 10^8$

b. 0.000051
   - **Step 1** $0.000051 \rightarrow 00005.1 \quad a = 00005.1$
   - **Step 2** The decimal point moved 5 places to the right, so $n = 5$.
   - **Step 3** $0.000051 = 00005.1 \times 10^{-5}$
   - **Step 4** $5.1 \times 10^{-5}$

**Check Your Progress**

1A. 68,700,000,000
1B. 0.0000725
You can also rewrite numbers in scientific notation in standard form.

### Key Concept: Scientific Notation to Standard Form

**Step 1** Note whether \( n > 0 \) or \( n < 0 \).

**Step 2** If \( n > 0 \), move the decimal point \( n \) places right.

If \( n < 0 \), move the decimal point \(-n\) places left.

**Step 3** Insert zeros, decimal point, and commas as needed for place value.

### Example 2: Scientific Notation to Standard Form

Express each number in standard form.

**a.** \( 6.32 \times 10^9 \)

- **Step 1** The exponent is 9, so \( n = 9 \).
- **Step 2** Since \( n > 0 \), move the decimal point 9 places to the right.
  
  \[
  6.32 \times 10^9 \rightarrow 6320000000
  \]

- **Step 3** 6.32 \times 10^9 = 6,320,000,000 \( \text{Rewrite; insert commas.} \)

**b.** \( 4 \times 10^{-7} \)

- **Step 1** The exponent is \(-7\), so \( n = -7 \).
- **Step 2** Since \( n < 0 \), move the decimal point 7 places to the left.
  
  \[
  4 \times 10^{-7} \rightarrow 0000004
  \]

- **Step 3** 4 \times 10^{-7} = 0.0000004 \( \text{Rewrite; insert a 0 before the decimal point.} \)

### Check Your Progress

**2A.** \( 3.201 \times 10^6 \)  
**2B.** \( 9.03 \times 10^{-5} \)

### Product and Quotients in Scientific Notation

You can use scientific notation to simplify multiplying and dividing very large and very small numbers.

### Example 3: Multiply with Scientific Notation

Evaluate \( (3.5 \times 10^{-3})(7 \times 10^5) \). Express the result in both scientific notation and standard form.

\[
(3.5 \times 10^{-3})(7 \times 10^5) = (3.5 \times 7)(10^{-3} \times 10^5)
\]

- Original expression
- Commutative and Associative Properties
- Product of Powers
- Product of Powers
- Standard form

\[
= 24.5 \times 10^2
\]

\[
= (2.45 \times 10^1) \times 10^2
\]

\[
= 2.45 \times 10^3
\]

\[
= 2450
\]

### Check Your Progress

Evaluate each product. Express the results in both scientific notation and standard form.

**3A.** \( (6.5 \times 10^{12})(8.7 \times 10^{-15}) \)

**3B.** \( (1.95 \times 10^{-8})(7.8 \times 10^{-2}) \)
**EXAMPLE 4** Divide with Scientific Notation

Evaluate \( \frac{3.066 \times 10^8}{7.3 \times 10^3} \). Express the result in both scientific notation and standard form.

\[
\frac{3.066 \times 10^8}{7.3 \times 10^3} = \left( \frac{3.066}{7.3} \right) \times \left( \frac{10^8}{10^3} \right) = 0.42 \times 10^5
\]

\[
= 4.2 \times 10^4
\]

\[
= 42,000
\]

**Check Your Progress**

Evaluate each quotient. Express the results in both scientific notation and standard form.

4A. \( \frac{2.3958 \times 10^3}{1.98 \times 10^8} \)  
4B. \( \frac{1.305 \times 10^3}{1.45 \times 10^{-4}} \)

**EXAMPLE 5** Use Scientific Notation

**MUSIC** In the United States, a CD reaches gold status once 500 thousand copies are sold. A CD reaches platinum status once 1 million or more copies are sold.

a. Express the number of copies of CD’s that need to be sold to reach each status in standard notation.

   - gold status: 500 thousand = 500,000
   - platinum status: 1 million = 1,000,000

b. Write each number in scientific notation.

   - gold status: 500,000 = \( 5 \times 10^5 \)
   - platinum status: 1,000,000 = \( 1 \times 10^6 \)

c. How many copies of a CD have sold if it has gone platinum 13 times? Write your answer in scientific notation and standard form.

   A CD reaches platinum status once it sells 1 million records. Since the CD has gone platinum 13 times, we need to multiply by 13.

   \[
   (13)(1 \times 10^6) = (13 \times 1)(10^6) = 13 \times 10^6 = (1.3 \times 10^1) \times 10^6 = 1.3 \times 10^7 = 13,000,000
   \]

**Check Your Progress**

5. **SATELLITE RADIO** Suppose a satellite radio company earned $125.4 million in one year.

   a. Write this number in standard form.

   b. Write this number in scientific notation.

   c. If the following year the company earned 2.5 times the amount earned the previous year, determine the amount earned. Write your answer in scientific notation and standard form.
Check Your Understanding

**Example 1**
Express each number in scientific notation.

1. 185,000,000
2. 1,902,500,000
3. 0.000564
4. 0.00000804

**MONEY** Express each number in scientific notation.
5. Teenagers spend $13 billion annually on clothing.
6. Teenagers have an influence on their families’ spending habit. They control about $1.5 billion of discretionary income.

**Example 2**
Express each number in standard form.

7. $1.98 \times 10^7$
8. $4.052 \times 10^6$
9. $3.405 \times 10^{-8}$
10. $6.8 \times 10^{-5}$

**Example 3**
Evaluate each product. Express the results in both scientific notation and standard form.

11. $(1.2 \times 10^3)(1.45 \times 10^{12})$
12. $(7.08 \times 10^{14})(5 \times 10^{-5})$
13. $(5.18 \times 10^2)(9.1 \times 10^{-5})$
14. $(2.9 \times 10^{-2})(5.2 \times 10^{-9})$

**Example 4**
Evaluate each quotient. Express the results in both scientific notation and standard form.

15. \[
\frac{1.035 \times 10^8}{2.3 \times 10^4}
\]
16. \[
\frac{2.542 \times 10^5}{4.1 \times 10^{-10}}
\]
17. \[
\frac{1.445 \times 10^{-7}}{1.7 \times 10^5}
\]
18. \[
\frac{2.05 \times 10^{-8}}{4 \times 10^{-2}}
\]

**Example 5**
Express each number in scientific notation.

19. **AIR FILTERS** Salvador bought an air purifier to help him deal with his allergies. The filter in the purifier will stop particles as small as one hundredth of a micron. A micron is one millionth of a millimeter.

a. Write one hundredth and one micron in standard form.

b. Write one hundredth and one micron in scientific notation.

C. What is the smallest size particle in meters that the filter will stop? Write the result in both standard form and scientific notation.

---

**Practice and Problem Solving**

**Example 1**
Express each number in scientific notation.

20. 1,220,000
21. 58,600,000
22. 1,405,000,000,000
23. 0.0000013
24. 0.000056
25. 0.000000000709

**E-MAIL** Express each number in scientific notation.

26. Approximately 100 million e-mails sent to the President are put into the National Archives.
27. By 2010, the e-mail security market will generate $5.5 billion.

**Example 2**
Express each number in standard form.

28. $1 \times 10^{12}$
29. $9.4 \times 10^7$
30. $8.1 \times 10^{-3}$
31. $5 \times 10^{-4}$
32. $8.73 \times 10^{11}$
33. $6.22 \times 10^{-6}$
INTERNET Express each number in standard form.
34. About $2.1 \times 10^7$ people, aged 12 to 17, use the Internet.
35. Approximately $1.1 \times 10^7$ teens go online daily.

**Examples 3 and 4** pp. 417–418

Evaluate each product or quotient. Express the results in both scientific notation and standard form.
36. $(3.807 \times 10^3)(5 \times 10^2)$
37. $\frac{9.6 \times 10^3}{1.2 \times 10^{-4}}$
38. $2.88 \times 10^3$
39. $(6.5 \times 10^7)(7.2 \times 10^{-2})$
40. $(9.5 \times 10^{-18})(9. \times 10^9)$
41. $\frac{8.8 \times 10^3}{4 \times 10^{-4}}$
42. $9.15 \times 10^{-3}$
43. $(2.01 \times 10^{-4})(8.9 \times 10^{-3})$
44. $2.58 \times 10^2$
45. $(2.58 \times 10^2)(3.6 \times 10^6)$
46. $\frac{1.363 \times 10^{16}}{2.9 \times 10^6}$
47. $(9.04 \times 10^6)(5.2 \times 10^{-4})$
48. $(1.6 \times 10^{-5})(2.3 \times 10^{-3})$
49. $\frac{5.6498 \times 10^{10}}{8.2 \times 10^4}$
49. $3.75 \times 10^{-9}$
50. $\frac{6.25 \times 10^{-4}}{1.25 \times 10^2}$
51. $(3.4 \times 10^4)(7.2 \times 10^{-15})$
52. $\frac{8.6 \times 10^4}{2 \times 10^{-6}}$
53. $(6.3 \times 10^{-2})(3.5 \times 10^{-4})$

**Example 5** p. 418

ASTRONOMY The distance between Earth and the Sun varies throughout the year. Earth is closest to the Sun in January when the distance is 91.4 million miles. In July, the distance is greatest at 94.4 million miles.

a. Write 91.4 million in both standard form and in scientific notation.

b. Write 94.4 million in both standard form and in scientific notation.

c. What is the percent increase in distance from January to July? Round to the nearest tenth of a percent.

Evaluate each product or quotient. Express the results in both scientific notation and standard form.
54. $(4.65 \times 10^{-2})(5 \times 10^6)$
55. $\frac{2.548 \times 10^5}{2.8 \times 10^{-2}}$
56. $2.135 \times 10^5$
57. $(4.8 \times 10^5)(3.16 \times 10^{-5})$
58. $\frac{5.184 \times 10^{-5}}{7.2 \times 10^3}$
59. $(4.3 \times 10^{-3})(4.5 \times 10^4)$
60. $1.032 \times 10^{-4}$
61. $(5 \times 10^3)(1.8 \times 10^{-7})$
62. $\frac{8.6 \times 10^{-15}}{8.6 \times 10^{-5}}$

**Real-World Link**
The distance from Earth to the Sun does not determine the seasons. The seasons are determined by the tilt of the Earth’s axis and the elliptical orbit around the Sun.

Source: University of British Columbia Okanagan

**LIGHT** The speed of light is approximately $3 \times 10^8$ meters per second.

63. Write an expression to represent the speed of light in kilometers per second.
64. Write an expression to represent the speed of light in kilometers per hour.
65. Make a table to show how many kilometers light travels in a day, a week, a 30-day month, and a 365-day year. Express your results in scientific notation.
66. The distance from Earth to the Moon is approximately $3.844 \times 10^5$ kilometers. How long would it take light to travel from Earth to the Moon?
Lesson 7-3
Scientific Notation

The population of Earth is about $6.623 \times 10^9$. The land surface of Earth is $1.483 \times 10^8$ square kilometers. What is the population density for the land surface area of Earth?

**RIVERS** A drainage basin separated from adjacent basins by a ridge, hill, or mountain, is known as a watershed. The watershed of the Amazon River is 2,300,000 square miles. The watershed of the Mississippi River is 1,200,000 square miles.

a. Write each of these numbers in scientific notation.

b. How many times as large is the Amazon River watershed as the Mississippi River watershed?

**AGRICULTURE** In a recent year, farmers planted approximately 92.9 million acres of corn. They also planted 64.1 million acres of soybeans and 11.1 million acres of cotton.

a. Write each of these numbers in scientific notation and in standard form.

b. How many times as much corn was planted as soybeans? Write your results in standard form and in scientific notation. Round your answer to four decimal places.

c. How many times as much corn was planted as cotton? Write your results in standard form and in scientific notation. Round your answer to four decimal places.

**H.O.T. Problems**

**Reasoning** Which is greater, $100^{10}$ or $10^{100}$? Explain your reasoning.

**Find the Error** Syreeta and Pete are solving a division problem with scientific notation. Is either of them correct? Explain your reasoning.

![Syreeta](3.65 \times 10^{-12}) / (5 \times 10^5) = 0.73 \times 10^{-17}

![Pete](3.65 \times 10^{-12}) / (5 \times 10^5) = 0.73 \times 10^{-17}

**Challenge** Order these numbers from least to greatest without converting them to standard form.

$5.46 \times 10^{-3}$, $6.54 \times 10^3$, $4.56 \times 10^{-4}$, $-5.64 \times 10^4$, $-4.65 \times 10^5$

**Reasoning** Determine whether the statement is always, sometimes, or never true. Give examples or a counterexample to verify your reasoning.

When multiplying two numbers written in scientific notation, the resulting number can have no more than two places to the left of the decimal point.

**Open Ended** Write two numbers in scientific notation with a product of $1.3 \times 10^{-3}$. Then name two numbers in scientific notation with a quotient of $1.3 \times 10^{-3}$.

**Writing in Math** Write the steps that you would use to divide two numbers written in scientific notation. Then describe how you would write the results in standard form.

*Source: Encarta Encyclopedia*
76. Which number represents \(0.05604 \times 10^8\) written in standard form?
   A 0.0000000005604  
   B 560,400  
   C 5,604,000  
   D 50,604,000

77. Toni left school and rode her bike home. The graph below shows the relationship between her distance from the school and time.

Which explanation could account for the section of the graph from \(t = 30\) to \(t = 40\)?
   F Toni rode her bike down a hill.  
   G Toni ran all the way home.  
   H Toni stopped at a friend’s house on her way home.  
   J Toni returned to school to get her mathematics book.

78. SHORT RESPONSE In his first four years of coaching football, Coach Delgato’s team won 5 games the first year, 10 games the second year, 8 games the third year, and 7 games the fourth year. How many games does the team need to win during the fifth year to have an average of 8 wins per year?

79. The table shows the relationship between Calories and fat grams contained in an order of fried chicken from various restaurants.

<table>
<thead>
<tr>
<th>Calories</th>
<th>305</th>
<th>410</th>
<th>320</th>
<th>500</th>
<th>510</th>
<th>440</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fat (g)</td>
<td>28</td>
<td>34</td>
<td>28</td>
<td>41</td>
<td>42</td>
<td>38</td>
</tr>
</tbody>
</table>

Assuming that the data can best be described by a linear model, how many fat grams would you expect to be in a 275-Calorie order of fried chicken?
   A 28  
   B 27  
   C 25  
   D 22

80. \(\frac{8^9}{8^6}\)
81. \(\frac{6^5}{6^3}\)
82. \(\frac{r^{8t^{12}}}{r^{4t^7}}\)
83. \(\frac{3d^4b^4}{8c^2}\)^4
84. \(\frac{5d^3-8c^2}{3h^4}\)^2
85. \(\frac{(4n^2p^4)^3}{8p^3}\)

86. CHEMISTRY Lemon juice is \(10^2\) times as acidic as tomato juice. Tomato juice is \(10^3\) times as acidic as egg whites. How many times as acidic is lemon juice as egg whites? (Lesson 7-1)

Write each equation in slope-intercept form. (Lesson 4-2)
87. \(y - 2 = 3(x - 1)\)
88. \(y - 5 = 6(x + 1)\)
89. \(y + 2 = -2(x + 5)\)
90. \(y + 3 = \frac{1}{2}(x + 4)\)
91. \(y - 1 = \frac{2}{3}(x + 9)\)
92. \(y + 3 = -\frac{1}{4}(x + 2)\)

83. \(4w^2 + w + 15w^2\)
84. \(13(5 + 4a)\)
85. \(6m^2 - 8m\)
86. \((4t - 6)16\)
Algebra tiles can be used to model polynomials. A polynomial is a monomial or the sum of monomials. The diagram at the right shows the models.

### ACTIVITY
Use algebra tiles to model each polynomial.

- **5x**
  
  To model this polynomial, you will need 5 green x-tiles.

- **3x^2 - 1**
  
  To model this polynomial, you will need 3 blue x^2-tiles and 1 red -1-tile.

- **-2x^2 + x + 3**
  
  To model this polynomial, you will need 2 red -x^2-tiles, 1 green x-tile, and 3 yellow 1-tiles.

### Model and Analyze
Use algebra tiles to model each polynomial. Then draw a diagram of your model.

1. -4x^2
2. 3x - 5
3. 2x^2 - 3x
4. x^2 + 2x + 1

Write an algebraic expression for each model.

5. 

6. 

7. 

8. 

9. **MAKE A CONJECTURE** Write a sentence or two explaining why algebra tiles are sometimes called *area tiles.*
Polynomials

Why?

In 2011, sales of digital audio players are expected to reach record numbers. The sales data can be modeled by the equation \( U = -2.7t^2 + 49.4t + 128.7 \), where \( U \) is the number of units shipped in millions and \( t \) is the number of years since 2005.

The expression \(-2.7t^2 + 49.4t + 128.7\) is an example of a polynomial. Polynomials can be used to model situations.

Degree of a Polynomial

A polynomial is a monomial or the sum or difference of monomials, each called a term of the polynomial. Some polynomials have special names. A binomial is the sum or difference of two monomials, and a trinomial is the sum or difference of three monomials.

EXAMPLE 1 Identify Polynomials

Determine whether each expression is a polynomial. If so, identify the polynomial as a monomial, binomial, or trinomial.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Is it a polynomial?</th>
<th>Monomial, binomial, or trinomial?</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( 4y - 5xz )</td>
<td>Yes; ( 4y - 5xz ) is the difference of two monomials.</td>
<td>binomial</td>
</tr>
<tr>
<td>b. (-6.5)</td>
<td>Yes; (-6.5) is a real number.</td>
<td>monomial</td>
</tr>
<tr>
<td>c. ( 7a^{-3} + 9b )</td>
<td>No; ( 7a^{-3} = \frac{7}{a^3} ), which is not a monomial.</td>
<td>none of these</td>
</tr>
<tr>
<td>d. ( 6x^3 + 4x + x + 3 )</td>
<td>Yes; ( 6x^3 + 4x + x + 3 = 6x^3 + 5x + 3 ), the sum of three monomials.</td>
<td>trinomial</td>
</tr>
</tbody>
</table>

Check Your Progress

1A. \( x \)
1B. \(-3y^2 - 2y + 4y - 1\)
1C. \( 5rx + 7tuv \)
1D. \( 10x^{-4} - 8x^2 \)

The degree of a monomial is the sum of the exponents of all its variables. A nonzero constant has degree 0. Zero has no degree.

The degree of a polynomial is the greatest degree of any term in the polynomial. To find the degree of a polynomial, you must find the degree of each term. Some polynomials have special names based on their degree.

<table>
<thead>
<tr>
<th>Degree</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>constant</td>
</tr>
<tr>
<td>1</td>
<td>linear</td>
</tr>
<tr>
<td>2</td>
<td>quadratic</td>
</tr>
<tr>
<td>3</td>
<td>cubic</td>
</tr>
<tr>
<td>4</td>
<td>quartic</td>
</tr>
<tr>
<td>5</td>
<td>quintic</td>
</tr>
<tr>
<td>6 or more</td>
<td>6th degree, 7th degree, and so on</td>
</tr>
</tbody>
</table>
EXAMPLE 2  Degree of a Polynomial

Find the degree of each polynomial.

a. \(3a^2b^3 + 6\)
   - **Step 1** Find the degree of each term.
     - \(3a^2b^3\): degree = \(2 + 3\) or 5
     - 6: degree 0
   - **Step 2** The degree of the polynomial is the greatest degree, 5.

b. \(2d^3 - 5e^5d - 7\)
   - \(2d^3\): degree = 3
   - \(5e^5d\): degree = 5 + 1 or 6
   - 7: degree 0
   - The degree of the polynomial is 6.

Check Your Progress

2A. \(7xy^5z\)  2B. \(2rs - 3rs^2 - 7r^2s^2 - 13\)

Polynomials in Standard Form  The terms of a polynomial may be written in any order. Polynomials written in only one variable are usually written in standard form.

The standard form of a polynomial is written with the terms in order from greatest degree to least degree. When a polynomial is written in standard form, the coefficient of the first term is called the leading coefficient.

EXAMPLE 3  Standard Form of a Polynomial

Write each polynomial in standard form. Identify the leading coefficient.

a. \(3x^2 + 4x^5 - 7x\)
   - **Step 1** Find the degree of each term.
     - Polynomial: \(\begin{array}{c} 2 \\ 5 \\ 1 \end{array}\)
     - Degree: \(\begin{array}{c} 3x^2 \\ 4x^5 \\ -7x \end{array}\)
   - **Step 2** Write the terms in descending order: \(4x^5 + 3x^2 - 7x\)
     - The leading coefficient is 4.

b. \(5y - 9 - 2y^4 - 6y^3\)
   - **Step 1** Polynomial: \(\begin{array}{c} 1 \\ 0 \\ 4 \\ 3 \end{array}\)
     - Degree: \(\begin{array}{c} 5y \\ -9 \\ -2y^4 \\ -6y^3 \end{array}\)
   - **Step 2** \(-2y^4 - 6y^3 + 5y - 9\)
     - The leading coefficient is \(-2\).

Check Your Progress

3A. \(8 - 2x^2 + 4x^4 - 3x\)  3B. \(y + 5y^3 - 2y^2 - 7y^6 + 10\)
We can use polynomials to estimate values between two points. We can also use them to predict values of events before they occur.

**Real-World Example 4** Use a Polynomial

**BUSINESS** From 2000 through 2006, the number \( U \) of skateboards (in thousands) produced at a manufacturing plant can be modeled by the equation \( U = 3t^2 - 2t + 10 \), where \( t \) is the number of years since 2000.

How many skateboards were produced in 2002?

Find the value of \( t \), and substitute the value of \( t \) to find the number of skateboards produced.

Since \( t \) is the number of years since 2000, \( t \) equals 2002 - 2000 or 2.

\[
U = 3t^2 - 2t + 10
\]

\[
= 3(2)^2 - 2(2) + 10
\]

\[
= 3(4) - 4 + 10
\]

\[
= 12 - 4 + 10
\]

\[
= 18
\]

The number of skateboards produced was 18 thousand or 18,000.

**Check Your Progress**

4A. How many skateboards were produced in 2005?

4B. If this trend continues, how many skateboards will be produced in 2015?

---

**Check Your Understanding**

**Example 1** Determine whether each expression is a polynomial. If so, identify the polynomial as a monomial, binomial, or trinomial.

1. \( 7ab + 6b^2 - 2a^3 \)
2. \( 2y - 5 + 3y^2 \)
3. \( 3x^2 \)
4. \( \frac{4m}{3p} \)
5. \( 5m^2p^3 + 6 \)
6. \( 5q^{-4} + 6q \)

**Example 2** Find the degree of each polynomial.

7. \( -3 \)
8. \( 6p^3 - p^4 \)
9. \( -7z \)
10. \( \frac{3}{4} \)
11. \( 12 - 7q^2t + 8r \)
12. \( 2a^2b^5 + 5 - ab \)
13. \( 6df^2 + 3d^2j^2 + 2d + 1 \)
14. \( 9hjk - 4h^2j^3 + 5j^2k^2 - h^3k^3 \)

**Example 3** Write each polynomial in standard form. Identify the leading coefficient.

15. \( 2x^5 - 12 + 3x \)
16. \( -y^3 + 3y - 3y^2 + 2 \)
17. \( 4z - 2z^2 - 5z^4 \)
18. \( 2a + 4a^3 - 5a^2 - 1 \)

**Example 4** Suppose the amount \( N \) (in hundreds) of students projected to attend a high school from 1998 to 2007 can be modeled by the equation \( N = t^2 + 1.5t + 0.5 \), where \( t \) is the number of years since 1998.

a. How many students were enrolled in the high school in 2003?

b. How many students were enrolled in the high school in 2005?
Example 1  
Determine whether each expression is a polynomial. If so, identify the polynomial as a monomial, binomial, or trinomial.

20. \( \frac{5y^3}{x^2} + 4x \)  
21. \( 21 \)  
22. \( c^4 - 2c^2 + 1 \)

23. \( d + 3d^{-c} \)  
24. \( a - a^2 \)  
25. \( 5n^3 + 4n^3 \)

Example 2  
Find the degree of each polynomial.

26. \( 13 - 4ab + 5a^3b \)  
27. \( 3x - 8 \)  
28. \( -4 \)

29. \( 17g^2h \)  
30. \( 10 + 2cd^4 - 6d^2g \)  
31. \( 2z^2y^2 - 7 + 5y^3w^4 \)

Example 3  
Write each polynomial in standard form. Identify the leading coefficient.

32. \( 5x^2 - 2 + 3x \)  
33. \( 8y + 7y^3 \)

34. \( 4 - 3c - 5c^2 \)  
35. \(-4d^4 + 1 - d^2 \)

36. \( 11t + 2t^2 - 3 + t^5 \)  
37. \( 2 + r - r^3 \)

38. \( \frac{1}{2}x - 3x^4 + 7 \)  
39. \(-9b^2 + 10b - b^6 \)

Example 4  
40. FIREWORKS A firework shell is launched two feet from the ground at a speed of 150 feet per second. The height \( H \) of the firework shell is modeled by the equation \( H = -16t^2 + 150t + 2 \), where \( t \) is time in seconds.

a. How high will the firework be after 3 seconds?

b. How high will the firework be after 5 seconds?

41. \( 4x - 3x^2 + 5 \)  
42. \( 11z^3 \)  
43. \( 9 + y^4 \)

44. \( 3x^3 - 7 \)  
45. \( -2z^5 - x^2 + 5x - 8 \)  
46. \( 10t - 4t^2 + 6t^3 \)

47. ICE CREAM An ice cream shop is changing the size of their ice cream cone. The volume of a cone is the product of \( \frac{1}{3} \pi \), the square of the radius \( r \), and the height \( h \).

a. Write a polynomial that represents the volume of the ice cream cone.

b. How much will the ice cream cone hold if the radius is 4 inches and the height is 3 inches?

c. If the volume of the ice cream cone must be 63 cubic inches and the radius of the cone is 3 inches, how tall is the ice cream cone?

48. GEOMETRY Write two expressions that represent the perimeter and area of the rectangle below.

\[ 4x^2 + 2x - 1 \]

\[ 2x^2 - x + 3 \]

49. GEOMETRY Write a polynomial to represent the area of the shaded region of the rectangle below.
50. **PROJECT**  Rocky and Arturo are designing a rocket for a competition. The top must be cone-shaped and the body of the rocket must be cylindrical. The volume of a cone is the product of \( \frac{1}{3} \pi \), the height \( h \), and the square of the radius \( r \). The volume of a cylinder is the product of \( \pi \), the height \( t \), and the square of the radius \( r \).

a. Write a polynomial that represents the volume of the rocket.

b. If the height of the body of the rocket is 8 inches, the height of the top is 6 inches, and the radius is 3 inches, find the volume of the rocket.

c. If the height of the body of the rocket is 9 inches, the height of the top is 5 inches, and the radius is 4 inches, find the volume of the rocket.

51. **MULTIPLE REPRESENTATIONS**  In this problem, you will explore perimeter and area.

a. **GEOMETRIC**  Draw three rectangles that each have a perimeter of 400 feet.

b. **TABULAR**  Record the width and length of each rectangle in a table like the one shown below. Find the area of each rectangle.

<table>
<thead>
<tr>
<th>Rectangle</th>
<th>Length</th>
<th>Width</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100 ft</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>50 ft</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>75 ft</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>( x ) ft</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c. **GRAPHICAL**  On a coordinate system, graph the area of rectangle 4 in terms of the length, \( x \). Use the graph to determine the largest area possible.

d. **ANALYTICAL**  Determine the length and width that produce the largest area possible.

52. **H.O.T. Problems**  Use Higher-Order Thinking Skills

52. **FIND THE ERROR**  Chuck and Claudio are writing \( 2x^2 - 3 + 5x \) in standard form. Is either of them correct? Explain your reasoning.

Chuck  
\[
2x^2 = \text{degree 2} \\
-3 = \text{degree 0} \\
5x = \text{degree 1} \\
2x^2 - 5x + 3
\]

Claudio  
\[
2x^2 = \text{degree 2} \\
-3 = \text{degree 0} \\
5x = \text{degree 1} \\
2x^2 + 5x - 3
\]

53. **CHALLENGE**  Write a polynomial that represents any odd integer if \( x \) is an integer. Explain.

54. **REASONING**  Determine whether the following statement is sometimes, always, or never true. Explain your reasoning.

\[ A \text{ binomial can have a degree of zero.} \]

55. **OPEN ENDED**  Write an example of a cubic trinomial.

56. **WRITING IN MATH**  Explain how to write a polynomial in standard form and how to identify the leading coefficient.
57. Matrices \( P \) and \( Q \) are given below.

\[
P = \begin{bmatrix}
3 & 2 \\
6 & 9 \\
1 & 0
\end{bmatrix} \quad Q = \begin{bmatrix}
-3 & -2 \\
-6 & -9 \\
4 & 0
\end{bmatrix}
\]

What is \( P - Q \)?

58. You have a coupon from The Really Quick Lube Shop for an $8 off oil change this month. An oil change costs $19.95, and a new oil filter costs $4.95. You use the coupon for an oil change and filter. Before adding tax, how much should you pay?

- F $11.95
- G $16.90
- H $24.90
- J $27.95

59. SHORT RESPONSE In a recent poll, 3000 people were asked to pick their favorite baseball team. The accompanying circle graph shows the results of that poll. How many people polled picked the Black Sox as their favorite team?

60. What value for \( y \) satisfies the system of equations below?

\[
\begin{align*}
2x + y &= 19 \\
4x - 6y &= -2
\end{align*}
\]

- A 5
- B 7
- C 8
- D 10

### Spiral Review

Express each number in standard notation. (Lesson 7-3)

- 61. \( 6 \times 10^{-7} \)
- 62. \( 7.2 \times 10^{-10} \)
- 63. \( 8.1 \times 10^5 \)
- 64. \( 7 \times 10^6 \)
- 65. \( 0.132 \times 10^{-6} \)
- 66. \( 1.88 \times 10^0 \)

Simplify. Assume that no denominator is equal to zero. (Lesson 7-2)

- 67. \( a^0(a^4)(a^{-8}) \)
- 68. \( \frac{(4m^{-3}c^6)^0}{mc} \)
- 69. \( \frac{(3^23^6)^0}{(18^63^2)^0} \)
- 70. \( 12^{-1} \)
- 71. \( \frac{k^{-4}}{m^2p^{-8}} \)
- 72. \( \frac{(nq^{-1})^3}{(n^4q^8)^{-1}} \)

73. BUSINESS The owners of a new restaurant have hired enough servers to handle 17 tables of customers. The fire marshal has approved the restaurant for a limit of 56 customers. How many two-seat tables and how many four-seat tables should the owners buy? (Lesson 6-4)

### Skills Review

Simplify each expression. If not possible, write simplified. (Lesson 1-5)

- 74. \( 7b^2 + 14b - 10b \)
- 75. \( 5t + 12t^2 - 8t \)
- 76. \( 3y^4 + 2y^4 + 2y^5 \)
- 77. \( 7h^5 - 7j^5 + 8k^5 \)
- 78. \( n + \frac{n}{3} + \frac{2}{3}n \)
- 79. \( 2u + \frac{u}{2} + u^2 \)
Simplify each expression. (Lesson 7-1)

1. \((x^3)(4x^5)\)
2. \((m^2p^5)^3\)
3. \(\left[(2xy^3)^2\right]^3\)
4. \((6ab^3c^4)(-3a^2b^3c)\)

5. **MULTIPLE CHOICE** Express the volume of the solid as a monomial. (Lesson 7-1)

   \[\text{volume} = \frac{2x^2}{4x^4} = \frac{1}{2x^2}\]

   A. \(6x^9\)
   B. \(8x^9\)
   C. \(8x^{24}\)
   D. \(7x^{24}\)

Simplify each expression. Assume that no denominator equals 0. (Lesson 7-2)

6. \(\left(\frac{2a^3b^3}{c^6}\right)^3\)
7. \(\frac{2xy^0}{6x}\)
8. \(\frac{m^7n^4p}{m^3n^3p}\)
9. \(\frac{p^4t^{-2}}{r^{-5}}\)

10. **ASTRONOMY** Physicists estimate that the number of stars in the universe has an order of magnitude of \(10^{21}\). The number of stars in the Milky Way galaxy is around 100 billion. Using orders of magnitude, how many more stars are in the universe than the Milky Way? (Lesson 7-2)

   \(\text{number of stars in the universe} = 10^{21}\)
   \(\text{number of stars in the Milky Way} = 10^{11}\)
   \(\text{difference} = 10^{10}\)

Express each number in scientific notation. (Lesson 7-3)

11. 0.000000054
12. 0.0042
13. 234,000
14. 418,000,000

Express each number in standard form. (Lesson 7-3)

15. \(4.1 \times 10^{-3}\)
16. \(2.74 \times 10^5\)
17. \(3 \times 10^9\)
18. \(9.1 \times 10^{-5}\)

Evaluate each product or quotient. Express the results in scientific notation. (Lesson 7-3)

19. \((2.13 \times 10^2)(3 \times 10^5)\)
20. \((7.5 \times 10^6)(2.5 \times 10^{-2})\)
21. \(\frac{7.5 \times 10^8}{2.5 \times 10^4}\)
22. \(\frac{6.6 \times 10^5}{2 \times 10^{-3}}\)

Determine whether each expression is a polynomial. If so, identify the polynomial as a monomial, binomial, or trinomial. (Lesson 7-4)

23. \(3y^2 - 2\)
24. \(4t^5 + 3t^2 + t\)
25. \(\frac{3x}{5y}\)
26. \(ax^{-3}\)
27. \(3b^2\)
28. \(2x^{-3} - 4x + 1\)

29. **POPULATION** The table shows the population density for Nevada for various years. (Lesson 7-4)

<table>
<thead>
<tr>
<th>Year</th>
<th>Years Since 1930</th>
<th>People/Square Mile</th>
</tr>
</thead>
<tbody>
<tr>
<td>1930</td>
<td>0</td>
<td>0.8</td>
</tr>
<tr>
<td>1960</td>
<td>30</td>
<td>206</td>
</tr>
<tr>
<td>1980</td>
<td>50</td>
<td>7.3</td>
</tr>
<tr>
<td>1990</td>
<td>60</td>
<td>10.9</td>
</tr>
<tr>
<td>2000</td>
<td>70</td>
<td>18.2</td>
</tr>
</tbody>
</table>

   a. The population density \(d\) of Nevada from 1930 to 2000 can be modeled by \(d = 0.005y^2 - 0.127y + 1\), where \(y\) represents the number of years since 1930. Identify the type of polynomial for 0.005\(y^2\) - 0.127\(y\) + 1.

   b. What is the degree of the polynomial?

   c. Predict the population density of Nevada for 2020. Explain your method.

   d. Predict the population density of Nevada for 2030. Explain your method.
Monomials such as $3x$ and $-2x$ are called \textit{like terms} because they have the same variable to the same power. When you use algebra tiles, you can recognize like terms because the individual tiles have the same size and shape.

**Polynomial Models**

- Like terms are represented by tiles that have the same shape and size.
- A zero pair may be formed by pairing one tile with its opposite. You can remove or add zero pairs without changing the polynomial.

**ACTIVITY 1**

\textbf{Add Polynomials}

Use algebra tiles to find $(2x^2 - 3x + 5) + (x^2 + 6x - 4)$.

**Step 1** Model each polynomial.

1. $2x^2 - 3x + 5$
2. $x^2 + 6x - 4$

**Step 2** Combine like terms and remove zero pairs.

**Step 3** Write the polynomial for the tiles that remain.

$$(2x^2 - 3x + 5) + (x^2 + 6x - 4) = 3x^2 + 3x + 1$$
ACTIVITY 2  Subtract Polynomials

Use algebra tiles to find \((4x + 5) - (-3x + 1)\).

**Step 1**  Model the polynomial \(4x + 5\).

**Step 2**  To subtract \(-3x + 1\), you must remove 3 red \(-x\)-tiles and 1 yellow 1-tile. You can remove the yellow 1-tile, but there are no red \(-x\)-tiles. Add 3 pairs of \(x\)-tiles. Then remove the 3 red \(-x\)-tiles.

**Step 3**  Write the polynomial for the tiles that remain. \((4x + 5) - (-3x + 1) = 7x + 4\)

Recall that you can subtract a number by adding its additive inverse or opposite. Similarly, you can subtract a polynomial by adding its opposite.

ACTIVITY 3  Subtract Polynomials Using Additive Inverse

Use algebra tiles to find \((4x + 5) - (-3x + 1)\).

**Step 1**  To find the difference of \(4x + 5\) and \(-3x + 1\), add \(4x + 5\) and the opposite of \(-3x + 1\).

The opposite of \(-3x + 1\) is \(3x - 1\).

**Step 2**  Write the polynomial for the tiles that remain. \((4x + 5) - (-3x + 1) = 7x + 4\). Notice that this is the same answer as in Activity 2.

Model and Analyze

Use algebra tiles to find each sum or difference.

1. \((x^2 + 5x - 2) + (3x^2 - 2x + 6)\)
2. \((2x^2 + 8x + 1) - (x^2 - 4x - 2)\)
3. \((-4x^2 + x) - (x^2 + 5x)\)
4. **WRITING IN MATH**  Find \((4x^2 - x + 3) - (2x + 1)\) using each method from Activity 2 and Activity 3. Illustrate with drawings, and explain in writing how zero pairs are used in each case.
Adding and Subtracting Polynomials

**Why?**

From 2000 to 2003, sales (in millions of dollars) of rap/hip-hop music \( R \) and country music \( C \) in the United States can be modeled by the following equations, where \( t \) is the number of years since 2000.

\[
R = -132.3t^3 + 624.7t^2 - 773.6t + 1847.7
\]

\[
C = -3.4t^3 + 8.6t^2 - 95t + 1532.6
\]

The total music sales \( T \) of rap/hip-hop music and country music is \( R + C \).

**Add Polynomials** Adding polynomials involves adding like terms. You can group like terms by using a horizontal or vertical format.

**EXAMPLE 1**

Find each sum.

**a.** \( (2x^2 + 5x - 7) + (3 - 4x^2 + 6x) \)

**Horizontal Method**

\[
(2x^2 + 5x - 7) + (3 - 4x^2 + 6x) = [2x^2 + (-4x^2)] + [5x + 6x] + [-7 + 3]
\]

\[
= -2x^2 + 11x - 4
\]

**Vertical Method**

\[
\begin{align*}
2x^2 & \quad 5x & \quad -7 \\
+ & (-4x^2) & + 6x & + 3 \\
\hline
-2x^2 & \quad 11x & \quad -4
\end{align*}
\]

**b.** \( (3y + y^3 - 5) + (4y^2 - 4y + 2y^3 + 8) \)

**Horizontal Method**

\[
(3y + y^3 - 5) + (4y^2 - 4y + 2y^3 + 8) = [y^3 + 2y^3] + 4y^2 + [3y + (-4y)] + [(-5) + 8]
\]

\[
= 3y^3 + 4y^2 - y + 3
\]

**Vertical Method**

\[
\begin{align*}
y^3 & \quad 0y^2 & \quad 3y & \quad -5 \\
+ & 2y^3 & + 4y^2 & - 4y & + 8 \\
\hline
3y^3 & \quad 4y^2 & \quad -y & \quad + 3
\end{align*}
\]

**Check Your Progress**

1A. Find \( (5x^2 - 3x + 4) + (6x - 3x^2 - 3) \).

1B. Find \( (y^4 - 3y + 7) + (2y^3 + 2y - 2y^4 - 11) \).
**Subtract Polynomials**  Recall that you can subtract a real number by adding its opposite or additive inverse. Similarly, you can subtract a polynomial by adding its additive inverse.

To find the additive inverse of a polynomial, write the opposite of each term in the polynomial.

\[-(3x^2 + 2x - 6) = -3x^2 - 2x + 6\]

**Study Tip**

Additive Inverse

When finding the additive inverse of a polynomial, you are multiplying every term by \(-1\).

**Example 2**  Subtract Polynomials

Find each difference.

**a.** \((3 - 2x + 2x^2) - (4x - 5 + 3x^2)\)

**Horizontal Method**

Subtract \(4x - 5 + 3x^2\) by adding its additive inverse.

\[
(3 - 2x + 2x^2) - (4x - 5 + 3x^2) = (3 - 2x + 2x^2) + (-4x + 5 - 3x^2)
\]

\[
= [2x^2 + (-3x^2)] + [(-2x) + (-4x)] + [3 + 5]
\]

\[
= -x^2 - 6x + 8
\]

**Vertical Method**

Align like terms in columns and subtract by adding the additive inverse.

\[
\begin{array}{c}
2x^2 - 2x + 3 \\
-3x^2 + 4x - 5 \\
\hline
\end{array}
\]

\[
\begin{array}{c}
\text{Add the opposite.} \\
\hline
2x^2 - 2x + 3 \\
+3x^2 - 4x + 5 \\
\hline
-x^2 - 6x + 8
\end{array}
\]

Thus, \((3 - 2x + 2x^2) - (4x - 5 + 3x^2) = -x^2 - 6x + 8\).

**b.** \((7p + 4p^3 - 8) - (3p^2 + 2 - 9p)\)

**Horizontal Method**

Subtract \(3p^2 + 2 - 9p\) by adding its additive inverse.

\[
(7p + 4p^3 - 8) - (3p^2 + 2 - 9p) = (7p + 4p^3 - 8) + (-3p^2 - 2 + 9p)
\]

\[
= [7p + 9p] + 4p^3 + (-3p^2) + [(-8) + (-2)]
\]

\[
= 16p + 4p^3 - 3p^2 - 10
\]

**Vertical Method**

Align like terms in columns and subtract by adding the additive inverse.

\[
\begin{array}{c}
4p^3 + 0p^2 + 7p - 8 \\
-3p^2 - 9p + 2 \\
\hline
\end{array}
\]

\[
\begin{array}{c}
\text{Add the opposite.} \\
\hline
4p^3 + 0p^2 + 7p - 8 \\
+3p^2 + 9p - 2 \\
\hline
4p^3 - 3p^2 + 16p - 10
\end{array}
\]

Thus, \((7p + 4p^3 - 8) - (3p^2 + 2 - 9p) = 4p^3 - 3p^2 + 16p - 10\).

**Check Your Progress**

**2A.** Find \((4x^3 - 3x^2 + 6x - 4) - (-2x^3 + x^2 - 2)\).

**2B.** Find \((8y - 10 + 5y^2) - (7 - y^3 + 12y)\).
CONSUMER ELECTRONICS  An electronics store sells cell phones and digital cameras. The equations below represent the monthly sales \( m \) of cell phones \( P \) and digital cameras \( C \).

\[
P = 7m + 137 \quad C = 4m + 78
\]

a. Write an equation that represents the total monthly sales \( T \) of phones and cameras.

Add the polynomial for \( P \) with the polynomial for \( C \).

\[
T = 7m + 137 + 4m + 78 \quad \text{Substitution}
\]

\[
T = 11m + 215 \quad \text{Combine like terms.}
\]

An equation is \( T = 11m + 215 \).

b. Use the equation to predict the number of cell phones and digital cameras sold in 10 months.

\[
T = 11(10) + 215 \quad \text{Substitute 10 for} \ m.
\]

\[
= 110 + 215 \quad \text{Simplify.}
\]

\[
= 325
\]

Thus, a total of 325 cell phones and digital cameras will be sold in 10 months.

3. Use the information above to write an equation that represents the difference in the monthly sales of cell phones and the monthly sales of digital cameras. Use the equation to predict the difference in monthly sales in 24 months.
Find each sum or difference.

10. 
\[(y + 5) + (2y + 4y^2 - 2)\]

11. 
\[(2x + 3x^2) - (7 - 8x^2)\]

12. 
\[(3c^3 - c + 11) - (c^2 + 2c + 8)\]

13. 
\[(z^2 + z) + (z^2 - 11)\]

14. 
\[(2x - 2y + 1) - (3y + 4x)\]

15. 
\[(4a - 5b^2 + 3) + (6 - 2a + 3b^2)\]

16. 
\[(x^2y - 3x^2 + y) - (3y - 2x^2y)\]

17. 
\[(-8xy + 3x^2 - 5y) + (4x^2 - 2y + 6xy)\]

18. 
\[(5n - 2p^2 + 2np) - (4p^2 + 4n)\]

19. 
\[(4rx - 8ry^2x + x^2) - (6rx^2 + 5rxt - 2x^2)\]

20. 
\[(6ab^2 + 2ab) + (3a^2b - 4ab + ab^2)\]

21. 
\[(cd^2 + 2cd - 4) + (-6 + 4cd - 2cd^2)\]

Example 3

PETS

From 1997 through 2007, the number of dogs $D$ and the number of cats $C$ (in hundreds) adopted at animal shelters in the United States are modeled by the following equations, where $n$ is the number of years since 1997.

\[D = 2n + 3 \quad C = n + 4\]

a. Write an equation that models the total number $T$ of dogs and cats adopted from animal shelters in hundreds for this time period.

b. If this trend continues, how many dogs and cats will be adopted from animal shelters in 2011?

Find each sum or difference.

22. 
\[(4x + 2y - 6z) + (5y - 2z + 7x) + (-9z - 2x - 3y)\]

23. 
\[(5a^2 - 4) + (a^2 - 2a + 12) + (4a^2 - 6a + 8)\]

24. 
\[(3c^2 - 7) + (4c + 7) - (c^2 + 5c - 8)\]

25. 
\[(3n^3 + 3n - 10) - (4n^2 - 5n) + (4n^3 - 3n^2 - 9n + 4)\]

26. 
\[2x^2 + 3x\]

27. GEOMETRY

Write a polynomial that represents the perimeter of the figure at the right.

28. PAINTING

Kin is painting two walls of her bedroom. The area of one wall can be modeled by $3x^2 + 14$, and the area of the other wall can be modeled by $2x - 3$. What is the total area of the walls that Kin will paint?

29. GEOMETRY

The perimeter of the figure at the right is represented by the expression $7x^2 - 2x + 11$. Write a polynomial that represents the measure of the third side.

30. FOOTBALL

The National Football League is divided into two conferences, the American $A$ and the National $N$. From 1996 through 2004, the total attendance $T$ (in thousands) for both conferences and for the American Conference games are modeled by the following equations, where $y$ is the number of years since 1996.

\[T = 35y^3 + 27y^2 + 1899 \quad A = 16y^3 + 13y^2 + 2y + 905\]

Determine how many people attended a National Conference football game in 2002.

31. GEOMETRY

The width of a rectangle is represented by the polynomial $5x + 2y$, and the length is represented by the polynomial $6y - 2x$. Write a polynomial that represents the perimeter of the rectangle.
32. **GARDENING** Candida is planting flowers on the perimeter of a rectangular patio.
   a. If the perimeter of the patio is $210x$ and one side measures $32x$, find the length of the other side.
   b. Write a polynomial that represents the area of the rectangular patio.

33. **GEOMETRY** The sum of the measures of the angles in a triangle is $180^\circ$.
   a. Write an expression to represent the measure of the third angle of the triangle.
   b. If $x = 23$, find the measures of the three angles of the triangle.

34. **SALES** An electronics store estimates that the cost, in dollars, of selling $t$ units of LCD televisions is given by the expression $0.002t^2 + 4t + 400$. The revenue from the sales of $t$ LCD televisions is $8t$.
   a. Write a polynomial that represents the profit of selling $t$ units.
   b. If 750 LCD televisions are sold, how much did the store earn?
   c. If 575 LCD televisions are sold, how much did the store earn?

35. **CAR RENTAL** The cost to rent a car for a day is $15 plus $0.15 for each mile driven.
   a. Write a polynomial that represents the cost of renting a car for $m$ miles.
   b. If a car is driven 145 miles, how much would it cost to rent?
   c. If a car is driven 105 miles each day for four days, how much would it cost to rent a car?
   d. If a car is driven 220 miles each day for seven days, how much would it cost to rent a car?

36. **FIND THE ERROR** Cheyenne and Sebastian are finding $(2x^2 - x) - (3x + 3x^2 - 2)$. Is either of them correct? Explain your reasoning.

37. **OPEN ENDED** Write two trinomials with a difference of $2x^3 - 7x + 8$.

38. **CHALLENGE** Write a polynomial that represents the sum of an odd integer $2n + 1$ and the next two consecutive odd integers.

39. **REASONING** Find a counterexample to the following statement.
   
   The order in which polynomials are subtracted does not matter.

40. **OPEN ENDED** Write three trinomials with a sum of $4x^4 + 3x^2$.

41. **WRITING IN MATH** Describe how to add and subtract polynomials using both the vertical and horizontal formats. Which one do you think is easier to use? Why?

---

**Real-World Link**

On average, an LCD television lasts about 60,000 hours. This means the lifespan of an LCD television can last 20 years or more if the television is used less than 8 hours per day.

Source: LCD TV Buying Guide
42. Three consecutive integers can be represented by \( x, x + 1, \) and \( x + 2 \). What is the sum of these three integers?

\[ A \ x(x + 1)(x + 2) \quad C \ 3x + 3 \]

\[ B \ x^3 + 3 \quad D \ x + 3 \]

43. SHORT RESPONSE What is the perimeter of a square with sides that measure \( 2x + 3 \) units?

44. Jim cuts a board in the shape of a regular hexagon and pounds in a nail at each vertex, as shown. How many rubber bands will he need to stretch a rubber band across every possible pair of nails?

\[ F \ 15 \quad H \ 12 \quad G \ 14 \quad J \ 9 \]

45. Which coordinate point is in the solution set for the system of inequalities shown in the accompanying graph?

\[ A \ (-3, 0) \quad C \ (5, 0) \]

\[ B \ (0, -3) \quad D \ (0, 5) \]

---

Spiral Review

Find the degree of each polynomial. (Lesson 7-4)

46. \( 6b^4 \)  
47. \( 10t \)  
48. \( 5s^2h \)  
49. \( 7np^4 \)  
50. \( 25 \)  
51. \( t^3 + 6u \)  
52. \( 2 + 3ab^3 - a^2b + 4a^6 \)  
53. \( 6 - v^4 + v^2z^3 + 6v^3 \)  

54. POPULATION The 2005 population of North Carolina’s Beaufort County was approximately 46,000. Express this number in scientific notation. (Lesson 7-3)

55. JOBS Kimi received a job offer in which she will receive $600 per month plus a commission of 2% of the total price of her sales. At her current job she receives $1000 per month and 1.5% commission of her total sales. What is total amount of sales that Kimi must sell each month to make the same income at either job? (Lesson 6-2)

Determine whether each sequence is an arithmetic sequence. If it is, state the common difference. (Lesson 3-5)

56. 24, 16, 8, 0, …  
57. \( 3\frac{1}{2}, 6\frac{1}{2}, 13, 26, … \)  
58. 7, 6, 5, 4, …  
59. 10, 12, 15, 18, …  
60. \(-15, -11, -7, -3, … \)  
61. \(-0.3, 0.2, 0.7, 1.2, … \)

Skills Review

Simplify. (Lesson 7-1)

62. \( t(t^5)(t^2) \)  
63. \( n^3(n^2)(-2n^3) \)  
64. \( (5t^5v^2)(10t^3v^4) \)  
65. \( (-8u^4z^5)(5uz^4) \)  
66. \( \left(3^2\right)^3 \)  
67. \( \left(2^{-3}\right)^2 \)  
68. \( (2m^4k^3)^2(-3mk^2)^3 \)  
69. \( (6xy^2)^2(2x^2y^2z^2)^3 \)
Then
You multiplied monomials.
(Lesson 7-1)

Now
- Multiply a polynomial
  by a monomial.
- Solve equations
  involving the products
  of monomials and
  polynomials.

KY Program
of Studies
HS-AT-S-VE06 Students
will add, subtract and
multiply polynomials. Also
addresses HS-NPO-S-NO10.

KY Math Online
glencoe.com
- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

**Then**

**Why?**

Charmaine Brooks is opening a fitness club. She tells the contractor doing the renovations that the length of the fitness room should be three times the width plus 8 feet.

To cover the floor with mats for exercise classes, Ms. Brooks needs to know the area of the floor. So she multiplies the width times the length, \( w(3w + 8) \).

**Polynomial Multiplied by Monomial**

To find the product of a polynomial and a monomial, you can use the Distributive Property.

**EXAMPLE 1**

**Multiply a Polynomial by a Monomial**

Find \(-3x^2(7x^2 - x + 4)\).

**Horizontal Method**

\[
\begin{align*}
-3x^2(7x^2 - x + 4) &= -3x^2(7x^2) - (-3x^2)(x) + (-3x^2)(4) \\
&= -21x^4 + 3x^3 - 12x^2
\end{align*}
\]

**Vertical Method**

\[
\begin{array}{c}
7x^2 - x + 4 \\
(x)
\end{array}
\]

\[
\begin{align*}
-3x^2 &
\end{align*}
\]

\[
\begin{align*}
-21x^4 + 3x^3 - 12x^2 \\
\end{align*}
\]

**Check Your Progress**

Find each product.

**1A.** \(5a^2(-4a^2 + 2a - 7)\)

**1B.** \(-6d^3(3d^4 - 2d^3 - d + 9)\)

We can use this same method more than once to simplify large expressions.

**EXAMPLE 2**

**Simplify Expressions**

Simplify \(2p(-4p^2 + 5p) - 5(2p^2 + 20)\).

\[
\begin{align*}
2p(-4p^2 + 5p) - 5(2p^2 + 20) &= (2p)(-4p^2) + (2p)(5p) + (-5)(2p^2) + (-5)(20) \\
&= -8p^3 + 10p^2 - 10p^2 - 100 \\
&= -8p^3 + (-10p^2 - 10p^2) - 100 \\
&= -8p^3 - 100
\end{align*}
\]
Simplify each expression.

2A. \(3(5x^2 + 2x - 4) - x(7x^2 + 2x - 3)\)

2B. \(15t(10y^3t^5 + 5y^2t) - 2y(yt^2 + 4y^2)\)

We can use the Distributive Property to multiply monomials by polynomials and solve real world problems.

The theme for a school dance is “Solid Gold.” For one decoration, Kana is covering a trapezoid-shaped piece of poster board with metallic gold paper to look like a bar of gold. If the height of the poster board is 18 inches, how much metallic paper will Kana need in square inches?

Read the Test Item
The question is asking you to find the area of the trapezoid with bases of \(h + 1\) and \(2h + 4\).

Solve the Test Item
Write an equation to represent the area of the trapezoid.
Let \(b_1 = h + 1\), let \(b_2 = 2h + 4\) and let \(h\) = height of the trapezoid.

\[
A = \frac{1}{2}h(b_1 + b_2) \\
= \frac{1}{2}h[(h + 1) + (2h + 4)] \\
= \frac{1}{2}h(3h + 5) \\
= \frac{3}{2}h^2 + \frac{5}{2}h \\
= \frac{3}{2}(18)^2 + \frac{5}{2}(18) \\
= 531
\]

Kana will need 531 square inches of metallic paper. Grid in your response of 531.

3. **GRIDDED RESPONSE** Kachima is making triangular bandanas for the dogs and cats in her pet club. The base of the bandana is the length of the collar with 4 inches added to each end to tie it on. The height is \(\frac{1}{2}\) of the collar length.

A. If Kachima’s dog has a collar length of 12 inches, how much fabric does she need in square inches?

B. Kachima is making a bandana for her friend’s cat that is in the pet club. The cat has a collar length of 6 inches. How much fabric does Kachima need in square inches?
Solve Equations with Polynomial Expressions We can use the Distributive Property to solve equations that involve the products of monomials and polynomials.

**Example 4** Equations with Polynomials on Both Sides

Solve $2a(5a - 2) + 3a(2a + 6) + 8 = a(4a + 1) + 2a(6a - 4) + 50$.

Original equation

Distributive Property

Combine like terms.

Subtract $16a^2$ from each side.

Add $7a$ to each side.

Subtract 8 from each side.

Divide each side by 21.

CHECK

$2a(5a - 2) + 3a(2a + 6) + 8 = a(4a + 1) + 2a(6a - 4) + 50$

$2(2)[5(2) - 2] + 3(2)[2(2) + 6] + 8 \div 2[4(2) + 1] + 2[2(6)(2) - 4] + 50$

$4(8) + 6(10) + 8 \div 2(9) + 4(8) + 50$

$32 + 60 + 8 \div 18 + 32 + 50$

$100 = 100 \checkmark$

Check Your Progress

Solve each of the following.

4A. $2x(x + 4) + 7 = (x + 8) + 2x(x + 1) + 12$

4B. $d(d + 3) - d(d - 4) = 9d - 16$

Check Your Understanding

**Example 1**

Find each product.

1. $5w(-3w^2 + 2w - 4)$

2. $6g^2(3g^3 + 4g^2 + 10g - 1)$

3. $4kn(8km^2 + 2k^2m + 5k)$

4. $-3p^4r^3(2p^2r^4 - 6p^6r^3 - 5)$

5. $2ab(7a^4b^2 + a^3b - 2a)$

6. $c^2d^3(5cd^7 - 3c^3d^2 - 4d^3)$

**Example 2**

Simplify each expression.

7. $t(4t^2 + 15t + 4) - 4(3t - 1)$

8. $x(3x^2 + 4) + 2(7x - 3)$

9. $-2d(d^3c^2 - 4dc^2 + 2d^2c) + c^2(dc^2 - 3d^4)$

10. $-5w^2(8w^2x - 11wx^2) + 6x(9wx^4 - 4w - 3x^2)$

**Example 3**

11. GRIDDED RESPONSE Marlene is buying a new plasma television. The height of the screen of the television is one half the width plus 5 inches. The width is 30 inches. Find the height of the screen in inches.

**Example 4**

Solve each equation.

12. $-6(11 - 2c) = 7(-2 - 2c)$

13. $t(2t + 3) + 20 = 2t(t - 3)$

14. $-2(w + 1) + w = 7 - 4w$

15. $3(y - 2) + 2y = 4y + 14$

16. $a(a + 3) + a(a - 6) + 35 = a(a - 5) + a(a + 7)$

17. $n(n - 4) + n(n + 8) = n(n - 13) + n(n + 1) + 16$
### Example 1

Find each product.

18. \( b(b^2 - 12b + 1) \)
19. \( f(f^2 + 2f + 25) \)
20. \( -3m^3(2m^3 - 12m^2 + 2m + 25) \)
21. \( 2j^2(5j^3 - 15j^2 + 2j + 2) \)
22. \( 2pr^2(2pr + 5p^2r - 15p) \)
23. \( 4t^3u(2t^2u^2 - 10tu^4 + 2) \)

### Example 2

Simplify each expression.

24. \( -3(5x^2 + 2x + 9) + x(2x - 3) \)
25. \( a(-8a^2 + 2a + 4) + 3(6a^2 - 4) \)
26. \( -4d(5d^2 - 12) + 7(d + 5) \)
27. \( -9g(-2g + g^2) + 3(g^2 + 4) \)
28. \( 2j(7j^2k^2 + jk^2 + 5k) - 9k(-2j^2k^2 + 2k^2 + 3j) \)
29. \( 4n(2n^3p^2 - 3np^2 + 5n) + 4p(6n^2p - 2np^2 + 3p) \)

### Example 3

**DAMS** A new dam being built has the shape of a trapezoid. The base at the bottom of the dam is 2 times the height. The base at the top of the dam is \( \frac{1}{5} \) times the height minus 30 feet.

a. Write an expression to find the area of the trapezoidal cross section of the dam.

b. If the height of the dam is 180 feet, find the area of this cross section.

### Example 4

Solve each equation.

31. \( 7(t^2 + 5t - 9) + t = t(7t - 2) + 13 \)
32. \( w(4w + 6) + 2w = 2(2w^2 + 7w - 3) \)
33. \( 5(4z + 6) - 2(z - 4) = 7z(z + 4) - z(7z - 2) - 48 \)
34. \( 9c(c - 11) + 10(5c - 3) = 3c(c + 5) + c(6c - 3) - 30 \)
35. \( 2f(5f - 2) - 10(f^2 - 3f + 6) = -8(f + 4) + 4(2f^2 - 7f) \)
36. \( 2k(-3k + 4) + 6(k^2 + 10) = k(4k + 8) - 2k(2k + 5) \)

Simplify each expression.

37. \( \frac{2}{3}n^2p(30p^2 + 9n^2p - 12) \)
38. \( \frac{3}{5}r^2t(10r^3 + 5rt^3 + 15t^2) \)
39. \( -5q^2w^3(4q + 7w) + 4qw^2(7q^2w + 2q) - 3qw(3q^2w^2 + 9) \)
40. \( -x^2z(2x^2 + 4xz^3) + xz^2(xz + 5x^3z) + x^2z^3(3x^2z + 4xz) \)

**PARKING** A parking garage charges $30 per month plus $0.50 per daytime hour and $0.25 per hour during nights and weekends. Suppose Trent parks in the garage for 47 hours in January and \( h \) of those are night and weekend hours.

a. Find an expression for Trent’s January bill.

b. Find the cost if Trent had 12 hours of night and weekend hours.

**PETS** Che is building a dog house for his new puppy. The upper face of the dog house is a trapezoid. If the height of the trapezoid is 12 inches, find the area of the face of this piece of the dog house.
**TENNIS** The tennis club is building a new tennis court with a hedge around it. Each bush of the hedge is planted 3 feet from each other.

a. Write an expression for the area of the tennis court.

b. Write an expression for the area of the hedge.

c. How many bushes will be planted if \( x = 36 \)?

**MULTIPLE REPRESENTATIONS** In this problem, you will investigate the degree of the product of a monomial and a polynomial.

a. **TABULAR** Write three monomials of different degrees and three polynomials of different degrees. Determine the degree of each monomial and polynomial. Multiply the monomials by the polynomials. Determine the degree of each product. Record your results in a table like the one shown below.

<table>
<thead>
<tr>
<th>Monomial</th>
<th>Degree</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Polynomial</th>
<th>Degree</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Product of Monomial and Polynomial</th>
<th>Degree</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. **VERBAL** Make a conjecture about the degree of the product of a monomial and a polynomial. What is the degree of the product of a monomial of degree \( a \) and a polynomial of degree \( b \)?

**H.O.T. Problems**

**FIND THE ERROR** Pearl and Ted both worked on this problem. Is either of them correct? Explain your reasoning.

Pearl

\[ 2x^2(3x^2 + 4x + 2) \]

\[ 6x^4 + 8x^2 + 4x^2 \]

\[ 6x^4 + 12x^2 \]

Ted

\[ 2x^2(3x^2 + 4x + 2) \]

\[ 6x^4 + 8x^3 + 4x^2 \]

**CHALLENGE** Find \( p \) such that \( 3x^p(4x^{2p} + 3 + 2x^{3p} - 2) = 12x^{12} + 6x^{10} \).

**CHALLENGE** Find \( 4x^{-3}y^2(2x^5y^{-4} + 6x^{-7}y^6 - 4x^0y^{-2}) \).

**REASONING** Is there a value for \( x \) that makes the statement \( (x + 2)^2 = x^2 + 2^2 \) true? If so, find a value for \( x \). Explain your reasoning.

**OPEN ENDED** Write a monomial and a polynomial using \( n \) as the variable. Find their product.

**WRITING IN MATH** Describe the steps to multiply a polynomial by a monomial.
51. Every week a store sells \( j \) jeans and \( t \) T-shirts. The store makes $8 for each T-shirt and $12 for each pair of jeans. Which of the following expression represents the total amount of money, in dollars, the store makes every week?

A \( 8j + 12t \)  
B \( 12j + 8t \)  
C \( 20(j + t) \)  
D \( 96jt \)

52. If \( a = 5x + 7y \) and \( b = 2y - 3x \), then what is the value of \( a + b \)?

F \( 2x - 9y \)  
H \( 2x + 9y \)  
G \( 3y + 4x \)  
J \( 2x - 5y \)

53. GEOMETRY A triangle has sides of length 5 inches and 8.5 inches. Which of the following cannot be the length of the third side?

A 3.5 inches  
B 4 inches  
C 5.5 inches  
D 12 inches

54. SHORT RESPONSE Write an equation in which \( x \) varies directly as the cube of \( y \) and inversely as the square of \( z \).

55. Find each sum or difference. (Lesson 7-5)

\[
55. (2x^2 - 7) + (8 - 5x^2) \quad 56. (3z^2 + 2z - 1) + (z^2 - 6) \quad 57. (2a - 4a^2 + 1) - (5a^2 - 2a - 6)
\]

\[
58. (a^3 - 3a^2 + 4) - (4a^2 + 7) \quad 59. (2ab - 3a + 4b) + (5a + 4ab) \quad 60. (8c^3 - 3c^2 + c - 2) - (3c^3 + 9)
\]

Find the degree of each polynomial. (Lesson 7-4)

\[
61. 12y \quad 62. -10 \quad 63. 2x^2 - 5 \quad 64. 9a - 8a^3 + 6 \quad 65. 7b^2c^3 \quad 66. -3p^4r^5t^2
\]

67. TRAVEL In 1990, about 3.6 million people took cruises. Between 1990 and 2000, the number of people taking cruises increased by about 300,000 each year. Write the point-slope form of an equation to find the total number of people \( y \) taking a cruise for any year \( x \). Estimate the number of people who will take a cruise in 2010. (Lesson 4-3)

Write an equation in function notation for each relation. (Lesson 3-6)

\[
68. \quad 69.
\]

Skills Review

Simplify. (Lesson 7-1)

\[
70. b(b^2)(b^3) \quad 71. 2y(3y^2) \quad 72. -y^4(-2y^3)
\]

\[
73. -3z^3(-5z^4 + 2z) \quad 74. 2m(-4m^4) - 3(-5m^3) \quad 75. 4p^2(-2p^3) + 2p^4(5p^6)
\]
You can use algebra tiles to find the product of two binomials.

**ACTIVITY 1 Multiply Binomials**

**Use algebra tiles to find \((x + 3)(x + 4)\).**

The rectangle will have a width of \(x + 3\) and a length of \(x + 4\). Use algebra tiles to mark off the dimensions on a product mat. Then complete the rectangle with algebra tiles.

The rectangle consists of 1 blue \(x^2\)-tile, 7 green \(x\)-tiles, and 12 yellow 1-tiles. The area of the rectangle is \(x^2 + 7x + 12\). So, \((x + 3)(x + 4) = x^2 + 7x + 12\).

**ACTIVITY 2 Multiply Binomials**

**Use algebra tiles to find \((x - 2)(x - 5)\).**

**Step 1** The rectangle will have a width of \(x - 2\) and a length of \(x - 5\). Use algebra tiles to mark off the dimensions on a product mat. Then begin to make the rectangle with algebra tiles.

**Step 2** Determine whether to use 10 yellow 1-tiles or 10 red \(-1\)-tiles to complete the rectangle. The area of each yellow tile is the product of \(-1\) and \(-1\). Fill in the space with 10 yellow 1-tiles to complete the rectangle.

The rectangle consists of 1 blue \(x^2\)-tile, 7 red \(x\)-tiles, and 10 yellow 1-tiles. The area of the rectangle is \(x^2 - 7x + 10\). So, \((x - 2)(x - 5) = x^2 - 7x + 10\).
ACTIVITY 3  Multiply Binomials

Use algebra tiles to find \((x - 4)(2x + 3)\).

**Step 1** The rectangle will have a width of \(x - 4\) and a length of \(2x + 3\). Use algebra tiles to mark off the dimensions on a product mat. Then begin to make the rectangle with algebra tiles.

**Step 2** Determine what color \(x\)-tiles and what color 1-tiles to use to complete the rectangle. The area of each red \(x\)-tile is the product of \(x\) and \(-1\). The area of each red 1-tile is represented by the product of 1 and \(-1\) or 1.

Complete the rectangle with 4 red \(x\)-tiles and 12 red \(-1\)-tiles.

**Step 3** Rearrange the tiles to simplify the polynomial you have formed. Notice that a 3 zero pair are formed by three positive and three negative \(x\)-tiles.

There are 2 blue \(x^2\)-tiles, 5 red \(-x\)-tiles, and 12 red \(-1\)-tiles left. In simplest form, \((x - 4)(2x + 3) = 2x^2 - 5x - 12\).

Model and Analyze

Use algebra tiles to find each product.

1. \((x + 1)(x + 4)\)  
2. \((x - 3)(x - 2)\)  
3. \((x + 5)(x - 1)\)  
4. \((x + 2)(2x + 3)\)  
5. \((x - 1)(2x - 1)\)  
6. \((x + 4)(2x - 5)\)

Is each statement true or false? Justify your answer with a drawing.

7. \((x - 4)(x - 2) = x^2 - 6x + 8\)  
8. \((x + 3)(x + 5) = x^2 + 15\)

9. **WRITING IN MATH** You can also use the Distributive Property to find the product of two binomials. The figure at the right shows the model for \((x + 4)(x + 5)\) separated into four parts. Write a sentence or two explaining how this model shows the use of the Distributive Property.
Then
You multiplied polynomials by monomials. (Lesson 7-6)

Now
- Multiply polynomials by using the Distributive Property.
- Multiply binomials by using the FOIL method.

KY Program of Studies
HS-AT-S-VEO6 Students will add, subtract and multiply polynomials. Also addresses HS-NPO-S-NO10.

New Vocabulary
- FOIL method
- quadratic expression

KY Math Online
- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help
- Math in Motion

glencoe.com

Why?
Bodyboards, which are used to ride waves, are made of foam and are more rectangular than surfboards. A bodyboard’s dimensions are determined by the height and skill level of the user.

The length of Ann’s bodyboard should be Ann’s height \( h \) minus 32 inches or \( h - 32 \). The board’s width should be half of Ann’s height plus 11 inches or \( \frac{1}{2}h + 11 \). To approximate the area of the bodyboard, you need to find \((h - 32)(\frac{1}{2}h + 11)\).

Multiply Binomials To multiply two binomials such as \( h - 32 \) and \( \frac{1}{2}h + 11 \), the Distributive Property is used. Binomials can be multiplied horizontally or vertically.

EXAMPLE 1 The Distributive Property

Find each product.

a. \((2x + 3)(x + 5)\)

Vertical Method

Multiply by 5. Multiply by \( x \).

\[
\begin{align*}
2x + 3 & \quad \quad \quad \quad \quad \quad \quad (x) x + 5 \\
10x + 15 & \quad \quad \quad \quad \quad \quad \quad (x) + 5 \\
2x^2 + 3x & \quad \quad \quad \quad \quad \quad \quad 10x + 15 \\
5(2x + 3) = 10x + 15 & \quad \quad x(2x + 3) = 2x^2 + 3x \\
& \quad \quad \quad \quad \quad \quad \quad 2x^2 + 3x
\end{align*}
\]

Horizontal Method

\[
(2x + 3)(x + 5) = 2x(x + 5) + 3(x + 5)
\]

Rewrite as the sum of two products. Distributive Property

Combine like terms.

b. \((x - 2)(3x + 4)\)

Vertical Method

Multiply by 4. Multiply by 3x. Combine like terms.

\[
\begin{align*}
x - 2 & \quad \quad \quad \quad \quad \quad \quad (x) 3x + 4 \\
4x - 8 & \quad \quad \quad \quad \quad \quad \quad 4x - 8 \\
x^2 - 6x & \quad \quad \quad \quad \quad \quad \quad 4x - 8 \\
4(x - 2) = 4x - 8 & \quad \quad 3x(x - 2) = 3x^2 - 6x \\
& \quad \quad \quad \quad \quad \quad \quad 3x^2 - 2x - 8
\end{align*}
\]

Horizontal Method

\[
(x - 2)(3x + 4) = x(3x + 4) - 2(3x + 4)
\]

Rewrite as the difference of two products. Distributive Property

Combine like terms.
A shortcut version of the Distributive Property for multiplying binomials is called the **FOIL method**.

**Key Concept**

**FOIL Method**

**Words**
To multiply two binomials, find the sum of the products of **F** the **First** terms, **O** the **Outer** terms, **I** the **Inner** terms, **L** and the **Last** terms.

**Example**

\[(x + 4)(x - 2)\]

<table>
<thead>
<tr>
<th>Product of First Terms</th>
<th>Product of Outer Terms</th>
<th>Product of Inner Terms</th>
<th>Product of Last Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>((x)(x))</td>
<td>((x)(-2))</td>
<td>((4)(x))</td>
<td>((4)(-2))</td>
</tr>
</tbody>
</table>

\[= x^2 - 2x + 4x - 8\]

\[= x^2 + 2x - 8\]

**EXAMPLE 2**

**FOIL Method**

Find each product.

a. \((2y - 7)(3y + 5)\)

\[(2y - 7)(3y + 5) = (2y)(3y) + (2y)(5) + (-7)(3y) + (-7)(5)\]

\[= 6y^2 + 10y - 21y - 35\]

\[= 6y^2 - 11y - 35\]

b. \((4a - 5)(2a - 9)\)

\[(4a - 5)(2a - 9) = (4a)(2a) + (4a)(-9) + (-5)(2a) + (-5)(-9)\]

\[= 8a^2 - 36a - 10a + 45\]

\[= 8a^2 - 46a + 45\]

**Check Your Progress**

2A. \((x + 3)(x - 4)\)

2B. \((4b - 5)(3b + 2)\)

2C. \((2y - 5)(y - 6)\)

2D. \((5a + 2)(3a - 4)\)

Notice that when two linear expressions are multiplied, the result is a quadratic expression. A **quadratic expression** is an expression in one variable with a degree of 2. When three linear expressions are multiplied, the resulting expression has a degree of 3.

The FOIL method can be used to find an expression that represents the area of an object when the lengths of the sides are given as binomials.
**Real-World Link**

The cost of a swimming pool depends on many factors, including the size of the pool, whether the pool is an above-ground or an in-ground pool, and the material used.

*Source: American Dream Homes*

---

**FOIL Method**

**SWIMMING POOL** A contractor is building a deck around a rectangular swimming pool. The deck is $x$ feet from every side of the pool. Write an expression for the total area of the pool and deck.

**Understand** We need to find an expression for the total area of the pool and deck.

**Plan** Use the formula for the area of a rectangle and determine the length and width of the pool with the deck.

**Solve** Since the deck is the same distance from every side of the pool, the length and width of the pool is $2x$ longer. So, the length can be represented by $2x + 20$ and the width can be represented by $2x + 15$.

Use these expressions to determine the area.

\[
\text{Area} = \text{length} \times \text{width}
\]

\[
= (2x + 20)(2x + 15)
\]

\[
= (2x)(2x) + (2x)(15) + (20)(2x) + (20)(15)
\]

\[
= 4x^2 + 30x + 40x + 300
\]

\[
= 4x^2 + 70x + 300
\]

So, the total area of the deck and pool is $4x^2 + 70x + 300$.

**Check** Choose a value for $x$. Substitute this value into $(2x + 20)(2x + 15)$ and $4x^2 + 70x + 300$. If the result is the same for both expressions, then they are equivalent.

---

**Check Your Progress**

3. If the pool is 25 feet long and 20 feet wide, find the total area of the pool and deck.

---

**Multiply Polynomials** The Distributive Property can also be used to multiply any two polynomials.

---

**EXAMPLE 4** The Distributive Property

Find each product.

a. $\quad (6x + 5)(2x^2 - 3x - 5)$$\quad (6x + 5)(2x^2 - 3x - 5)$$\quad (6x + 5)(2x^2 - 3x - 5)$$\quad (6x + 5)(2x^2 - 3x - 5)$

\[
= 6x(2x^2 - 3x - 5) + 5(2x^2 - 3x - 5)
\]

\[
= 12x^3 - 18x^2 - 30x + 10x^2 - 15x - 25
\]

\[
= 12x^3 - 8x^2 - 45x - 25
\]

b. $\quad (2y^2 + 3y - 1)(3y^2 - 5y + 2)$$\quad (2y^2 + 3y - 1)(3y^2 - 5y + 2)$

\[
= 2y^2(3y^2 - 5y + 2) + 3y(3y^2 - 5y + 2) - 1(3y^2 - 5y + 2)
\]

\[
= 6y^4 - 10y^3 + 4y^2 + 9y^3 - 15y^2 + 6y - 3y^2 + 5y - 2
\]

\[
= 6y^4 - y^3 - 14y^2 + 11y - 2
\]

---

**Check Your Progress**

4A. $(3x - 5)(2x^2 + 7x - 8)$

4B. $(m^2 + 2m - 3)(4m^2 - 7m + 5)$

---

**Study Tip**

**Multiplying Polynomials**

If a polynomial with $c$ terms and a polynomial with $d$ terms are multiplied together, there will be $c \cdot d$ terms before simplifying. In Example 4a, there are $2 \cdot 3$ or 6 terms before simplifying.
Check Your Understanding

Examples 1 and 2
pp. 447—448

Find each product.

1. \( (x + 5)(x + 2) \)
2. \( (y - 2)(y + 4) \)
3. \( (b - 7)(b + 3) \)
4. \( (4n + 3)(n + 9) \)
5. \( (8h - 1)(2h - 3) \)
6. \( (2a + 9)(5a - 6) \)

Example 3
p. 449

7. **FRAME** Hugo is designing a frame to surround the picture shown at the right. The frame is the same distance all the way around. Write an expression that represents the total area of the picture and frame.

Example 4
p. 449

Find each product.

8. \( (2a - 9)(3a^2 + 4a - 4) \)
9. \( (4y^2 - 3)(4y^2 + 7y + 2) \)
10. \( (x^2 - 4x + 5)(5x^2 + 3x - 4) \)
11. \( (2n^2 + 3n - 6)(5n^2 - 2n - 8) \)

Practice and Problem Solving

Examples 1 and 2
pp. 447—448

Find each product.

12. \( (3c - 5)(c + 3) \)
13. \( (g + 10)(2g - 5) \)
14. \( (6a + 5)(5a + 3) \)
15. \( (4x + 1)(6x + 3) \)
16. \( (5y - 4)(3y - 1) \)
17. \( (6d - 5)(4d - 7) \)
18. \( (3m + 5)(2m + 3) \)
19. \( (7n - 6)(7n - 6) \)
20. \( (12t - 5)(12t + 5) \)
21. \( (5r + 7)(5r - 7) \)
22. \( (8w + 4x)(5w - 6x) \)
23. \( (11z - 5y)(3z + 2y) \)

Example 3
p. 449

24. **GARDEN** A walkway surrounds a rectangular garden. The width of the garden is 8 feet, and the length is 6 feet. The width \( x \) of the walkway around the garden is the same on every side. Write an expression that represents the total area of the garden and walkway.

Example 4
p. 449

Find each product.

25. \( (2y - 11)(y^2 - 3y + 2) \)
26. \( (4a + 7)(9a^2 + 2a - 7) \)
27. \( (m^2 - 5m + 4)(m^2 + 7m - 3) \)
28. \( (x^2 + 5x - 1)(5x^2 - 6x + 1) \)
29. \( (3b^3 - 4b - 7)(2b^2 - b - 9) \)
30. \( (6z^2 - 5z - 2)(3z^3 - 2z - 4) \)

Simplify.

31. \( (m + 2)[(m^2 + 3m - 6) + (m^2 - 2m + 4)] \)
32. \( [(t^2 + 3t - 8) - (t^2 - 2t + 6)](t - 4) \)

**GEOMETRY** Find an expression to represent the area of each shaded region.

33.

34.
Lesson 7-7 Multiplying Polynomials

VOLLEYBALL The dimensions of a sand volleyball court are represented by a width of $6y - 5$ feet and a length of $3y + 4$ feet.

a. Write an expression that represents the area of the court.

b. The length of a sand volleyball court is 31 feet. Find the area of the court.

GEOMETRY Write an expression for the area of a triangle with a base of $2x + 3$ and a height of $3x - 1$.

Find each product.

37. $(a - 2b)^2$  38. $(3c + 4d)^2$  39. $(x - 5y)^2$

40. $(2r - 3t)^3$  41. $(5g + 2h)^3$  42. $(4y + 3z)(4y - 3z)^2$

CONSTRUCTION A sandbox kit allows you to build a square sandbox with sides that are $x - 2$ feet long or a rectangular sandbox with a length of $x$ feet and width of $x - 4$ feet.

a. What are the possible values of $x$? Explain.

b. Which shape has the greatest area?

c. What is the difference in areas between the square sandbox and the rectangular sandbox?

MULTIPLE REPRESENTATIONS In this problem, you will investigate the square of a sum.

a. TABULAR Copy and complete the table for each sum.

<table>
<thead>
<tr>
<th>Expression</th>
<th>$(Expression)^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x + 5$</td>
<td></td>
</tr>
<tr>
<td>$3y + 1$</td>
<td></td>
</tr>
<tr>
<td>$z + q$</td>
<td></td>
</tr>
</tbody>
</table>

b. VERBAL Make a conjecture about the terms of the square of a sum.

c. SYMBOLIC For a sum of the form $a + b$, write an expression for the square of the sum.

H.O.T. Problems Use Higher-Order Thinking Skills

45. REASONING Determine if the following statement is sometimes, always, or never true. Explain your reasoning.

The FOIL method can be used to multiply a binomial and a trinomial.

46. CHALLENGE Find $(x^m + x^p)(x^m - 1 - x^1 - x - p + x^p)$.

47. OPEN ENDED Write a binomial and a trinomial involving a single variable. Then find their product.

48. REASONING Compare and contrast the procedure used to multiply a trinomial by a binomial using the vertical method with the procedure used to multiply a three-digit number by a two-digit number.

49. WRITING IN MATH Summarize the methods that can be used to multiply polynomials.

On May 20, 2007, Misty May-Treanor won her 73rd professional beach volleyball title. May-Treanor has more wins than any other woman.

Source: Association of Volleyball Professionals
50. What is the product of $2x - 5$ and $3x + 4$?
   A $5x - 1$
   B $6x^2 - 7x - 20$
   C $6x^2 - 20$
   D $6x^2 + 7x - 20$

51. Which statement is correct about the symmetry of this design?

   F The design is symmetrical only about the $y$-axis.
   G The design is symmetrical only about the $x$-axis.
   H The design is symmetrical about both the $y$- and the $x$-axes.
   J The design has no symmetry.

52. Which point on the number line represents a number that, when cubed, will result in a number greater than itself?

   \[ (-5, -4) \quad (-3, -2) \quad (-1, 0) \quad (1, 2) \quad (3, 4) \quad (5, 6) \]

   A $P$
   B $Q$
   C $R$
   D $T$

53. **SHORT RESPONSE** For a science project, Jodi selected three bean plants of equal height. Then, for five days, she measured their heights in centimeters and plotted the values on the graph below.

   She drew a line of best fit on the graph. What is the slope of the line that she drew?

54. **SAVINGS** Carrie has $6000 to invest. She puts $x$ dollars of this money into a savings account that earns 2% interest per year. She uses the rest of the money to purchase a certificate of deposit that earns 4% interest. Write an equation for the amount of money that Carrie will have in one year. (Lesson 7-6)

   Find each sum or difference. (Lesson 7-5)

   55. $(7a^2 - 5) + (-3a^2 + 10)$
   56. $(8n - 2n^2) + (4n - 6n^2)$
   57. $(4 + n^3 + 3n^2) + (2n^3 - 9n^2 + 6)$
   58. $(-4u^2 - 9 + 2u) + (6u + 14 + 2u^2)$
   59. $(b + 4) + (c + 3b - 2)$
   60. $(3a^3 - 6a) - (3a^3 + 5a)$
   61. $(-4m^3 - m + 10) - (3m^3 + 3m^2 - 7)$
   62. $(3a + 4ab + 3b) - (2b + 5a + 8ab)$

**Skills Review**

Simplify. (Lesson 7-1)

63. $(-2t^4)^3 - 3(-2t^3)^4$
64. $(-3h^2)^3 - 2(-h^3)^2$
65. $2(-5y^3)^2 + (-3y^3)^3$
66. $3(-6n^4)^2 + (-2n^2)^2$
Then
You multiplied binomials by using the FOIL method. 
(Lesson 7-7)

Now
Find squares of sums and differences.
Find the product of a sum and a difference.

KY Program of Studies
HS-AT-S-VEO6 Students will add, subtract and multiply polynomials. Also addresses HS-NPO-S-NO10.

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- Personal Tutor
- Self-Check Quiz
- Homework Help
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**Special Products**

**Why?**

Colby wants to attach a dartboard to a square piece of corkboard. If the radius of the dartboard is \( r + 12 \), how large does the square corkboard need to be?

Colby knows that the diameter of the dartboard is \( 2(r + 12) \) or \( 2r + 24 \). Each side of the square also measures \( 2r + 24 \). To find how much corkboard is needed, Colby must find the area of the square:

\[
A = (2r + 24)^2.
\]

**Squares of Sums and Differences**

Some pairs of binomials, such as squares like \((2r + 24)^2\), have products that follow a specific pattern. Using the pattern can make multiplying easier. The square of a sum, \((a + b)^2\) or \((a + b)(a + b)\), is one of those products.

\[
(a + b)^2 = a^2 + 2ab + b^2
\]

**Key Concept**

**Square of a Sum**

**Words**

The square of \(a + b\) is the square of \(a\) plus twice the product of \(a\) and \(b\) plus the square of \(b\).

**Symbols**

\[(a + b)^2 = (a + b)(a + b)\]

**Example**

\[(x + 4)^2 = (x + 4)(x + 4) = x^2 + 8x + 16\]

**EXAMPLE 1**

**Square of a Sum**

Find \((3x + 5)^2\).

\[
(a + b)^2 = a^2 + 2ab + b^2
\]

\[
(3x + 5)^2 = (3x)^2 + 2(3x)(5) + 5^2
= 9x^2 + 30x + 25
\]

**Check Your Progress**

Find each product.

1A. \((8c + 3d)^2\)  
1B. \((3x + 4y)^2\)

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There is also a pattern for the square of a difference. Write \(a - b\) as \(a + (-b)\) and square it using the square of a sum pattern.

\[
(a - b)^2 = [a + (-b)]^2
= a^2 + 2(a)(-b) + (-b)^2
= a^2 - 2ab + b^2
\]

The square of a difference is the square of a sum minus twice the product of the two terms plus the square of the second term.

### For Your Words
The square of \(a - b\) is the square of \(a\) minus twice the product of \(a\) and \(b\) plus the square of \(b\).

### Symbols
\[
(a + b)^2 = (a + b)(a + b)
\]

### Example
\[
(x - 3)^2 = (x - 3)(x - 3)
= x^2 - 6x + 9
\]

### Example 2
Find \((2x - 5y)^2\).

\[
(a - b)^2 = a^2 - 2ab + b^2
\]

\[
(2x - 5y)^2 = (2x)^2 - 2(2x)(5y) + (5y)^2
= 4x^2 - 20xy + 25y^2
\]

### Check Your Progress
Find each product.

2A. \((6p - 1)^2\)

2B. \((a - 2b)^2\)

### Example 3
PHYSICAL SCIENCE Each edge of a cube of aluminum is 4 centimeters less than each edge of a cube of copper. Write an equation to model the surface area of the aluminum cube.

Let \(c\) = the length of each edge of the cube of copper.

So, each edge of the cube of aluminum is \(c - 4\).

\[
SA = 6s^2
\]

\[
= 6(c - 4)^2
\]

\[
= 6[c^2 - 2(4)(c) + 4^2]
\]

\[
= 6(c^2 - 8c + 16)
\]

### Check Your Progress
3. Alano has a garden that is \(g\) feet long and \(g\) feet wide. He wants to add 3 feet to the length and the width.

A. Show how the new area of the garden can be modeled by the square of a binomial.

B. Find the square of this binomial.
**Product of a Sum and a Difference**  Now we will see what the result is when we multiply a sum by a difference with the same $b$ value, or $(a + b)(a - b)$. Recall that $a - b$ can be written as $a + (-b)$.

Notice that the middle terms are opposites and add to a zero pair. So $(a + b)(a - b) = a^2 - ab + ab - b^2 = a^2 - b^2$.

**Key Concept**  **Product of a Sum and a Difference**

**Words**  The product of $a + b$ and $a - b$ is the square of $a$ minus the square of $b$.

**Symbols**  $(a + b)(a - b) = (a - b)(a + b) = a^2 - b^2$

**Example 4**  **Product of a Sum and a Difference**

Find $(2x^2 + 3)(2x^2 - 3)$.

$(a + b)(a - b) = a^2 - b^2$

$(2x^2 + 3)(2x^2 - 3) = (2x^2)^2 - (3)^2$

$= 4x^4 - 9$

Simplify.

**Check Your Progress**

Find each product.

4A. $(3n + 2)(3n - 2)$

4B. $(4c - 7d)(4c + 7d)$

**Check Your Understanding**

Find each product.

1. $(x + 5)^2$

2. $(11 - a)^2$

3. $(2x + 7y)^2$

4. $(3m - 4)(3m - 4)$

5. $(g - 4h)(g - 4h)$

6. $(3c + 6d)^2$

7. **GENETICS**  The color of a Labrador retriever’s fur is genetic. Dark genes $D$ are dominant over yellow genes $y$. A dog with genes $DD$ or $Dy$ will have dark fur. A dog with genes $yy$ will have yellow fur. Pepper’s genes for fur color are $Dy$, and Ramiro’s are $yy$.

   a. Write an expression for the possible fur colors of Pepper’s and Ramiro’s puppies.

   b. What is the probability that a puppy will have yellow fur?
Example 4 p. 455

Find each product.
8. \((a - 3)(a + 3)\)
9. \((x + 5)(x - 5)\)
10. \((6y - 7)(6y + 7)\)

Practice and Problem Solving

Examples 1 and 2 pp. 453–454

Find each product.
12. \((a + 10)(a + 10)\)
13. \((b - 6)(b - 6)\)
14. \((h + 7)^2\)
15. \((x + 6)^2\)
16. \((8 - m)^2\)
17. \((9 - 2y)^2\)
18. \((2b + 3)^2\)
19. \((5t - 2)^2\)
20. \((8h - 4n)^2\)

Example 3 p. 454

21. GENETICS The ability to roll your tongue is inherited genetically from parents if either parent has the dominant trait \(T\). Children of two parents without the trait will not be able to roll their tongues.
   a. Show how the combinations can be modeled by the square of a sum.
   b. Predict the percent of children that will have both dominant genes, one dominant gene, and both recessive genes.

Example 4 p. 455

Find each product.
22. \((u + 3)(u - 3)\)
23. \((b + 7)(b - 7)\)
24. \((2 + x)(2 - x)\)
25. \((4 - x)(4 + x)\)
26. \((2q + 5r)(2q - 5r)\)
27. \((3a^2 + 7b)(3a^2 - 7b)\)
28. \((5y + 7)^2\)
29. \((8 - 10a)^2\)
30. \((10x - 2)(10x + 2)\)
31. \((3t + 12)(3t - 12)\)
32. \((a + 4b)^2\)
33. \((3q - 5r)^2\)
34. \((2c - 9d)^2\)
35. \((g + 5h)^2\)
36. \((6y - 13)(6y + 13)\)
37. \((3a^4 - b)(3a^4 + b)\)
38. \((5x^2 - y^2)^2\)
39. \((8a^2 - 9b^3)(8a^2 + 9b^3)\)
40. \(\left(\frac{3x}{4} + 8\right)^2\)
41. \(\left(\frac{2}{5}y - 4\right)^2\)
42. \(\left(7z^2 + 5y^2\right)^2\)
43. \((2m + 3)(2m - 3)(m + 4)\)
44. \((r + 2)(r - 5)(r - 2)(r + 5)\)

45. GEOMETRY Write a polynomial that represents the area of the figure at the right.

46. FLYING DISKS A flying disk shaped like a circle has a radius of \(x + 3\) inches.
   a. Write an expression representing the area of the flying disk.
   b. If the diameter of the flying disk is 8 inches, what is its area?

GEOMETRY Find the area of each shaded region.

47. \(\frac{x - 1}{x + 2}\)
48. \(\frac{x + 6}{x - 3}\)

Real-World Link

In the 1870s, a baker named William Frisbie put his name on the bottom of tin pie pans. In the 1940s, students from Yale University began throwing the pie pans through the air. Eventually, the pie tins became known as Frisbees.

Source: Idea Finder
Find each product.

49. \((c + d)(c + d)(c + d)\)  
50. \((2a - b)^3\)

52. \((k - m)(k + m)(k - m)\)  
53. \((n - p)^2(n + p)\)

55. **WRESTLING** A high school wrestling mat must be a square with 38-foot sides and contain two circles as shown. Suppose the inner circle has a radius of \(r\) feet, and the radius of the outer circle is nine feet longer than the inner circle.

a. Write an expression for the area of the larger circle.

b. Write an expression for the area of the square outside the larger circle.

56. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate a pattern. Begin with a square piece of construction paper. Label each edge of the paper \(a\). In any of the corners, draw a smaller square and label the edges \(b\).

a. **NUMERICAL** Find the area of each of the squares.

b. **CONCRETE** Cut the smaller square out of the corner. What is the area of the shape?

c. **ANALYTICAL** Remove the smaller rectangle on the bottom. Turn it and slide it next to the top rectangle. What is the length of the new arrangement? What is the width of the new arrangement? What is the area of the new arrangement?

d. **ANALYTICAL** What pattern does this verify?

57. **WHICH ONE DOESN’T BELONG?** Identify the expression that does not belong with the other three. Explain your reasoning.

\[(2c - d)(2c - d)\]  
\[(2c + d)(2c - d)\]  
\[(2c + d)(2c + d)\]  
\[(c + d)(c + d)\]

58. **CHALLENGE** Does a pattern exist for the cube of a sum, \((a + b)^3\)?

a. Investigate this question by finding the product \((a + b)(a + b)(a + b)\).

b. Use the pattern you discovered in part a to find \((x + 2)^3\).

c. Draw a diagram of a geometric model for the cube of a sum.

d. What is the pattern for the cube of a difference, \((a - b)^3\)?

59. **REASONING** Find \(c\) that makes \(25x^2 - 90x + c\) a perfect square trinomial.

60. **OPEN ENDED** Write two binomials with a product that is a binomial and two binomials with a product that is not a binomial.

61. **WRITE ABOUT IT** Describe how to square the sum of a binomial, square the difference of a binomial, and how to find the product of a sum of a binomial and a difference of a binomial.
62. **GRIDDED RESPONSE** In the right triangle, $\overline{DB}$ bisects $\angle B$. What is the measure of $\angle ADB$ in degrees?

![Diagram of right triangle with angle bisector $DB$]

**63.** What is the product of $(2a - 3)$ and $(2a - 3)$?

- **A** $4a^2 + 12a + 9$
- **B** $4a^2 + 9$
- **C** $4a^2 - 12a - 9$
- **D** $4a^2 - 12a + 9$

64. Myron can drive 4 miles in $m$ minutes. At this rate, how many minutes will it take him to drive 19 miles?

- **F** $76m$
- **H** $\frac{4m}{19}$
- **G** $\frac{19m}{4}$
- **J** $\frac{4}{19m}$

65. What property is illustrated by the equation $2x + 0 = 2x$?

- **A** Commutative Property of Addition
- **B** Additive Inverse Property
- **C** Additive Identity Property
- **D** Associative Property of Addition

---

**Spiral Review**

**Find each product.** (Lesson 7-7)

66. $(y - 4)(y - 2)$
67. $(2c - 1)(c + 3)$
68. $(d - 9)(d + 5)$
69. $(4h - 3)(2h - 7)$
70. $(3x + 5)(2x + 3)$
71. $(5m + 4)(8m + 3)$

**Simplify.** (Lesson 7-6)

72. $x(2x - 7) + 5x$
73. $c(c - 8) + 2c(c + 3)$
74. $8y(-3y + 7) - 11y^2$
75. $-2d(5d) - 3d(d + 6)$
76. $5m(2m^3 + m^2 + 8) + 4m$
77. $3p(6p - 4) + 2\left(\frac{1}{2}p^2 - 3p\right)$

**Use substitution to solve each system of equations.** (Lesson 6-2)

78. $4c = 3d + 3$
   \[ c = d - 1 \]
79. $c - 5d = 2$
   \[ 2c + d = 4 \]
80. $5r - t = 5$
   \[ -4r + 5t = 17 \]

81. **BIOLOGY** Each type of fish thrives in a specific range of temperatures. The best temperatures for sharks range from 18°C to 22°C, inclusive. Write an inequality to represent temperatures where sharks will not thrive. (Lesson 6-2)

Write an equation of the line that passes through each pair of points. (Lesson 4-2)

82. $(1, 1), (7, 4)$
83. $(5, 7), (0, 6)$
84. $(5, 1), (8, -2)$

85. **COFFEE** A coffee store wants to create a mix using two coffees, one priced at $6.40 per pound and the other priced at $7.28 per pound. How many pounds of the $7.28 coffee should be mixed with 9 pounds of the $6.40 coffee to sell the mixture for $6.95 per pound? (Lesson 2-9)

**Skills Review**

Find the prime factorization of each number. (Concepts and Skills Bank Lesson 6)

86. 40
87. 120
88. 900
89. 165
Chapter Summary

Key Concepts
For any real numbers \(a\) and \(b\) and any integers \(m, n,\) and \(p,\) the following are true.

Multiplying Monomials (Lesson 7-1)
- Product of Powers: \(a^m \cdot a^n = a^{m+n}\)
- Power of a Power: \((a^m)^n = a^{m \cdot n}\)
- Power of a Product: \((ab)^m = a^m b^m\)

Dividing Monomials (Lesson 7-2)
- Quotient of Powers: \(\frac{a^m}{a^n} = a^{m-p}\)
- Power of a Quotient: \(\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}\)
- Zero Exponent: \(a^0 = 1\)
- Negative Exponent: \(a^{-n} = \frac{1}{a^n}\) and \(\frac{1}{a^{-n}} = a^n\)

Scientific Notation (Lesson 7-3)
- A number is in scientific notation if it is in the form \(a \times 10^n\), where \(1 \leq a < 10\).
- To write in standard form:
  - If \(n > 0\), move the decimal \(n\) places right.
  - If \(n < 0\), move the decimal \(n\) places left.

Operations with Polynomials (Lessons 7-5 through 7-8)
- To add or subtract polynomials, add or subtract like terms. To multiply polynomials, use the Distributive Property.
- Special products: \((a + b)^2 = a^2 + 2ab + b^2\)
  \((a - b)^2 = a^2 - 2ab + b^2\)
  \((a + b)(a - b) = a^2 - b^2\)

Foldables® Study Organizer
Be sure the Key Concepts are noted in your Foldable.

Key Vocabulary
- binomial (p. 424)
- constant (p. 401)
- degree of a monomial (p. 424)
- degree of a polynomial (p. 424)
- FOIL method (p. 448)
- leading coefficient (p. 425)
- monomial (p. 401)
- negative exponent (p. 410)
- Negative Exponent Property (p. 410)
- order of magnitude (p. 411)
- polynomial (p. 424)
- Power of a Power (p. 402)
- Power of a Product (p. 403)
- Power of a Quotient (p. 409)
- Product of Powers (p. 402)
- Product of a Sum and Difference (p. 455)
- quadratic expression (p. 448)
- Quotient of Powers (p. 408)
- scientific notation (p. 416)
- Square of a Difference (p. 454)
- Square of a Sum (p. 453)
- standard form of a polynomial (p. 425)
- trinomial (p. 424)
- zero exponents (p. 410)
- Zero Exponent Property (p. 410)

Vocabulary Check
Choose a term from the Key Vocabulary list above that best describes each expression or equation.
1. \(x^2 + 1\)
2. \(5^0 = 1\)
3. \(x^2 - 3x + 2\)
4. \((xy^3)(x^2y^4) = x^3y^7\)
5. \((a^2)^3 = a^6\)
6. \(5^{-2} = \frac{1}{25}\)
7. \(6.2 \times 10^5\)
8. \((x + 2)(x - 5) = x^2 - 3x - 10\)
9. \(x^3 + 2x^2 - 3x - 1\)
10. \(7xy^4\)


Lesson-by-Lesson Review

7-1 Multiplying Monomials (pp. 401–407)

Simplify each expression.

11. \( x \cdot x^3 \cdot x^5 \)
12. \( (2xy)(-3x^2y^3) \)
13. \( (-4ab^4)(-5a^5b^2) \)
14. \( (6x^3y^2)^2 \)
15. \( [(2r^3t)^3]^2 \)
16. \( (-2n^3)(5u) \)
17. \( (2x^2)^3(x^3)^3 \)
18. \( \frac{1}{2}(2x^3)^3 \)

19. GEOMETRY Use the formula \( V = \pi r^2h \) to find the volume of the cylinder.

\[ \text{Volume} = \pi \left( \frac{3x}{2} \right)^2 \left( \frac{5x^2}{5} \right) \]

EXAMPLE 1

Simplify \( (5x^2y^3)(2x^4y) \).

\[
(5x^2y^3)(2x^4y) = (5 \cdot 2)(x^2 \cdot x^4)(y^3 \cdot y) = 10x^6y^4
\]

EXAMPLE 2

Simplify \( (3a^2b^4)^3 \).

\[
(3a^2b^4)^3 = 3^3(a^2)^3(b^4)^3 = 27a^6b^{12}
\]

EXAMPLE 3

Simplify \( \frac{2k^4m^3}{4k^2m} \).

\[
\frac{2k^4m^3}{4k^2m} = \left( \frac{2}{4} \right) \left( \frac{k^4}{k^2} \right) \left( \frac{m^3}{m} \right) = \left( \frac{1}{2} \right) k^{4-2} m^{3-1} = \frac{k^2m^2}{2}
\]

EXAMPLE 4

Simplify \( \frac{t^4u^v}{t^{-3}u^7} \).

\[
\frac{t^4u^v}{t^{-3}u^7} = \left( \frac{t^4}{t^{-3}} \right) \left( \frac{u}{u^7} \right) (u^{v-2}) = (t^{4+3})(u^{1-7})(u^{-2}) = t^7u^{-6}v^{-2} = \frac{t^7}{u^6v^2}
\]
Scientific Notation (pp. 416–422)

Express each number in scientific notation.

29. 2,300,000
30. 0.0000543
31. **ASTRONOMY** Earth has a diameter of about 8000 miles. Jupiter has a diameter of about 88,000 miles. Write in scientific notation the ratio of Earth’s diameter to Jupiter’s diameter.

**EXAMPLE 5**

Express 300,000,000 in scientific notation.

**Step 1**
300,000,000

**Step 2**
The decimal point moved 8 places to the left, so \( n = 8 \).

**Step 3**
300,000,000 = \( 3 \times 10^8 \)

Polynomials (pp. 424–429)

Write each polynomial in standard form.

32. \( x^2 + 3x^2 \)
33. \( 1 - x^4 \)
34. \( 2 + 3x + x^2 \)
35. \( 3x^5 - 2 + 6x - 2x^2 + x^3 \)
36. **GEOMETRY** Write a polynomial that represents the perimeter of the figure.

**EXAMPLE 6**

Write \( 3 - x^2 + 4x \) in standard form.

**Step 1** Find the degree of each term.
- 3: degree 0
- \(-x^2\): degree 2
- 4x: degree 1

**Step 2** Write the terms in descending order.
\( 3 - x^2 + 4x = -x^2 + 4x + 3 \)

Adding and Subtracting Polynomials (pp. 433–438)

Find each sum or difference.

37. \((x^3 + 2) + (-3x^3 - 5)\)
38. \(a^2 + 5a - 3 - (2a^2 - 4a + 3)\)
39. \((4x - 3x^2 + 5) + (2x^2 - 5x + 1)\)
40. \((6ab + 3b^2) - (3ab - 2b^2)\)
41. **PICTURE FRAMES** Jean is framing a painting that is a rectangle that is \(2x^2 - 3x + 1\) units long and \(5x + 3\) wide. What is the perimeter of the frame?

**EXAMPLE 7**

Find \( (8r^2 + 3r) - (10r^2 - 5) \).

\begin{align*}
(8r^2 + 3r) - (10r^2 - 5) &= (8r^2 + 3r) + (-10r^2 + 5) \\
&= (8r^2 - 10r^2) + 3r + 5 \\
&= -2r^2 + 3r + 5
\end{align*}
Solve each equation.
42. \(x^2(x + 2) = x(x^2 + 2x + 1)\)
43. \(2x(x + 3) = 2(x^2 + 3)\)
44. \(2(4w + w^2) - 6 = 2w(w - 4) + 10\)
45. \(6k(k + 2) = 6(k^2 + 4)\)
46. **GEOMETRY** Find the area of the rectangle.

\[
x^2 + x - 7
\]

\[
3x
\]

\[
2x + 5
\]

\[
5x - 4
\]

Find each product.
47. \((x - 3)(x + 7)\)
48. \((3a - 2)(6a + 5)\)
49. \((3r - 7t)(2r + 5t)\)
50. \((2x + 5)(5x + 2)\)
51. **PARKING LOT** The parking lot shown is to be paved. What is the area to be paved?

\[
2x + 3
\]

\[
5x - 4
\]

52. \((x + 5)(x - 5)\)
53. \((3x - 2)^2\)
54. \((5x + 4)^2\)
55. \((2x - 3)(2x + 3)\)
56. \((2r + 5t)^2\)
57. \((3m - 2)(3m + 2)\)
58. **GEOMETRY** Write an expression to represent the area of the shaded region.

\[
\begin{array}{c}
2x + 5 \\
\hline
x + 2 \\
\hline
x - 2 \\
\hline
2x - 5
\end{array}
\]

EXAMPLE 8
Solve \(m(2m - 5) + m = 2m(m - 6) + 16\).
\[
m(2m - 5) + m = 2m(m - 6) + 16
\]
\[
2m^2 - 5m + m = 2m^2 - 12m + 16
\]
\[
2m^2 - 4m = 2m^2 - 12m + 16
\]
\[
-4m = -12m + 16
\]
\[
8m = 16
\]
\[
m = 2
\]

EXAMPLE 9
Find \((6x - 5)(x + 4)\).
\[
(6x - 5)(x + 4)
\]
\[
= (6x)(x) + (6x)(4) + (-5)(x) + (-5)(4)
\]
\[
= 6x^2 + 24x - 5x - 20
\]
\[
= 6x^2 + 19x - 20
\]

Multiply. Combine like terms.

EXAMPLE 10
Find \((x - 7)^2\).
\[
(a - b)^2 = a^2 - 2ab + b^2
\]
\[
(x - 7)^2 = x^2 - 2(x)(7) + (-7)^2
\]
\[
= x^2 - 14x + 49
\]

Square of a Difference \(a = x\) and \(b = 7\) Simplify.

EXAMPLE 11
Find \((5a - 4)(5a + 4)\).
\[
(a + b)(a - b) = a^2 - b^2
\]
\[
(5a - 4)(5a + 4) = (5a)^2 - (4)^2
\]
\[
= 25a^2 - 16
\]

Product of a Sum and Difference \(a = 5a\) and \(b = 4\) Simplify.
Simplify each expression.

1. \((x^2)(7x^8)\)

2. \((5a^7b^2c)(-6a^2b^3c)\)

3. **MULTIPLE CHOICE** Express the volume of the solid as a monomial.

   \[x \times x \times x\]

   A. \(x^3\)
   
   B. \(6x^3\)
   
   C. \(6x\)
   
   D. \(x^6\)

Simplify each expression. Assume that no denominator equals 0.

4. \(\frac{x^6y^8}{x^2}\)

5. \(\left(\frac{2a^4b^3}{c^6}\right)^0\)

6. \(\frac{2xy^{-7}}{8x}\)

Express each number in scientific notation. (Lesson 7-3)

7. 0.00021

8. 58,000

Express each number in standard form.

9. \(2.9 \times 10^{-5}\)

10. \(9.1 \times 10^6\)

Evaluate each product or quotient. Express the results in scientific notation.

11. \((2.5 \times 10^3)(3 \times 10^4)\)

12. \(\frac{8.8 \times 10^2}{4 \times 10^{-4}}\)

13. **ASTRONOMY** The average distance from Mercury to the Sun is 35,980,000 miles. Express this distance in scientific notation.

Find each sum or difference.

14. \((x + 5) + (x^2 - 3x + 7)\)

15. \((7m - 8n^2 + 3n) - (-2n^2 + 4m - 3n)\)

16. **MULTIPLE CHOICE** Antonia is carpeting two of her rooms in her house. The dimensions are shown in the figure. What is the total area to be carpeted?

   \[x + 3\]

   \[x + 5\]

   \[x - 2\]

   A. \(x^2 + 3x\)
   
   B. \(x^2 + 3x - 5\)
   
   C. \(2x^2 + 6x - 10\)
   
   D. \(8x + 12\)

Find each product.

17. \(a(a^2 + 2a - 10)\)

18. \((2a - 5)(3a + 5)\)

19. \((x - 3)(x^2 + 5x - 6)\)

20. \((x + 3)^2\)

21. \((2b - 5)(2b + 5)\)

22. **GEOMETRY** A rectangular prism has dimensions \(x, x + 3,\) and \(2x + 5\).

   a. Find the volume of the prism in terms of \(x\).
   
   b. Choose two values for \(x\). How do the volumes compare?

Solve each equation.

23. \(5(t^2 - 3t + 2) = t(5t - 2)\)

24. \(3x(x + 2) = 3(x^2 - 2)\)

25. **MONEY** Money invested in a certificate of deposit (CD) collects interest once per year. Suppose you invest $4000 in a 2-year CD.

   a. If the interest rate is 5\% per year, the expression \(4000(1 + 0.05)^2\) can be evaluated to find the total amount of money after two years. Explain the numbers in this expression.
   
   b. Find the amount at the end of two years.
   
   c. Suppose you invest $10,000 in a CD for 4 years at an annual rate of 6.25\%. What is the total amount of money you will have after 4 years?
Using a Scientific Calculator

Scientific calculators are powerful problem-solving tools. There are times when using a scientific calculator can be used to make computations faster and easier, such as computations with very large numbers. However, there are times when using a scientific calculator is necessary, like the estimation of irrational numbers.

Strategies for Using a Scientific Calculator

**Step 1**

Familiarize yourself with the various functions of a scientific calculator as well as when they should be used:

- **Exponents** scientific notation, calculating with large or small numbers
- **Pi** solving circle problems, like circumference and area
- **Square roots** distance on a coordinate plane, Pythagorean theorem, Quadratic Formula
- **Graphs** analyzing paired data in a scatter plot, graphing functions, finding roots of equations

**Step 2**

Use your scientific or graphing calculator to solve the problem.

- Remember to work as efficiently as possible. Some steps may be done mentally or by hand, while others should be completed using your calculator.
- If time permits, check your answer.

**EXAMPLE**

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

The distance from the Sun to Jupiter is approximately \(7.786 \times 10^{11}\) meters. If the speed of light is about \(3 \times 10^8\) meters per second, how long does it take for light from the Sun to reach Jupiter? Round to the nearest minute.

A about 43 minutes  
B about 51 minutes  
C about 1,876 minutes  
D about 2,595 minutes
Read the problem carefully. You are given the approximate distance from the Sun to Jupiter as well as the speed of light. Both quantities are given in scientific notation. You are asked to find how many minutes it takes for light from the Sun to reach Jupiter. Use the relationship distance = rate × time to find the amount of time.

\[ d = r \times t \]
\[ \frac{d}{r} = t \]

To find the amount of time, divide the distance by the rate. Notice, however, that the units will be in seconds.

\[ \frac{7.786 \times 10^{11} \text{ m}}{3 \times 10^{8} \text{ m/s}} = t \text{ seconds} \]

Use a scientific calculator to quickly find the quotient. On most scientific calculators, the EE key is used to enter numbers in scientific notation.

**KEYSTROKES:** (7.786 \(\text{EE}\) 11) / (3 \(\text{EE}\) 8)

The result is 2595.33333333 seconds. To convert this number to minutes, use your calculator to divide the result by 60. This gives an answer of about 43.2555 minutes. The answer is A.

**Exercises**

Read each problem. Identify what you need to know. Then use the information in the problem to solve.

1. Since its creation 5 years ago, approximately \(2.504 \times 10^7\) items have been sold or traded on a popular online website. What is the average daily number of items sold or traded over the 5-year period?
   - A about 9640 items per day
   - B about 13,720 items per day
   - C about 1,025,000 items per day
   - D about 5,008,000 items per day

2. Evaluate \(\sqrt{ab}\) if \(a = 121\) and \(b = 23\).
   - F 5.260869565
   - G 9.899494937
   - H 12
   - J 52.75414676

3. The population of the United States is about \(3.034 \times 10^8\) people. The land area of the country is about \(3.54 \times 10^6\) square miles. What is the average population density (number of people per square mile) of the United States?
   - A about 136.3 people per square mile
   - B about 112.5 people per square mile
   - C about 94.3 people per square mile
   - D about 85.7 people per square mile

4. Eleece is making a cover for the marching band’s bass drum. She needs to find the area of the face of the drum. The drum has a diameter of 20 inches. Estimate the area of the face of the bass drum.
   - F 31.41 square inches
   - G 62.83 square inches
   - H 78.54 square inches
   - J 314.16 square inches
Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Express the area of the triangle below as a monomial.

![Triangle Image]

A $12b^5d^7$
B $12b^6d^{10}$
C $6b^6d^{10}$
D $6b^5d^7$

2. Simplify the following expression.

$$\left(\frac{2w^2z^8}{3y^4}\right)^3$$

F $\frac{2w^5z^8}{3y^7}$
G $\frac{8w^6z^{15}}{27y^{12}}$
H $\frac{8w^5z^8}{27y^7}$
J $\frac{2w^6z^{15}}{3y^{12}}$

3. Which equation of a line is perpendicular to $y = \frac{3}{5}x - 3$?

A $y = -\frac{5}{3}x + 2$
B $y = -\frac{3}{5}x + 2$
C $y = \frac{5}{3}x - 2$
D $y = \frac{3}{5}x - 2$

Test-Taking Tip
Question 2 Use the laws of exponents to simplify the expression. Remember, to find the power of a power, multiply the exponents.

4. Express the perimeter of the rectangle below as a polynomial.

![Rectangle Image]

$$x^2 - 3x + 4$$

F $3x^2 - 4x + 7$
G $3x^2 + x + 7$
H $6x^2 - 8x + 14$
J $6x^2 - 4x + 7$

5. Subtract the polynomials below.

$$(7a^2 + 6a - 2) - (-4a^3 + 3a^2 + 5)$$

A $4a^3 + 4a^2 - 6a - 7$
B $11a^2 + 3a - 7$
C $4a^3 + 10a^2 + 6a + 3$
D $4a^3 + 7a^3 - 3a$

6. Which inequality is shown in the graph?

![Graph Image]

F $y \leq -\frac{2}{3}x - 1$
G $y \leq -\frac{3}{4}x - 1$
H $y \leq -\frac{2}{3}x + 1$
J $y \leq -\frac{3}{4}x + 1$
Short Response/Gridded Response

7. Mickey has 180 feet of fencing that she wants to use to enclose a play area for her puppy. She will use her house as one of the sides of the region.

a. If she makes the play area \( x \) feet deep as shown in the figure, write a polynomial in standard form to represent the area of the region.

b. How many square feet of area will the puppy have to play in if Mickey makes it 40 feet deep?

8. Identify the expression below that does not belong with the other two. Explain.

\[
(3m - 2n)(3m + 2n) \\
(3m + 2n)(3m + 2n) \\
(3m + 2n)(3m - 2n)
\]

9. What is the solution to the following system of equations? Show your work.

\[
\begin{align*}
y &= 6x - 1 \\
y &= 6x + 4
\end{align*}
\]

10. GRIDDED RESPONSE At a family fun center, the Wilson and Sanchez families each bought video game tokens and batting cage tokens as shown in the table.

<table>
<thead>
<tr>
<th>Family</th>
<th>Wilson</th>
<th>Sanchez</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Video Game Tokens</td>
<td>25</td>
<td>30</td>
</tr>
<tr>
<td>Number of Batting Cage Tokens</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>Total Cost</td>
<td>$26.50</td>
<td>$25.50</td>
</tr>
</tbody>
</table>

What is the cost in dollars of a batting cage token at the family fun center?

Extended Response

Record your answers on a sheet of paper. Show your work.

11. The table below shows the distances from the Sun to Mercury, Earth, Mars, and Saturn. Use the data to answer each question.

<table>
<thead>
<tr>
<th>Planet</th>
<th>Distance from Sun (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>(5.79 \times 10^7)</td>
</tr>
<tr>
<td>Earth</td>
<td>(1.50 \times 10^8)</td>
</tr>
<tr>
<td>Mars</td>
<td>(2.28 \times 10^8)</td>
</tr>
<tr>
<td>Saturn</td>
<td>(1.43 \times 10^9)</td>
</tr>
</tbody>
</table>

a. Of the planets listed, which one is the closest to the Sun?

b. About how many times as far from the Sun is Mars as Earth?